Department of Materials Science and Engineering
Massachusetts Institute of Technology
3.044 Materials Processing – Spring 2009

Final Exam

Monday, May 18, 2008

The Rules:

1) No books allowed; no computers allowed; etc.

2) A simple calculator is allowed

3) THREE hand written 3x5 index cards may be prepared as a crutch

4) Complete 8 out of the 10 problems. If you do more than 8 problems, I will grade the first 8 that are not crossed out.

5) Make sure that you READ THE QUESTIONS CAREFULLY

6) Supplementary materials are attached to the end of the test (eqns., etc.)

7) WRITE YOUR NAME HERE:

SOLUTIONS

Problem #1: More Fun Than a Laser in the Eye

The process of laser surface hardening involves a travelling laser source that deposits energy at the upper surface of a sheet of material. For this problem, let's say that no local melting or phase transformations occur.

Part A:

Let's take the system to be a simple 2-D situation as shown, where the laser beam is very wide (infinite into the page) compared to its thickness. Let's also assume that the beam has been travelling for a long time, so we are in a steady state condition (with a moving coordinate system; direction of travel is shown). Write down the form of the conduction equation that you would use to solve for the heat distribution in this problem.
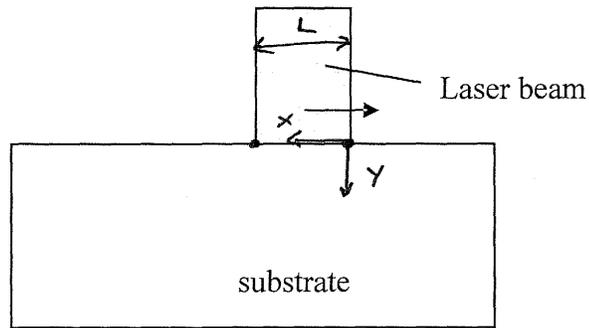
$$\underline{\text{2-D Conduction}}: \quad \frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0$$

moving coordinate system: x travels with velocity v ✓

2

Part B:

Write down the set of boundary conditions that are appropriate for the upper surface of the sheet.

$Y=0$
$\begin{cases} \text{on } x = 0 \ldots L, & \text{constant radiant laser flux} \quad \dot{q} = -k \frac{\partial T}{\partial y} \\ x \geq L, & \text{constant convection loss}: \quad -k \frac{\partial T}{\partial y} = h(T - T_{ambient}) \end{cases}$

2

Part C:

Write down any other boundary conditions that you think you would need to solve this problem.

$y \to \infty \qquad T = T_{ambient}$

$x \to \infty \qquad T = T_{ambient}$

1

Problem #2: Swimming for Their Lives (the particles, not the students)

In continuous casting, one of the major problems is that small inclusion particles can be caught in the casting. Let's analyze this problem, for small inclusions of Al_2O_3 (which are actually quite common in steel).

— melt

Particle 'A'

solid metal
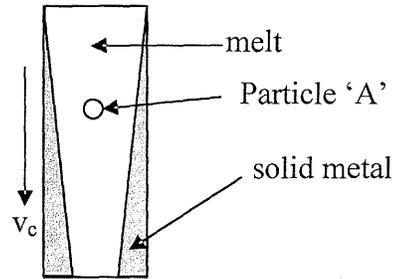
Part A:

The key consideration is that the alumina particles float in molten steel. For such a particle in the melt region at point 'A', the particle is (a) trying to float to the top, (b) but being propelled downward at the casting velocity v_c (= 1 m/min) at the same time. Write down a reasonable and simple condition that decides whether the particle will eventually escape at the top surface, or be carried down and get incorporated in the metal in the sump.

Partcle velocity, v, must be greater than v_c, or it will have net downward motion and get trapped!

$$\boxed{V \geq V_e}$$

Part B:

Let's calculate the velocity at which the particle would rise. Write down the force balance for particle 'A', and indicate with what velocity it rises. (For this part of the problem, ignore the v_c effect.)

— bouyancy force F_b
drag force F_d
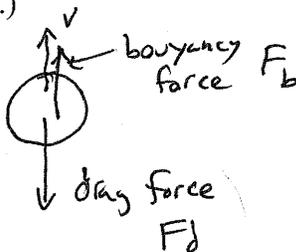
$$F_b = \Delta \rho g \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 \qquad \frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot m^3 = N$$

$$F_d = 3 \pi \mu V d \qquad (Pa \cdot s)(m/s)(m) = N$$

$$F_d = F_b \quad \text{in equilibrium:} \quad \Delta \rho g \frac{4}{3} \pi d^3 \frac{1}{8} = 3 \pi \mu V d$$

Part C:

$$\boxed{V = \frac{\Delta \rho g}{\mu} \frac{4}{18} d^2}$$

Some particles will escape, and some will not—calculate which size particles will escape and which will not. You may need the information at the back of the test, and the viscosity of the melt is $7 \cdot 10^{-3}$ Pa·s.

$$\Delta \rho = \left| \rho_{Al_2O_3} - \rho_{steel} \right| = \left| 3970 - 7800 \right| = 3830$$

$$g = 9.8$$

$$\mu = 7 \times 10^{-3}$$

$$V_c = 1 \, m/min = 0.017 \, m/s$$

$$\boxed{d_c = 140 \, \mu m}$$

smaller prtzles are pulled under

larger particles float out

Problem #3: Due Dilligence is Conducted on the Professor

In class, a long time ago, we looked at the problem of glass fiber drawing. We explicitly looked at the required cooling rate to keep a new fiber of diameter 20 μm, drawn from the fluid, from breaking up into droplets before it became rigid. A schematic of the process in shown at the right.

$\rightarrow\!\!\leftarrow$ 20 μm
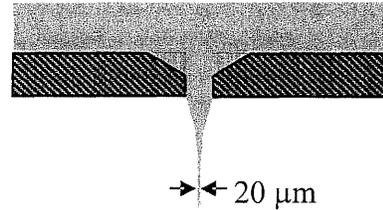
Now, back when we looked at this problem, we knew heat transfer but not fluid flow. Accordingly, you had to trust me that the kinetics of the fluid flow problem were known. Specifically, I told you that the time to break up the cylindrical fiber into droplets was on the order of milliseconds.

Now that you know a bit about fluid flow, please revisit this assertion. Can you convince yourself (and me) that the millisecond timescale is reasonable?

Information: the material is glass at 900° C, $\mu = 10^2$ Pa·s, $\rho = 2500$ kg/m³, $\gamma = 0.5$ J/m².

Part A:

Start by writing down the Newtonian flow equation, and rearranging it so that you have units of seconds on either side.

2

$$\tau = \mu \dot{\gamma} \implies \frac{1}{\dot{\gamma}} \left[\frac{1}{1/s} \right] = \frac{\mu}{\tau} \left[\frac{Pa \cdot s}{Pa} \right]$$

$$\frac{1}{\dot{\gamma}} \text{ is a characteristic time}$$

Part B:

Replace stress in your equation with a characteristic stress that is driving the break-up of the fiber into droplets.

2

driving force for break-up is <u>Surface tension</u>

use Young - LaPlace eqn. $\quad P \approx 2\gamma/r \quad$ (or γ/r for 1-axis of curvature)

$$\tau \rightarrow 2\gamma/r$$

Part C:

$$\frac{1}{\dot{\gamma}} = \frac{\mu r}{2\gamma}$$

Evaluate the characteristic time for droplet breakup. Is it <u>of millisecond scale?</u>

1

$$\frac{1}{\dot{\gamma}} = \frac{(10^2)(10 \times 10^{-6})}{2 (0.5)} = 0.001 \text{ s} \quad \underline{yep.}$$

Problem #4: Collapsing Under the Pressure (not the students, the voids)

In the final stage of sintering, isolated voids shrink to remove surface area. As we discussed in class, these voids will contain trapped gas if the sintering was conducted in air, and applied pressure can help to overcome the pressure contained in the voids. Let's look at how such voids shrink.

Part A:

Write down the pressure balance equation that would apply if we are pressing powders in the final stage of sintering under the conditions described above.

$$P_a + \frac{2\gamma}{r} = P_g$$

applied pressure   capillarity pressure   internal gas pressure

Part B:

If the pores first form at a size roughly that of the initial powder size, with radius r_p, and they initially trap gas at atmospheric pressure, then write down an equation that shows how the pressure in the pores increases as they shrink.

@ $r = r_p$ gas is at atm. pressure, P_{atm}

as $r\downarrow$, $P\uparrow$ by ideal gas law: $P_1 V_1 = P_2 V_2$

$$P_{atm} \cdot \tfrac{4}{3}\pi r_p^3 = P_g \cdot \tfrac{4}{3}\pi r^3 \quad \Rightarrow \quad \boxed{P_g = \frac{P_{atm} \cdot r_p^3}{r^3}}$$

Part C:

Combine Parts A and B to provide an expression for the equilibrium pore size in the final stage of sintering.

$$\boxed{P_a + \frac{2\gamma}{r} = P_{atm}\frac{r_p^3}{r^3}}$$

Solve for r:

Problem #5: The Fun of Multidimensional Diffusion Encounters the World of Chemical Vapor Deposition

In CVD, a chemical reaction in the vapor provides a source for deposition, and diffusion is responsible for bringing the reacted gases to the substrate surfaces. In class we discussed some possible geometries for this process. Let's try another one—imagine a long square tube, where we want to coat the sides of the tube. The reactive gasses are flowed in along the tube length, at a high rate so that flow is essentially uniform (plug flow) in the tube.
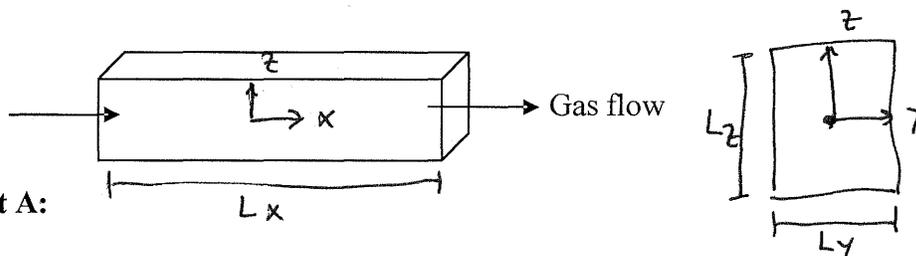
Part A:

Write down the form of the diffusion equation that you would use to analyze this process. Explain any simplifications that you would employ, and the set of boundary conditions you would use as well.

full 3-D model, plus generation! But steady state is ok.

$$\frac{\partial c}{\partial t} = D\left[\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}\right] + G = 0$$

Uniform generation throughout volume (gas rxn)

B.C.: at walls $y = \pm L_y/2$, $z = \pm L_z/2$, $c = 0$ (adsorption)

take x to be large, neglect $\frac{\partial c}{\partial x}$.

Part B:

Can this problem be addressed by a superposition approach? In other words, are there two or more 1-D diffusion solutions that we can put together to give the full solution? If the answer is yes, please write down an approximate solution to the problem, or indicate what the solution would consist of. If the answer is no, please explain why it cannot be done.

2-D problem, on y and on z, B is the same.

Solve: $\frac{\partial c}{\partial t} = 0 = \frac{\partial^2 c}{\partial y^2} + G$ and $\frac{\partial^2 c}{\partial z^2} + G = 0$

Same b.c. in each case

→ Can superimpose them.

$C = C_y C_z$

# Problem #6: A Spoonful of Slurry Will Cure What Ails You

Think about an unusual slurry, in which two different powder materials (call them A and B) are suspended. Each of the two powders has its own isoelectric point, and therefore its own zeta potential. The graph below shows how pH affects their zeta potentials.

Now, a series of experiments is conducted. First, a small amount of slurry is mixed up, with some concentration of A and B particles. The pH is adjusted to a specific value, and the particles are allowed to settle. The experiment is repeated five times, for the conditions listed below. For each experiment, draw a picture of what the settled particle packing might look like. In each case, write ONE SENTENCE explaining your thoughts and why they led to the picture you drew.
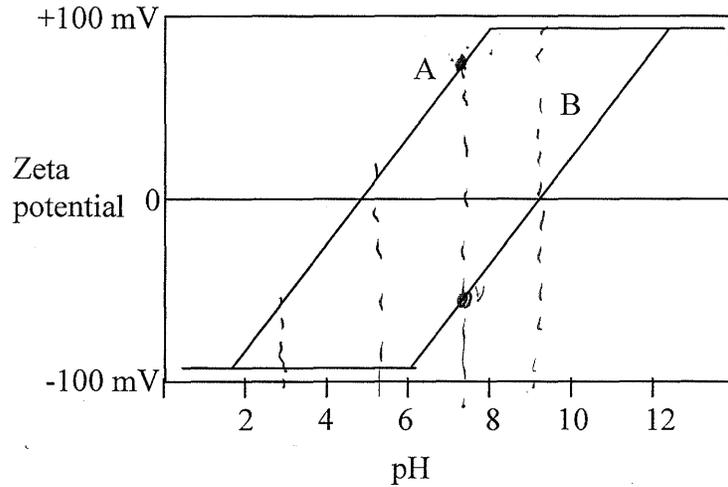
(1) 100% A, 0% B, pH = 5
(2) 0% A, 100% B, pH = 5
(3) 50% A, 50% B, pH = 3
(4) 50% A, 50% B, pH = 7
(5) 50% A, 50% B, pH = 9

① near zero ζ,
only one component.
Will floc./coag.
irregular packing
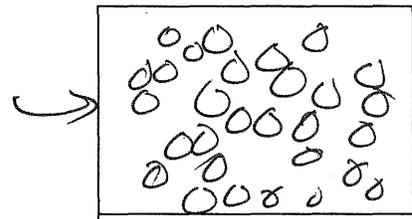
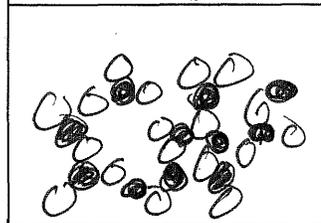② large negative ζ, one component
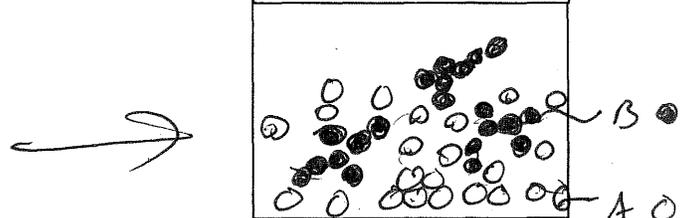→ will repel in suspension, settle into a regular pattern
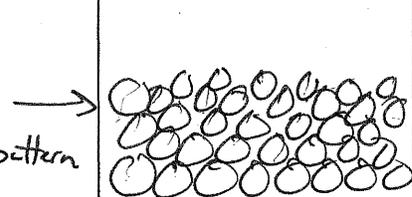
③ both components large negative ζ,
→ will repel in suspension and settle into →
regular pattern w/ random site occupation
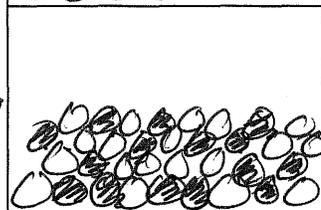
④ components have opposite sign ζ –
will attract each other in suspension.
irregular packing w/ near-neighbor →
unlike pairs dominant.

⑤ A high ζ,   B neutral ζ
    ↓   ↓
repel +   coag/ floc.
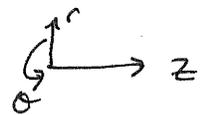settle well

Problem #7: Get Wired

The process of wire coating is commonly used to put the plastic (insulating) sheath around electrical wires of all kinds. It looks like an extrusion process, in the sense that polymer is forced through a die. However, in wire coating the polymer flow is induced by the motion of the wire, which is pulled at a fixed rate, v_0.

die channel

v_0
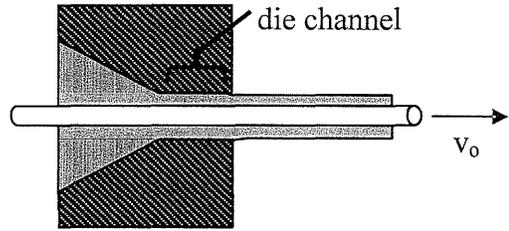
Part A:

Write down the proper form of the fluid flow equation for this problem, and explain any simplifications that you feel can be made in this regard.

no body forces or pressure — just velocity

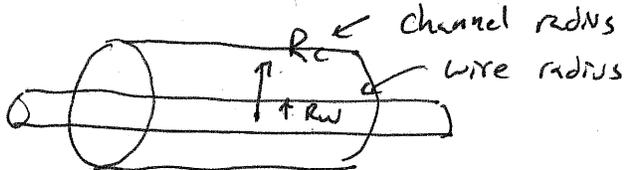only flow along z in the channel

neglect θ, Θ terms.

$$\frac{\partial v_z}{\partial t} = \boxed{\frac{\partial^2 v_z}{\partial r^2} \cdots} \leftarrow \begin{array}{c}\text{steady}\\\text{state!}\end{array}$$

2.5

r

θ → z

$$\nu \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right) = 0$$

Part B:

List the boundary conditions for flow through the die channel region. You may need to define some geometrical parameters—please be sure the meaning of any symbols you introduce is clear.

R_c ← channel radius

R_w — wire radius

@ $r = R_w$, $v = v_0$

@ $r = R_c$, $v = 0$ 1

Part C:

Write down the solution to the equation from part A, subject to the boundary conditions from part B. (You can solve it if you want to, or perhaps there is some other way to get the answer...)

steady-state, fixed B.C. → analogy to heat transfer!

$$\theta = \frac{T(r) - T(r_1)}{T(r_2) - T(r_1)} = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}$$

@ $r_2 = R_c$

$r_1 = R_w$

$$\boxed{\frac{v_z - v_0}{0 - v_0} = \frac{\ln(r/R_w)}{\ln(R_c/R_w)}}$$

1.5

Problem #8: The Students Face Down a Real Phase Diagram

You are given a rod of titanium that contains 1 at% cobalt, and asked to purify it. Looking at the Ti-Co phase diagram, you reason that this alloy could be zone refined, exactly as discussed in your 3.044 class. You place the rod into an induction coil and pull it through slowly, locally melting and resolidifying the rod as the hot zone passes over it.
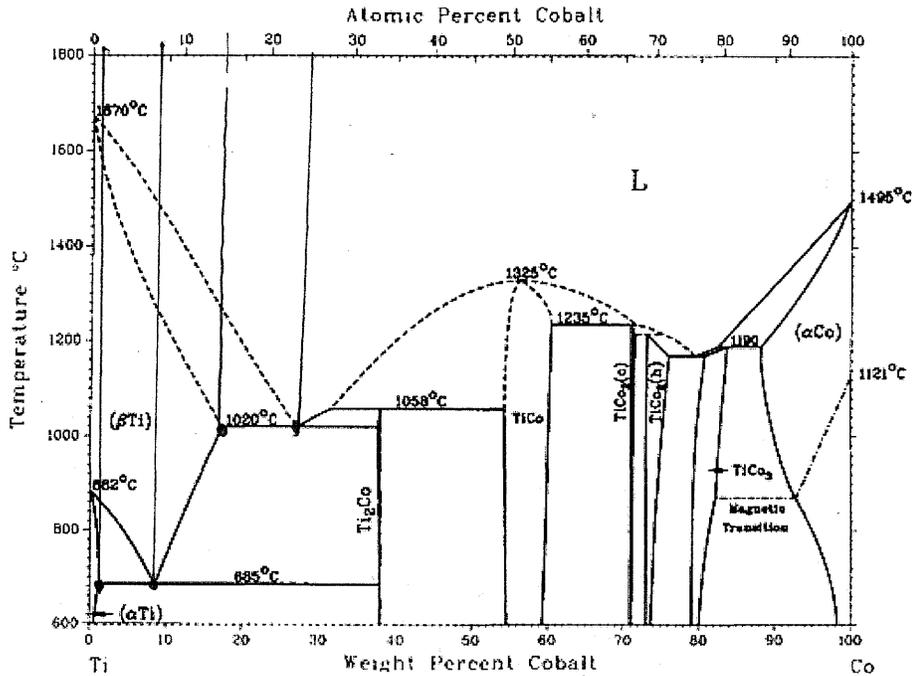
At the end of one pass, you analyze the leading end of the rod—this is the first part of the rod to be melted and resolidified.

Part A:

From the phase diagram, you estimate that the alloy composition at the end of the bar should be about 0.6 at% after one pass through the zone refiner. Explain this estimate.

$$K = \frac{C_s}{C_\ell} = \frac{15}{24} \approx 0.625$$

after one pass:

Part B:

$$K \cdot C_0 = 0.625 \times 1\% \approx 0.625 \approx 0.6\% \, !$$

You are shocked to find, however, that the end of the bar is quite substantially purer than you first suspected it might be! The composition is ~0.1 at% Co. Explain why the rod is so pure at this end, and why specifically it is at a value near 0.1 at% Co. (Note: your measurements are made after the rod is cooled back down to room temperature).

(I) After solidification there is another phase transformation! $\beta \to \alpha$ Ti and this trans also pushes solute around.

(II) $K' = \frac{C_\beta}{C_\alpha} \approx \frac{1}{7} \approx 0.14$

$C_0' = 0.6$ after solidification →
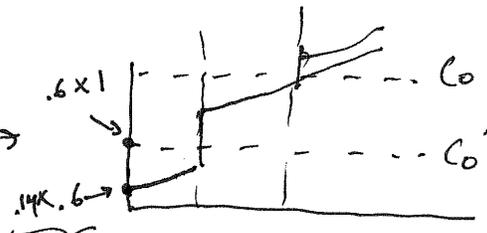
(III) $0.14 \times 0.6 \approx .09 \approx 0.1$ at%

Problem #9: Outside-In Heat Transfer

We have frequently talked about processes in which a solid was heated or cooled by virtue of its contact with a fluid (or vapor) that exchanged heat at the interface by convection. What if the problem went the other way around? How long does it take to heat a vapor with a hot solid? We have never thought through this problem, but I think you should be able to do it on the fly.

Consider a furnace where the heating elements are in the walls; the walls may be considered as instantaneously being raised to a temperature T_w at t = 0. Derive an equation that tells how the temperature rises in the furnace as a function of time. You may assume that the air flow in the air is rapid, so that the whole furnace interior is at a constant temperature.

Walls at T_w

Air inside furnace
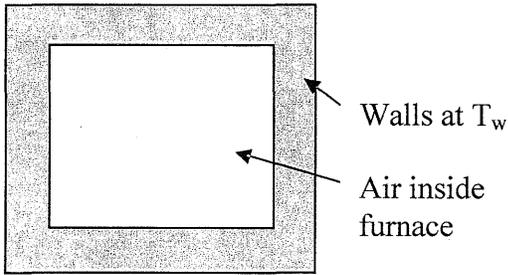
You may need to introduce some quantities/variables (such as the volume of air in the furnace, V, among other things). Please explain when you do and label the symbols so I know what you are thinking.

flux in = heat accumulated

total area between walls + air

$$h(T_w - T_f) \cdot A = \frac{\partial H}{\partial t}$$

$$\boxed{h(T_w - T_f) \cdot A = \rho \, C_p V \frac{\partial T_f}{\partial t}} \quad \checkmark$$

$\Downarrow$ integrate

int. const.

$$B + \frac{h \cdot A}{V \rho C_p} t = \int \frac{\partial T_f}{T_w - T_f} = - \ln(T_w - T_f) \quad \Big\backslash$$

Init. Cond.

$$t = 0, \quad T_f = T_o : \qquad B + 0 = - \ln(T_w - T_o) \quad \Big\backslash$$

$$\therefore \quad - \ln(T_w - T_o) + \frac{hA}{\rho C_p V} t = - \ln(T_w - T_f) \qquad \text{Newton heating!}$$

$$\ln\left(\frac{T_w - T_f}{T_w - T_o}\right) = -\frac{hA}{\rho C_p V} t \qquad \Big\} \quad \boxed{\frac{T_w - T_f}{T_w - T_o} = \exp\left(\frac{hAt}{V \rho C_p}\right)}$$

Problem #10: Blowing Hot and Cold

Below are five scenarios in shape forming. In each case, there is a question about how a change in temperature can impact the process. For each, indicate whether an increase or a decrease is the required change, and explain in ONE SENTENCE why this is the case.

A) A sheet being stretch-formed is developing a local thin region that leads to a rupture. The engineer in charge would like to introduce a local air jet that would either heat or cool the material near the thinning region.

Cool → local suppression of strain rate, slows down a region that otherwise is deforming too fast.

B) Aluminum is being shape formed at a stress of 1 MN/m^2 and 300° C; the ductility is insufficient to form the desired part geometry. A deformation mechanism map is provided for this material.

higher T will cross into power-law creep → more non-Newtonian!

∴ cooling more likely to get us to Newtonian conditions, larger strains.
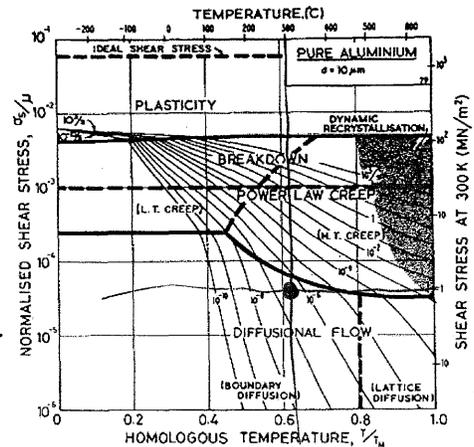
C) A thermoplastic is being continuously film-blown into a large tube. The plastic is coming out cloudy.

Must be crystallizing — too hot → cool

D) Glass is being poured continuously onto a bed of molten tin. The waves created on pouring are not damping fast enough, and the sheet is coming out non-planar.

from Eq. sheet → damping eqn.

$$\frac{A}{A_o} = exp\left(- \frac{\rho g \lambda t}{4\pi\mu}\right)$$

as μ ↑ longer time to damp.
heat → lower μ, better, faster levelling

E) A melt is being sprayed into an atomization chamber to create powders. These are coming out irregularly shaped instead of spherical.

Not enough time to spheroidize → heat up

3.044 Materials Processing
Spring 2013