

3.032 Class Questions Re: Quiz 2 Topics

Fall 2007

Note: These are responses to class members' questions in order of emails received, not necessarily importance. My responses are intended to supplement your class lecture notes and reading material.

1. Please go through the shrimp at the base of the ocean problem again. I am not clear the difference between stress and shear when it comes to the death of the shrimp. That was a great example you brought up.

A material or an animal within the ocean is subjected to pure compressive hydrostatic stress (normal stresses in x, y, and z are equal in magnitude). The maximum normal stress criterion for plastic deformation predicts that a material will yield if the normal stress reaches the value obtained when the material fails under uniaxial tension/compression. In other words, the material will fail if ANY of the normal stresses reaches yield strength s_y . The uniaxial compressive stress required to yield or kill a shrimp (0.1 MPa), so this criterion predicts that a shrimp would yield/die if it reached a depth in the ocean where the pressure reached 0.1 MPa. However, shrimp live happily at depths where pressure is 10 MPa, so this criterion does not accurately capture the "yield criterion" that predicts plastic failure of shrimp. We discussed that the reason for this failure is that many materials including the biopolymers that make up a shrimp fail not due to high normal stresses but to high shear stresses.

From Mohr's circle, it is easy to see that hydrostatic stress fails to generate shear stress on any plane inside the material: this is a principal stress state where all 3 normal stresses are located at the same point on the σ -axis, so you cannot draw a circle to define planes on which the shear stress is nonzero.

2. Principle [sic] stress, principle [sic] shear: What does this mean in an engineering system? If I have a ceramic I know has a yield stress of x MPa, how does a 2 x 2 matrix of stress and shear indicate how and in what orientation I can implement it?

Engineers need to be able to determine the principal state of stress or of strain in a material to identify the maximum/minimum values that normal/shear stresses/strains will attain inside the material. As we learn from the yield criteria, if these maximum values reach the magnitudes required to yield the material, the material will deform permanently (plastically) rather than reversibly (elastically). This often leads to functional failure of the structure or device, and computation of the principal stresses corresponding to a triaxial stress state is the fastest way to determine whether yielding will occur according to these criteria.

By "2 x 2 matrix of stress and shear", I think you mean a matrix σ_{ij} that includes both normal and shear stresses; if it is a 2x2 matrix then you are assuming plane stress: the only nonzero normal/shear stresses are contained in one plane (say, x-y plane), and there are no normal/shear stresses in the orthogonal plane of the material (say, the z-plane). This is a reasonable approximation for thin sheets of material, where z is the through-thickness direction.

If you had a ceramic of known yield stress x MPa and knew your application would place this ceramic under this plane stress state, you would want to immediately determine the principal stress state. Why? First, you could then determine whether these stresses are sufficient for yielding to occur, according to your choice of yield criteria. Second, this would tell you the orientation and magnitude of the maximum normal stress. Although we've not yet covered fracture in 3.032 yet, ceramics are typically brittle materials that fracture rather than plastically deform, and the maximum normal stress criterion does a good job at predicting the fracture stress of such materials. If the maximum normal stress in the ceramic reaches the fracture stress of your ceramic, you know right away that the ceramic device will fail and you would choose another ceramic of larger fracture stress.

If the maximum normal stress is below the fracture stress of your ceramic, you'd still want to consider modifying the ceramic via processing to either increase its fracture stress or adding fibers that would be aligned to block growth of cracks. Thinking back to the pressure vessels/hot dogs, you can intuit that the cracks would run perpendicular to the maximum normal stress, and you would then align these fibers parallel to the maximum normal stress orientation (the principal stress axis orientation) to be sure you'd block such cracks.

3. Do components like fibers on a silicon substrate really undergo stress tensors like in that problem set? How? Do engineers at intel and other device companies really look at stress states like that?

Yes, engineers at electronic device companies like Intel in fact consider such stress states on silicon. Typically, such engineers are concerned with triaxial thermal stresses when these devices reversibly heat and cool. If the silicon substrate is both heated and loaded mechanically, which is common in such devices, both normal and shear stresses are generated within the Si wafer. Interconnect lines of metal and device insulators of oxides are so thin on such devices that, at the interface where those tiny lines of materials join to the silicon substrate, the strains generated in those small-volume “fibers” or films are exactly equal to the strains in the Si.

4. Why do I want to rotate a stress state α degrees? What does this physically mean in an engineering system?

We’ve discussed this through several different examples in class and recitation, as well as in Lab 1. We are not rotating a stress state, but rather resolving the stresses or transforming the stresses onto a new set of coordinate axes. Coordinate axes are arbitrary choices to analyze a real engineering system; we often pick axes that align with the shape of the object or the orientation of the applied stresses/strains, but that is purely arbitrary. If we were to draw a different set of coordinate axes rotated by some angle θ , we’d come up with a different combination of stresses/strains.

This is the same exercise as resolving a force vector that is at some arbitrary angle into its “vertical” and “horizontal” components. We have not rotated the force, just expressed that force in terms of a new set of axes. This makes it easier for engineers to analyze all the complex forces acting on a device, e.g., by balancing all the vertical and horizontal components to make sure the device will not move.

In the case of stresses inside a material, we resolve stresses onto coordinate axes to easily identify planes inside the material that are under high normal stress/strains or high shear stresses/strains. These are locations that we identify with respect to our initial set of axes, so we say that these orientations of high normal or shear stress/strain are “rotated some angle θ with respect to our initial axes.” As a materials engineer, you know that materials will deform and/or fail under sufficiently high stress states, so you need to know the orientation of regions under high stress (likely locations of failure). You also need to know how a stress state is distributed onto planes defined by the unit cell planes of a crystal; or onto planes that define the orientation of precipitates, particles, grain boundaries, or fibers within the material.

5. Could you go over the dislocation stress fields and also the force vs. extension relationships for the three types of yield conditions.

(a) Yield conditions (covered first, as this is a continuum concept applicable to crystalline and noncrystalline materials):

1. Maximum normal stress criterion for yielding: material yields when any of the three principal normal stresses reach the normal stress attained when the material yields under uniaxial loading, which is exactly the yield stress.
2. Maximum shear stress criterion/Tresca: material yields when the shear stress reaches the maximum shear stress attained when the material yields under uniaxial loading. From Mohr’s circle, you can see that the shear stress attained upon yielding under uniaxial loading is $(\sigma_1 - \sigma_3)/2 = \sigma_y/2$.
3. Maximum distortional energy/von Mises: material yields when the effective stress reaches the effective stress when the material yields under uniaxial loading. The effective stress is defined by the 2nd invariant of the deviatoric stress tensor, and can be calculated from any stress tensor σ_{ij} or the principal stresses σ_1 , σ_2 , and σ_3 corresponding to that stress tensor.

(b) Dislocation stress fields: The stress generated inside the crystal due to the presence of a single screw or edge dislocation can be calculated from the displacement the dislocation creates: $u \rightarrow \epsilon \rightarrow \sigma$ in our usual way, where the arrows here indicate a partial derivative and a constitutive relation, respectively. We’ll cover this more extensively in the elective, 3.14. In this class, you should understand how these stress fields are derived (just as we derive stress in a material from any displacement applied to that material), and how these dislocation energies and stresses depend on elastic properties (G and ν) and length scales (b) of the crystal.

6. It would be nice to clarify the different types of elasticity we learned and the important differences between them, and what materials they are relevant for.

We'll do this in the quiz review sessions together. You should try to make a table of this for your crib sheet, using your notes and course reader.

7. Why do only gases readily obey Newton's viscosity law but not high molecular weight polymers?

When the motion of particles is uncorrelated, Newton's law of viscosity holds well: the resistance to shear flow or the shear strain rate varies linearly with the applied stress. For gases and low molecular weight polymers or fluids of low viscosity, the number of particles/chains per unit volume is so low that the motion of one particle/chain is really independent of the motion of the nearby particles/chains. As the molecular weight of a polymer increases, the chains are increasingly entangled and thus the motion of one chain depends in part on the motion of the nearby chains.

8. Overview of the graphs for the LVE solid. Specifically, why does the KV model fail to describe relaxation in polymers? why does the Maxwell model fail to accurately predict creep in solid polymers? (You gave an excellent explanation in class during those lectures but I can't remember fully:)

"Why" these models describe only creep or only relaxation well is attributed to what kind of material for which they were originally designed to match the experimental data. The Maxwell model describes a spring and dashpot in series, and was proposed to explain the time-dependent deformation of very viscous fluids like tar. If this model is subjected to an immediate, constant tensile stress (tensile creep), the spring can extend instantaneously to a strain $\epsilon = \sigma/E$; a real polymer does not deform instantaneously, but in fact exhibits a lagged increase in strain. The K-V model describes a spring and dashpot in parallel. If this model is subjected to an immediate, constant strain e , relaxation is the decrease in stress inside the material with time. The K-V model's constitutive law shows that for $de/dt = 0$, stress would be constant for all time (and equal to exactly Ee), and never relax; a real polymer does relax over time, so K-V poorly predicts polymer relaxation.

9. Graph of stress-strain curve showing the energy loss and the concept of mechanical damping. Can you please explain the aeroplane explanation concept again?

If you load a linear elastic material, the strain in the material would instantaneously be equal to the stress/E, no matter how fast or slow your loading frequency. In contrast, if you stress a polymer at time = 1/frequency much faster than its characteristic time ($\tau = \eta/E$ for the Maxwell or Kelvin-Voigt models) or much slower than its characteristic time, you'll observe the strain that you would predict from the constitutive models for short times ($t = 0$) or long time ($t = \text{infinity}$), respectively. If you stress a polymer at 1/frequency that is intermediate to these two extremes, the "viscous" or "dashpot" nature of the polymer will cause the strain to lag behind your loading frequency. The polymer is damping the input mechanical stress via this lag, often dissipated or as heat. You would see this in a stress-strain curve as a hysteresis, or gap between the loading part and the unloading part, where the area inside that hysteresis loop is the dissipated energy. The example I gave in class was actually of a tire material (a reinforced rubber, not really a good example of a LVE polymer governed by internal energy U) where the goal is to design the material to damp dissipate as much mechanical energy as possible at the operating frequency, which might be the angular frequency of an airplane or truck or car tire.

10. Are we responsible for the fine details of the derivations of the force and entropic parameters that characterize the polymer chains? Gaussian distribution function?

No, you do not need to memorize the form of a Gaussian distribution function. Yes, you should understand and be comfortable deriving the molecular origin of rubber elasticity and predicting the effects of changing L_p , L_c , T , etc on entropic spring constant. From previous Course 3 classes, you can relate the Helmholtz energy in terms of entropy as a function of accessible configurations that can be expressed by some probability distribution function.

11. Can you please address the main differences between the Tresca and Von Mises criteria? Also can you please discuss the concept of conservatism?

Tresca: material yields when max shear stress = max shear stress attained when material yields under uniaxial loading; this neglects the intermediate principal stress.

von Mises: material yields when the shear strain energy reaches a critical value, which can be expressed in terms of the effective stress σ_{eff} = effective stress attained when material yields under uniaxial loading; this considers all 3 principal (normal) stresses.

Conservative: you are conservative if you predict yielding at stresses that could be lower than the actual stresses required for yielding. This is a conservative estimate of when yielding will occur, because those stresses may actually be insufficient to cause yielding. If you want to ensure that yielding does not occur under some triaxial stress state, you typically want to err on the conservative side of your estimates so that you're quite sure yielding will not occur.

12. I remember writing in my lectures notes that yielding is defined as the stress needed to increase dislocations or cause the dislocations to multiply(during the guest lectures)-I may have written the point incorrectly. Can you please explain? In general, is it safe to assume that having dislocations in a material is advantageous because it provides strengthening mechanisms in the material and increase the yield strength of the material? That said, can you please emphasize the role of dislocations in plastic deformation/yielding?

Yielding means "plastically or permanently deforming without fracturing/breaking". The motion of dislocations is the microscopic origin of plasticity; dislocations are the carriers of plasticity in a crystal, just as electrons are the carriers of current. At the microscopic level, this happens as soon as the first dislocation moves. From an engineering point of view, motion of just one dislocation is usually inconsequential, so we define "yield stress" as the stress we can measure beyond which the material is plastically deformed. To measure such a stress practically, the displacements must be much greater than a single Burgers vector due to one dislocation. In fact, we need lots of dislocations to move, so we say that yielding is measureable when lots of dislocations move, which means the stress is sufficient to move existing and generate new dislocations.

Will having dislocations always increase the yield strength of a crystal? This is a tricky concept: as we discussed, a single crystal with zero dislocations will have a higher yield strength than a single crystal with some pre-existing dislocations; to yield the former case, we'd have to nucleate some dislocations at GPa-level stresses. On the other hand, a single or polycrystal with 1000 pre-existing dislocations would generally have a higher yield strength than one with 100 pre-existing dislocations, for the reasons related to work hardening that we discussed.

13. What is the relationship between the pinning points and plasticity? I am under the impression that the pinning points prevent plasticity from taking place by impeding the motion of the dislocations.

Pinning points "pin" segments of a moving dislocation, such that it takes additional stress to continue to move that segment of the dislocation (it must get longer to curve between those pinning points, and its energetic penalty increases per unit length of the dislocation). You are correct: since plasticity requires motion of dislocations, pinning impedes dislocation motion and thus increases the stress required for sustained plastic deformation. However, a point beyond what we've covered in class in detail (and will cover in 3.14) is that these pinning points also can serve as sites of dislocation multiplication: looking at the precipitate/particle strengthening, we see that when dislocations are bent sufficiently around an obstacle, parts of the dislocation "pinch off" and form new "child" dislocations that can move independently of the "parent" dislocation.

14. Do we need to qualitatively explain the strengthening mechanisms, i.e alloying elements, dislocations and grain boundaries or are supposed to quantitatively describe these concepts?

Both: You need to understand the physical explanation for why each of these approaches increases the yield strength, and quantitatively know how a change in the microstructure will affect the yield strength of the material. These equations are quite simple because they are based on empirical (experimental) observations, but are good engineering tools for material design.

15. The general concept that I am the most confused about is the worm-like chain model and the freely-jointed chain model, and the various parameters we can derive from these models.

FJC model: Force required to extend this chain depends on contour length L_c , which depends on the number and length of rigid segments that make up the chain, n and b , respectively.

WLC model: Force required to extend this chain depends on L_c and L_p . Here, L_p is like b from the FJC model, but reflects the fact that polymer chains are not made up of rigid segments b connected by joints, but flexible segments that fluctuate independently over some length L_p .

Both models are based on entropically dominated free energy, where entropy is defined by accessible configurations of the chain as derived in class. As in pset 5, one can apply these models to experimentally measured data to estimate L_c , L_p , and/or n and b of the chain, and the stiffness of that polymer chain.