

1. (25 Points) You're using electron beams to investigate the structure of multi-layered materials. Looking at the distribution of electrons in a material consisting of several layers you find that the wave function as a function of position in the layered material x for an electron is:

$$\psi(x) = \begin{cases} Ae^{\frac{kx}{2}} & x \leq 0 \quad \text{Region I} \\ A\left(1 + \frac{x}{L} - \left(\frac{x}{L}\right)^2\right) & 0 \leq x \leq L \quad \text{Region II} \\ Ae^{\frac{2-kx}{2}} & x \geq L \quad \text{Region III} \end{cases}$$

- a. Write 4 boundary condition equations that must be true for this system for the wave function to satisfy piecewise continuity (8 Points: 2 Points per equation).

- b. What value must k be equal to for these 4 equations to be satisfied (2 Points).

- c. What is the kinetic energy ($E - V(x)$) of the system in each region (5 Points, 2 for I, 1 for II, 2 for III)? (Use m for electron mass.)

- d. What must A equal (as a function of L) for this wave function to be a valid wave function (8 Points for setting up the equation, 2 Points for solving for A in terms of L)?

Bonus (10 Points): Find the Taylor expansion of $E - V(x)$ in region II to find a Taylor Series 2nd order polynomial approximation form of the potential energy in this region about $x = \frac{L}{2}$.

2. (25 Points) You're measuring momenta of electrons (mass m) in vacuum and zero gravity constrained to move in 1D. There are virtually no fields and no forces. To your surprise you find that half of your measurements return the momentum p_1 , another third return momentum $p_2 = 2p_1$, and yet another one sixths of your experiments return something really strange – an electron flying backwards with a momentum $p_3 = -p_2$.

Your experiment is very precise and you suspect that all electrons should be identical, i.e. be described by the same eigenfunction.

- a. What is the Hamiltonian for this system (6 Points)?
- b. What is the total wave function of an electron in your experiment (7 Points)?
Hint: Write the wave function in terms of eigenfunctions of the Hamiltonian.
- c. Is momentum of a given electron conserved in your system (3 Points)? What is the average momentum (expectation value) of the electron in your experiment (3 Points)?
- d. Is energy conserved in your system for a given electron (3 points)? What is the average energy (expectation value) of the electron in your experiment (3 Points)?

3. (25 Points) It's your first day of your extraterrestrial internship and your task is to design the toughest and most transparent (large band gap) semiconducting shield for your space ship. Unfortunately, the planet where your internship is has only three elements in abundance Cd, S and Se.
- a. Which semiconductor would you expect to have a larger elastic modulus CdS or CdSe? Base your answer on a detailed explanation. (For your reference, the atomic number of Se is 34, and the atomic number of S is 16) (12 Points)
- b. Which semiconductor would you expect to have a larger bandgap, CdS or CdSe? Base your answer on a detailed explanation. (13 Points)

4. (25 Points) It's your third year of graduate school and you've upgraded from studying electrons in vacuum to studying electrons inside crystals. By looking at the distribution of the electrons inside the crystal you have determined that in your experiment electron should be described by the wavefunction:

$$\psi(x) = e^{i\frac{\pi}{2d}x} \left[\sin\left(\frac{2\pi}{d}x\right) + \cos\left(\frac{8\pi}{d}x\right) \right], \text{ where } d = 2 \text{ \AA}.$$

- a. What is the lattice spacing a in your crystal (3 Points)?
- b. Is momentum conserved for a given electron inside the crystal (3 Points)? What is the expectation value of the momentum for this electron (3 Points)?
- c. Is energy conserved for this electron (3 Points)? Approximate the potential for the crystal with $V(x) = 2V_0 \cos\left(\frac{2\pi}{a}x\right)$, where " a " is equal to the lattice spacing you've found in part (a). What is the expectation value of energy for this electron (3 Points)?

- d. Qualitatively plot the band diagram $E \sim k$ for electrons in a potential from part (c) inside the Brillouin zone considering a 2X2 matrix approximation to the central equation (10 Points).

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