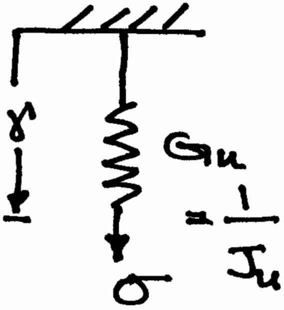
Spring - Dashpot Models

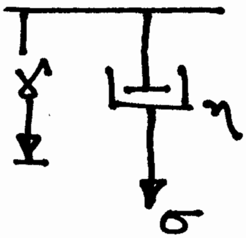
Hookean spring



$$\sigma = G_u \gamma, \quad \gamma = J_u \sigma$$

$\frac{N}{m^2}$ (Pa)

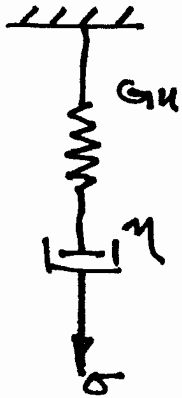
Newtonian dashpot

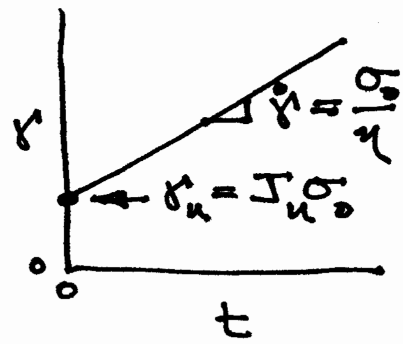


$$\sigma = \eta \dot{\gamma}$$

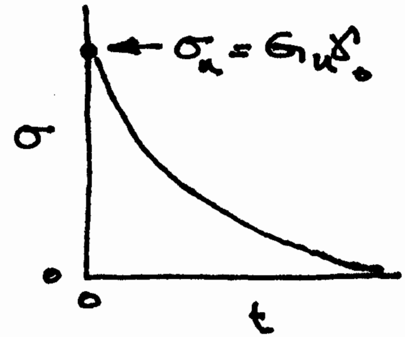
$\frac{N \cdot s}{m^2}$ (Pa. s)

Maxwell Model





creep



relaxation

Series connection: $\sigma_s = \sigma_d = \sigma$, $\gamma = \gamma_s + \gamma_d$

$$\dot\gamma = \dot\gamma_s + \dot\gamma_d = \frac{1}{G_u}\dot\sigma + \frac{1}{\eta}\sigma$$

$$G_u\dot\gamma = \dot\sigma + \frac{1}{\tau_\sigma}\sigma \qquad (\tau_\sigma = \eta/G_u)$$

Relaxation $\gamma = \gamma_0$, $\dot{\gamma} = 0$

$$0 = \dot{\sigma} + \frac{1}{\tau_\sigma}\sigma \longrightarrow \frac{d\sigma}{dt} = -\frac{1}{\tau_\sigma}\sigma$$

$$\frac{d\sigma}{\sigma} = -\frac{1}{\tau_\sigma}dt \longrightarrow \ln\sigma\Big|_{\sigma_0}^{\sigma} = -\frac{t}{\tau_\sigma}$$

$$\sigma = \sigma_0 e^{-t/\tau_\sigma}$$

$$G_{rel}(t) = \frac{\sigma_0}{\gamma_0} e^{-t/\tau_\sigma} = G_u e^{-t/\tau_\sigma}$$

$$\left(@\, t = \tau_\sigma, \quad G_{rel} = \frac{G_u}{e} \approx \frac{1}{3} G_u \right)$$

Laplace Method:

$$G_u \dot{\gamma} = \dot{\sigma} + \frac{1}{\tau_\sigma}\sigma \rightarrow G_u s \bar{\gamma} = s\bar{\sigma} + \frac{1}{\tau_\sigma}\bar{\sigma}$$

$$\bar{\sigma} = \frac{G_u s}{s + \frac{1}{\tau_\sigma}}\bar{\gamma} \qquad (\bar{\sigma} = \mathcal{G}\,\bar{\gamma})$$

relaxation: $\gamma(t) = \gamma_0\, u(t) \rightarrow \bar{\gamma} = \frac{\gamma_0}{s}$

$$\bar{\sigma} = \frac{G_u s}{s + \frac{1}{\tau_\sigma}} \cdot \frac{\gamma_0}{s}$$

$$\frac{\bar{\sigma}}{\gamma_0} = \bar{G}_{rel} = \frac{G_u}{s + \frac{1}{\tau_\sigma}} \rightarrow G_{rel} = G_u e^{-t/\tau_\sigma}$$