3.044 MATERIALS PROCESSING

LECTURE 16

Navier-Stokes Equation (1-D):

$$\frac{\partial v_x}{\partial t} = \nu \frac{\partial^2 v_x}{\partial y^2} - \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{F_x}{\rho}$$

$$\underbrace{\rho \frac{\partial v_x}{\partial t}}_{\text{Inertial Force} \frac{kg\,m}{s^2}\frac{1}{m^3}} = \underbrace{\mu \frac{\partial^2 v_x}{\partial y^2}}_{\text{Viscous Force}} - \underbrace{\frac{\partial P}{\partial x}}_{\text{Pressure Force}} + \underbrace{F_x}_{\text{Body Force} \frac{N}{m^3}}$$

Simplest Case:

$$\rho \frac{\partial v_x}{\partial t} = \mu \frac{\partial^2 v_x}{\partial y^2}$$

Non-Dimensional Length:

$$X = \frac{x}{L}, \text{ where } L = \frac{V}{A}$$
$$Y = \frac{y}{L}$$

Non-Dimensional Time:

$$\tau = t\frac{V_0}{L}$$

Date: April 11th, 2012.

Velocity:

$$V_x = \frac{v_x}{v_0}$$

$$\frac{\partial v_x}{\partial t} = \frac{\partial v_x}{\partial \tau}\frac{\partial \tau}{\partial t} = \frac{v_0}{L}\frac{\partial v_x}{\partial \tau} = \frac{v_0}{L}\frac{\partial V_x}{\partial \tau}\frac{\partial v_x}{\partial V_x} = \frac{v_0^2}{L}\frac{\partial V_x}{\partial \tau}$$

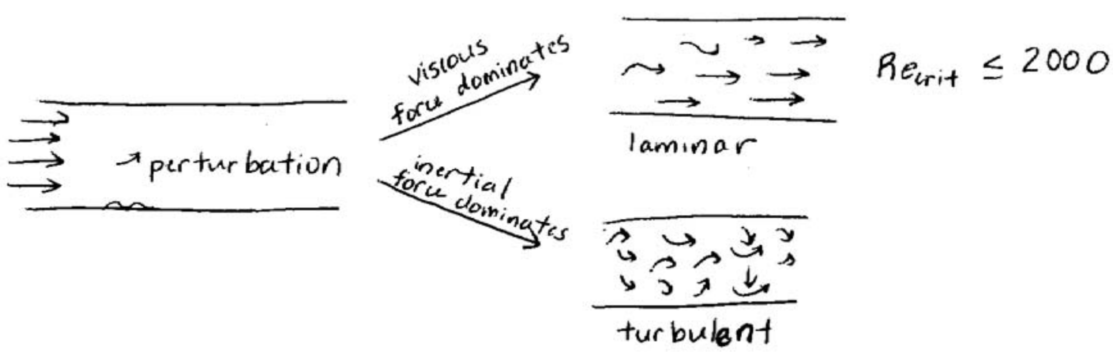
$$\frac{\partial^2 v_x}{\partial y^2} = v_0 \frac{\partial^2 V_x}{\partial y^2} = \frac{v_0}{L^2}\frac{\partial^2 V_x}{\partial Y^2}$$

$$\rho \frac{v_0^2}{L}\frac{\partial V_x}{\partial \tau} = \mu \frac{v_0}{L^2}\frac{\partial^2 V_x}{\partial Y^2}$$

$$\boxed{\frac{\partial V_x}{\partial \tau} = \left(\frac{\mu}{\rho L v_0}\right)\frac{\partial^2 V_x}{\partial Y^2}}$$

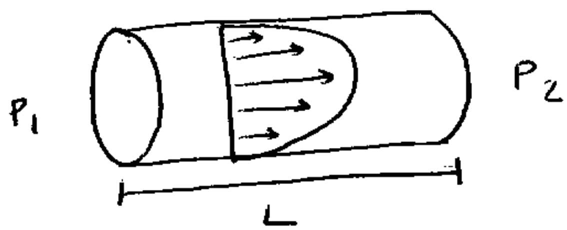
Reynold's Number: $$\boxed{\mathrm{Re} = \frac{\rho\, L\, v_0}{\mu} = \frac{\text{inertial force}}{\text{viscous force}}}$$

<u>Flow in a Tube:</u>



Geometry	Critical Re
channel	~ 1000
tube	~ 2100
1 free surface (e.g. falling film)	~ 20

Below Re^{crit}: definitely laminar
Above Re^{crit}: might be turbulent, needs perturbation



$$v_x(r) = \frac{\Delta P}{4L\mu}\left(R^2 - r^2\right)$$

$$\tau_{rx} = \frac{\Delta P}{2}\frac{R}{L}$$

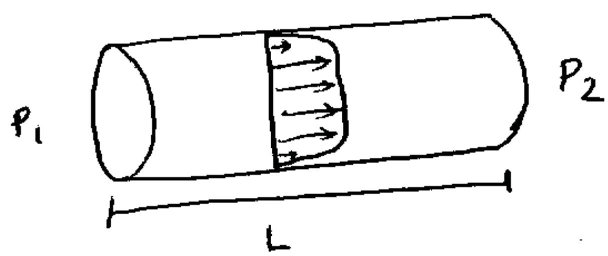
Kinetic Force:

$$F_k = \int \tau dA$$

$$F_k = \Delta P \pi R^2$$

What if Flow is Turbulent?

"Plug Flow":



$\Rightarrow$ Cannot solved explicitly for $v_x(r)$

Generalized Drag Force:

$$F_k = \underbrace{f}_{\text{friction factor}} \quad \underbrace{A}_{\text{char. area of momentum transfer}} \quad \underbrace{K}_{\text{char. kinetic energy of flow}}$$

Laminar: $f_L = \frac{16}{Re}$ $\quad$ $A = 2\pi RL$ $\quad$ KE of flow $= \frac{1}{2}\rho v_{\text{avg}}^2$

How do you use $\mathbf{F_k}$?

$(P_1 - P_2)\,\pi R^2 = fAK$

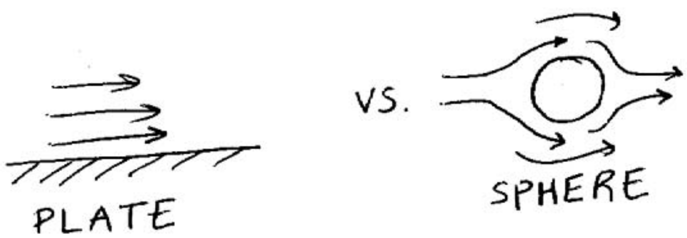
$\Rightarrow$ Provides a relationship between ΔP and v_{avg}

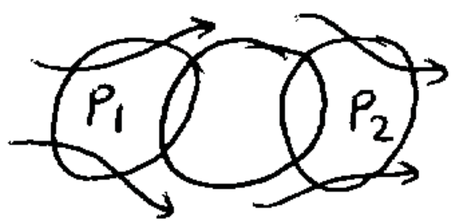
Summary:

If <u>Laminar:</u> F_k can be calculated exactly
$v_x(r) \to \tau_{rx} \to F_x = \int \tau dA$

<u>If Turbulent:</u> Use empirical f from experiment or simulation
$f = f\,(\text{Re, Geometry})$

<u>Flow Past Objects:</u>





$$F_k = \underbrace{F_{\text{friction}}}_{\substack{\text{shear griping}\\ \text{sides}}} + \underbrace{F_{\text{form}}}_{\substack{\text{dynamic pressure}\\ \text{of impinging flow}}}$$

$$F_k = (fAK)_{\text{friction}} + (fAK)_{\text{form}}$$

Solve for: $(fAK)_{\text{friction}}$

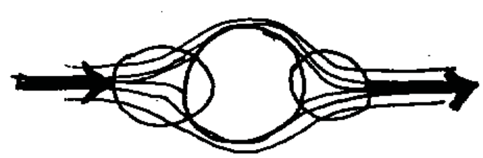
Laminar: Solve exactly with **Stokes Law**

$$f_{\text{fric}} = \frac{24}{Re}$$
$$A = \frac{\pi}{4}d^2$$
$$K = \frac{1}{2}\rho v^2$$
$$\boxed{F_{\text{k, fric}} = 3\pi\mu v d}$$

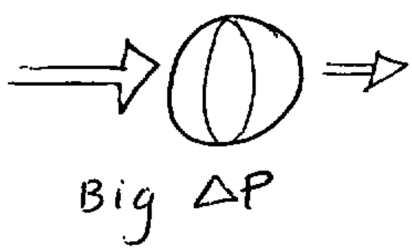
Turbulent: negligible

Solve for: $(fAK)_{\text{form}}$

Laminar: $F_{\text{form}} \approx 0$



Turbulent:



$$f_{\text{form}} \approxeq 0.44$$
$$A = \frac{\pi}{4}d^2$$
$$K = \frac{1}{2}\rho v^2$$
$$\boxed{F_{\text{form}} = f_{\text{form}}\frac{\pi}{8}d^2\rho v^2}$$

Summary:

$$F_k = \underbrace{F_{\text{fric}}}_{\substack{\text{laminar}\\ \text{term}}} + \underbrace{F_{\text{form}}}_{\substack{\text{turbulent}\\ \text{term}}}$$

where f_{fric} is a function of the **Reynolds Number**
and f_{form} is a function of **Geometry**

3.044 Materials Processing
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