3.024 Electrical, Optical, and Magnetic Properties of Materials Spring 2012
Recitation 9

Outline:

1. Intrinsic Semiconductors
2. Doped Semiconductors
3. Engineering Conductivity

1. Intrinsic Semiconductors

For an intrinsic semiconductor at a finite temperature, the conduction band has a number of electron charge carriers and the valence band has a number of hole charge carriers. Both the electrons and holes can be approximated as free electrons with negative and positive charge, respectively.

$$E_c = E_g + \frac{\hbar^2 k^2}{2m_c^*}$$
$$E_v = -\frac{\hbar^2 k^2}{2m_v^*}$$

$${m_{c,v}^*}^{-1} = \frac{1}{\hbar^2}\frac{d^2 E_{c,v}}{dk^2}$$

For 3D systems:

$$g(E) = \frac{m}{\pi^2\hbar^2}\sqrt{\frac{2mE}{\hbar^2}}$$

For the conduction band:

$$g_c(E) = \frac{m_c^*}{\pi^2\hbar^2}\sqrt{\frac{2m_c^*(E - E_c)}{\hbar^2}}$$

For the valence band:

$$g_v(E) = \frac{m_v^*}{\pi^2\hbar^2}\sqrt{\frac{2m_v^*(E_v - E)}{\hbar^2}}$$

Charge carrier density integrals for each band:

$$n_c(T) = \int_{E_c}^{\infty} f(E,T) g_c(E) dE$$
$$p_v(T) = \int_{-\infty}^{E_v} (1 - f(E,T)) g_v(E) dE$$

Non-Degenerate Semi-Conductor Approximation

$$E_c - \mu \gg k_B T$$
$$\mu - E_v \gg k_B T$$

$$f(E) \approx e^{\frac{-(E-\mu)}{k_B T}}, E \geq E_c$$

$$1 - f(E) \approx e^{\frac{(E-\mu)}{k_B T}}, E \leq E_v$$

$$n_c(T) \cong \int_{E_c}^{\infty} e^{\frac{-(E-\mu)}{k_B T}} g_c(E) dE \cong e^{\frac{-(E_c-\mu)}{k_B T}} \int_{E_c}^{\infty} e^{\frac{-(E-E_c)}{k_B T}} g_c(E) dE \cong N_c(T) e^{\frac{-(E_c-\mu)}{k_B T}}$$

$$N_c(T) = \int_{E_c}^{\infty} e^{\frac{-(E-E_c)}{k_B T}} g_c(E) dE \cong \int_{E_c}^{E_c+k_B T} e^{\frac{-(E-E_c)}{k_B T}} g_c(E) dE \cong \frac{1}{4}\left(\frac{2m_c^* k_B T}{\pi \hbar^2}\right)^{\frac{3}{2}}$$

$$p_v(T) \cong \int_{-\infty}^{E_v} e^{\frac{(E-\mu)}{k_B T}} g_v(E) dE \cong e^{\frac{(E_v-\mu)}{k_B T}} \int_{-\infty}^{E_v} e^{\frac{(E-E_v)}{k_B T}} g_v(E) dE \cong P_v(T) e^{\frac{(E_v-\mu)}{k_B T}}$$

$$P_v(T) = \int_{-\infty}^{E_v} e^{\frac{(E-E_v)}{k_B T}} g_v(E) dE \cong \int_{E_v-k_B T}^{E_v} e^{\frac{(E-E_v)}{k_B T}} g_v(E) dE \cong \frac{1}{4}\left(\frac{2m_v^* k_B T}{\pi \hbar^2}\right)^{\frac{3}{2}}$$

Law of Mass Action

$$n_c(T) p_v(T) = N_c(T) P_v(T) e^{\frac{-E_g}{k_B T}}$$

Intrinsic SC

$$n_c = p_v = n_i$$
$$n_c p_v = n_i^2 = N_c(T) P_v(T) e^{\frac{-E_g}{k_B T}}$$

$$\mu = {}_F = E_v + \frac{E_g}{2} + \frac{1}{2} k_B T \ln\left(\frac{P_v(T)}{N_c(T)}\right) = E_v + \frac{E_g}{2} + \frac{3}{4} k_B T \ln\left(\frac{m_v^*}{m_c^*}\right)$$

2. Doped Semiconductors
   p-type material (dopant is electron acceptor)

$$p_v \approx N_A$$
$$n_c \approx \frac{n_i^2}{N_A}$$
$${}_F = +\frac{E_g}{2} + \frac{3}{4} k_B T \ln\left(\frac{m_v^*}{m_c^*}\right) - k_B T \ln\left(\frac{N_A}{n_i}\right)$$

   n-type material (dopant is electron donor)

$$p_v \approx \frac{n_i^2}{N_D}$$
$$n_c \approx N_D$$
$${}_F = +\frac{E_g}{2} + \frac{3}{4} k_B T \ln\left(\frac{m_v^*}{m_c^*}\right) + k_B T \ln\left(\frac{N_D}{n_i}\right)$$

3. Engineering Conductivity

$$\sigma = n_c e \mu_e + p_v e \mu_h$$