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Presupposition Projection, Trivalence and Relevance[1]
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1. Goals

1 To argue that presuppositions project from quantificational sentences according to the predictions of certain trivalent theories of projection (see Peters 1979, Beaver and Krahmer 2001, George 2008).
2. To argue for a bivalent method of deriving the trivalent predictions. The method will involve a new assertability condition (Relevance, hinted at in Fox 2008).

> The condition will demand that the presupposition of an atomic sentence be met to the extent that the atomic sentence is *relevant* for determining the semantic value of the matrix sentence.

2. Projection from the Nuclear Scope – An Empirical Debate

(1) Some student [x drives x's car to school]$_{\text{x has a (unique) car}}$

(2) No student [x drives x's car to school]$_{\text{x has a (unique) car}}$

(3) Every student [x drives x's car to school]$_{\text{x has a (unique) car}}$

(4) **Competing Empirical Claims**:

Universal Projection (Heim 1983): A quantificational sentence of the form $Q(A)\lambda xB(x)_{p(x)}$ presupposes $\forall x(A(x)\rightarrow p(x))$
Existential Projection (Beaver 1992): A quantificational sentence of the form $Q(A)\lambda xB(x)_{p(x)}$ presupposes $\exists x(A(x)\wedge p(x))$
Nuanced Projection (Peters, George, Chemla): A quantificational sentence of the form $Q(A)\lambda xB(x)_{p(x)}$ presupposes different things depending on various properties of Q.

3. Trivalent Predictions (one version of Nuanced Projection)

(5) **Stalnaker's Bridging Principle**:
A sentence S is assertable given a context set C only if
$\forall w\in C$ [the denotation of S in w is either 0 or 1].

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[1] This work owes on obvious debt to Schlenker's work on presupposition projection (see Fox 2008). Many thanks to Emmanuel Chemla, Paul Egre, Kai von Fintel, Ben George, Irene Heim, Alejandro Pérez Carballo, Raj Singh, Benjamin Spector, Steve Yablo, and especially to Alexandre Cremers and Philippe Schlenker.

(6) Trivalent denotation of the nuclear scope in (1)a,b,c:

$$\lambda x.\begin{cases} 1 & \text{if x has a (unique) car and x drives it to school} \\ 0 & \text{if x has a (unique) car and x doesn't drive it to school} \\ \# & \text{if x has no car (or more than one car)} \end{cases}$$

(7) **Strong Kleene:**
The denotation of S in w is
(a) 1 if its denotation (in a bivalent system) would be 1 under every bivalent correction of sub-constituents.
(b) 0 if its denotation would be 0 under every bivalent correction of sub-constituents.
(c) # if neither (a) nor (b) hold

(8) a function $g:X \rightarrow \{0,1\}$ is a *bivalent correction* of a function $f:X \rightarrow \{0,1,\#\}$ if $\forall x[(f(x)=0 \vee f(x)=1) \rightarrow g(x)=f(x)]$

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(1)' Some student [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car (and doesn't drive it to school)].

(2)' No student [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [Every student has a car (and doesn't drive it to school)] or
[Some student has a car and doesn't drive it to school] or

(3)' Every student [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [Every student has a car (and drives it to school)] or
[Some student has a car and doesn't drive it to school].

4. Conflicting Evidence

4.1. Simple sentences appear to provide evidence for these nuanced predictions

(9) At least one of these 10 students [x drives x's car to school].
Leads only to an existential inference

(10) None of these 10 students [x drives x's car to school].
Leads to a universal inference

See Chemla (2009) for important experimental data.

4.2. Yes/no questions appear to provide counter evidence

(11) Does one of these 10 students [x drive x's car to school].
Leads to a universal inference (Schlenker 2009)

5. An argument for the Trivalent Predictions

Claim: The trivalent projection is always correct but is usually disguised by the assertion or by a particular form of pragmatic strengthening.

(12) QP_1 [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [QP_2 has a car and (doesn't) drive it to school] or
[Every student has a car] (where QP_2 can, though need not, be identical to QP_1)

Equivalently:
$\neg$[QP_2 has a car and (doesn't) drive it to school] $\rightarrow$
[Every student has a car]

Believing this disjunction without believing one of the disjuncts is odd. It suggests that there is a connection between the two (if one is false, the other is true). So (as in the discussion of proviso to which we will return) this could affect our ability to detect the formal presupposition in actual contexts of use.

5.1. Indicative *some*

(1)' Some student [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car]

It is odd for a speaker to believe the disjunction without believing one of the disjuncts.

Four scenarios to consider:

Scenario 1: The first disjunct *some student has a car and drives it to school* is part of the common ground, C, at the point of utterance. This could be a reasonable context, but probably one in which the sentence is not assertable for Stalnakarian reasons (it is a contextual tautology).

Scenario 2: The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the sentence is assertable.

Scenario 3: The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic scenario.

Scenario 4: The disjunction is not part of C at the point of utterance. Here accommodation is required. By a simple-minded model of accommodation (below), accommodation is minimal leading to the C from Scenario 3. If this was our last move, we would be left with an unrealistic C. However, accommodation is followed by update of the context by the assertion. The resulting C now entails the first disjunct (hence realistic).

Conclusion: there is a scenario (scenario 4) in which the sentence is acceptable without a resulting context that entails the universal statement (the second disjunct). Hence, speakers do not report a universal inference.

Presupposed Architecture:

Assertability Condition: When a sentence S is asserted in a context C it is associated with a formal presupposition p. When p is entailed by (the common ground in) C, the sentence is assertable. When p is not entailed by C, a repair strategy might come into play.

> **Accommodation:** When p is not entailed by C, it would either be judged as unacceptable or C might be modified minimally so that p is satisfied
>
> Accommodation (C,p) = C$\cap$p
>
> I.e. accommodation is always minimal.

Update: After S is asserted, the context will be updated
(Update (C, S) = C$\cap${w: S is true in w} if S is indicative)

5.2. Indicative *no*

(2)' No student [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car]

It is very odd for a speaker to believe the disjunction without believing one of these disjuncts.

Four scenarios to consider:

Scenario 1: The first disjunct *some student has a car and drives it to school* is part of C at the point of utterance. This could be a reasonable context, but

	probably one in which the sentence is not assertable for Stalnakarian reasons (it is a contextual contradiction).
Scenario 2:	The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the sentence is assertable.
Scenario 3:	The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic Scenario.
Scenario 4:	The disjunction is not part of C at the point of utterance. Here accommodation would be required. By assumption, it is minimal and is followed by update of the context by the assertion. In our particular case, the resulting C entails the second conjunct.

Conclusion: Under every scenario in which the sentence is acceptable, the resulting context entails the universal statement (the second conjunct). Hence, speakers report a universal inference.

5.3. Indicative *every*

(3)' Every student [x drives x's car to school]$_{\text{x has a (unique) car}}$
Presupposes:
Either [Some student has a car and doesn't drive it to school] or
[Every student has a car]

Again, it is odd for a speaker to believe the disjunction without believing one of these disjuncts.

Four scenarios to consider:

Scenario 1:	The first disjunct *some student has a car and doesn't drive it to school* is part of C at the point of utterance. This could be a reasonable context, but probably one in which the sentence is not assertable for Stalnakarian reasons (it is a contextual contradiction).
Scenario 2:	The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the sentence is assertable.
Scenario 3:	The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic scenario.
Scenario 4:	The disjunction is not part of C at the point of utterance. Here accommodation would be required. By assumption, it is minimal and is followed by update of the context by the assertion. In our particular case, the resulting context entails the second disjunct.

Conclusion: Under every scenario in which the sentence is acceptable, the resulting context entails the universal statement (the second conjunct). Hence, speakers report a universal inference.

5.4. Negated Universals

Reveal that a universal presupposition is wrong for universal statements:

(13) A: There are many students around, hence many cars.
B: No, half of the students don't have a car.
Furthermore, some don't drive their car to school.
Furthermore, not every student drives his car to school.
Furthermore, every student leaves his car at home

5.5. Questions

(14) Does one of these 10 students [x drive x's car to school].
Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car]

Scenario 1: The first disjunct *some student has a car and drives it to school* is part of C at the point of utterance. This could be a reasonable context, but probably one in which the question is not assertable (the answer is already part of the common ground).

Scenario 2: The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the question is assertable.

Scenario 3: The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic Scenario.

Scenario 4: The disjunction is not part of C at the point of utterance. Here accommodation would be required. By assumption, it is minimal and is followed by update of the context by the question. In this particular case (a question not an assertion), the resulting common ground is not affected. Since it is the unrealistic common ground from Scenario 3, we're in trouble (unless we have a method of strengthening presuppositions).[2]

Conclusion: Under every scenario in which the sentence is acceptable, the resulting context entails the universal statement (the second disjunct). Hence, speakers report a universal inference.

Prediction: A yes/no question will reveal weaker presuppositions if we make it plausible to believe the disjunction without believing one of the disjuncts.

[2] There are well known challenges for this line of reasoning that we will bring up in section 7 and attempt to address in section 12.

(15) John and Bill meet for a game of poker. The rules they set for their engagement are the following. They each give Jane 100 dollar and get chips in return. The game will continue until one of them has no more chips left. The moment this happens, the winner (the player that has 200 chips) goes to Jane and cashes his chips.

Fred (who knows the rules of engagement) is responsible for cleaning the room the moment the game is over. He calls Jane and asks one of the following questions:

Did one of the two players cash his chips?

(16) Did anyone of these bankers acquire his fortune by wiping out one of the others?
Presupposition: if none of these bankers acquired his fortune by wiping out one of the others, they all have a fortune.

Confound (Ben George p.c.): nominals can receive temporal interpretations independent of tense, Hence it is not clear that a universal presupposition will be wrong here.

Can be addressed by explicating the temporal interpretation of the nominal:

(17) Did anyone of these bankers acquire the fortune he deposited in the bank last week by wiping out one of the others?
Presupposition: if no banker acquired the fortune he deposited in the bank last week by wiping out one of the others, they each deposited a fortune last week.

Likewise for (15):

(18) Is any one of the two players allowed to cash the chips that he now has in his possession?

6. Charlow's Challenge

(19) Just five of these 100 boys smoke. They all smoke Nelson
#Unfortunately, some/at least two of these 100 boys also smoke Marlboro$_F$.

Charlow's Conclusion: *also* is a "strong trigger" and reveals the true projection properties which are universal.

6.1. Conflicting Data.

(20) More than 80% of the boys went to the party.
More than 40% of the boys also had a drink.
Available reading:
More than 40% of the boys (including those which didn't go to the party)…

(21) More than 50% of Americans think that Obama is bad for America.
More than 40% of Americans also think that he's a Muslim.

(22) More than 80% of the boys went to the party.
Luckily, fewer than 50% of the boys also had a drink.

(23) Imagine the following rather stupid game. Four players are each handed a card, and what happens next depends on whether or not one of the players gets an ace.
First possibility: No player gets an ace → Every player gets a cookie.

Second possibility: one or more player gets an ace → the player or players that get an ace get a cookie and a million dollars. No one else gets anything
So it is clear that some or all players will get a cookie.

The only reason anyone would watch the game is to find out whether someone also gets the million dollars.

6.2. Goal

To understand what distinguishes Charlow's examples from those discussed above.

Tentative Suggestion based on Magri (2009): A sentence φ is "odd" when it is contextually equivalent to a scalar alternative φ' which is logically stronger than φ.

(24) Just five of these 100 boys smoke. They all smoke Nelson
φ:#Unfortunately, some of these 100 boys also smokes Marlboro_F.
φ': Unfortunately, some of these 5 boys also smokes Marlboro_F.

7. Problems for the Trivalent Setup

7.1. The Proviso Problem

The type of explanation we gave for the presuppositions of questions (4.5.) is familiar from Karttunen and Heim, and much subsequent work.

(25) a. If John is a scuba diver, he will bring his wet suit.
Appears to presuppose: If John is a scuba diver, he has a wet suit.
b. If John flies to London, his sister will pick him up.
Appears to presuppose: John has a sister.

The Heim/Karttunen claim: Both sentences in (25) have a conditional presupposition. It is not plausible to believe the conditional *If John flies to London, he has a sister* without believing that he has a sister. Hence, one would tend to infer that John has a sister.

Criticism by Geurts (1997): By parity of reasoning, we would expect the presupposition of (26) to be strengthened, but it isn't.

(26) Bill knows that if John flies to London, he has a sister.

Conclusion reached by Singh (2008, 2010) and Schlenker (2010): if we want a mechanism that strengthens presuppositions, we need to say something that would predict when strengthening is possible.

The trivalent system would have to face the same challenge:

(27) Bill knows that either some student drives his car to school or every student has a car.

7.2. Presupposition of non truth denoting expressions

The trivalent system might work for describing presupposition projection in indicative sentences which have a truth value. But how do we extend it to deal with the presupposition of non indicative sentences, e.g. questions? (Thanks to A. Cremers)

My Goal for the remainder: to develop a new way of deriving the trivalent predictions in a bivalent system with the aid of an assertability condition (a modification of Stalanker's bridging principle). My condition (as we will see) is very much inspired by Schlenker's work.

The condition will have two advantages over the trivalent system:

a. It will not care what type of denotation a matrix sentence has and thus would have predictions for non-indicative sentences.
b. It will be possible to account for presupposition strengthening (of the sort we assumed for questions) based on recent thinking about the Proviso Problem.

8. The setup

First Ingredient: No projection beyond atomic sentences

Certain lexical items will have a two dimensional entry. Consequently, the S nodes immediately above them will have a two dimensional representation (one dimension is the "assertion" and the other the "presupposition", but, as we will see, the terminology is a bit misleading). In such cases we will write S as S_p, where p expresses the presupposition (possibly assignment dependent). There will be no partiality (or trivalence). (We will assume that the S part entails p but this is not crucial). Furthermore, there will be no rule of projection for p beyond S_p. That is, the semantics is classical (i.e. works as if p was not there).

Second Ingredient: An assertability condition

Presuppositions of complex sentences will be predicted (following Schlenker) by a pragmatic condition on an utterance of a sentence φ that has S_p as a constituent. The condition, again following Schlenker, will have a global version (that will have no left right asymmetry) that we will then incrementalize (to derive the asymmetries).

However, the pragmatic condition will be different from Schlenker's. It will bear some resemblance to Stalnaker's bridging principle in (5).

Let's start with the "propositional case" in which S_p has no free variables in it (which are not in the domain of the contextually given assignment function).

9. The Propositional Case

9.1. The Global Version

Let $\varphi(S_p)$ be a sentence dominating (or identical to) S_p.

(28) $\varphi(S_p)$ is assertable in C only if
$\forall w \in C$: Relevant(S_p, $\varphi(S_p)$, w) $\rightarrow$ p is true in w.[3]

(29) $\text{Rel}(S_p, \varphi(S_p), w) \Leftrightarrow_{\text{def}} ((\llbracket\varphi(T)\rrbracket^w \neq \llbracket\varphi(\perp)\rrbracket^w)$
Where $\llbracket T\rrbracket^w = 1$ for all w and $\llbracket\perp\rrbracket^w = 0$ for all w

9.1.1. Negation

$\varphi(S_p)$: $\neg S_p$

$\forall w\ \forall S$: Rel(S,$\neg$S, w).

Hence, $\neg S_p$ is assertable in C, by (28), only if $\forall w \in C$: p is true in w.

9.1.2. Symmetric theory of disjunction, conjunction

$\varphi(S_p)$: $S_1 \vee S_p$

$\forall w \in C$: S_1 is false in w $\rightarrow$ Rel(S_p, $\varphi(S_p)$, w).
$\forall w \in C$: S_1 is true in w $\rightarrow$ $\neg$Rel(S_p, $\varphi(S_p)$, w).

Hence $S_1 \vee S_p$ is assertable in C, by (28), only if
$\forall w \in C$: S_1 is false in w $\rightarrow$ p is true in w.

[3] 'Rel(S, φ(S), w)' should be read as the value of S is relevant for the value of φ in w.

$\varphi(S_p)$: $S_1 \wedge S_p$

$\forall w \in C$: S_1 is true in w $\rightarrow$ Rel(S_p, $\varphi(S_p)$, w)
$\forall w \in C$: S_1 is false in w $\rightarrow$ $\neg$Rel(S_p, $\varphi(S_p)$, w)

Hence $S_1 \wedge S_p$ is assertable in C, by (28), only if
$\forall w \in C$: S_1 is true in w $\rightarrow$ p is true in w.

9.1.3. (Material-)Conditionals

$\varphi(S_p)$: $S_1 \rightarrow S_p$

$\forall w \in C$: S_1 is true in w $\rightarrow$ Rel(S_p, $\varphi(S_p)$, w)
$\forall w \in C$: S_1 is false in w $\rightarrow$ $\neg$Rel(S_p, $\varphi(S_p)$, w)

Hence $S_1 \rightarrow S_p$ is assertable in C, by (28), only if
$\forall w \in C$: S_1 is true in w $\rightarrow$ p is true in w.

9.2. The Incremental Version

(30) $\varphi(S_p)$ is assertable in C only if
$\forall w \in C$: Rel_{inc} (S_p, $\varphi(S_p)$, w) $\rightarrow$ p is true in w.

(31) Rel_{inc} (S_p, $\varphi(S_p)$, w) $\Leftrightarrow_{def}$ $\exists \varphi' \in$GOOD-FINAL(S, φ) s.t. Rel(S_p, $\varphi'(S_p)$, w)

(32) GOOD-FINAL(S, φ) =
{φ': φ' can be derived from φ by replacing constituents in φ that follow S}

For more general statements, see Appendix A

10. Generalizing to an Extensional System with one Free Variable

Again, we will start with a global version which can then be incrementalized

(33) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound-variable in $S(x)_{p(x)}$ (i.e. a variable free in S_p and bound in φ).

φ is assertable in C only if
$\forall w \in C\ \forall a \in D_\alpha$ [Rel($S(x)_{p(x)}$, $\varphi(S(x)_{p(x)})$, w, *a*) $\rightarrow$ $[\![p(x)]\!]^{w,x\rightarrow a}$ =1)][4]

(34) Rel($S(x)_{p(x)}$, $\varphi(S(x)_{p(x)})$, w, *a*) $\Leftrightarrow_{def}$ $\exists T_a$, F_a
a. $\langle T_a, F_a \rangle$ is *a*-differing-extension of $S(x)_{p(x)}$ (an *a*-DE of $S(x)_{p(x)}$)
b. $[\![\varphi(T_a)]\!]^{w,g} \neq [\![\varphi(F_a)]\!]^{w,g}$

[4] 'Rel(S(x), φ(S(x)), w, a)' should be read as the value of S(x) is relevant for the value of φ in w given an individual a (or under an assignment function g, s.t. g(x)=a).

(35) $\langle T_a, F_a \rangle$ is an *a*-DE of $S(x)_{p(x)} \Leftrightarrow_{def}$
$\forall w\ [\![T_a]\!]^{w,x\to a} = 1\ \&\ [\![F_a]\!]^{w,x\to a} = 0\ \&$
$\forall \alpha \neq a\ [([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&\ [([\![p(x)]\!]^{w,x\to\alpha} = 1) \to ([\![T_a]\!]^{w,x\to\alpha} = [\![S]\!]^{w,x\to\alpha})]]$

Equivalently:

(35)' $\langle T_a, F_a \rangle$ is an *a*-DE of $S(x)_{p(x)} \Leftrightarrow_{def}$
$\exists \psi \exists T_a \exists F_a$
$\forall \alpha \neq a\ [([\![p(x)]\!]^{w,x\to\alpha} = 1) \to ([\![\psi]\!]^{w,x\to\alpha} = [\![S]\!]^{w,x\to\alpha})]\ \&$
$T_a = [x = a \vee \psi]$ and $F_a = [x \neq a \wedge \psi]$

Below we state results without proofs. For proofs, see appendix B:

10.1. Binding by an expression of type e

(36) φ: John λx [x likes x's mother]$_{\text{x has a (unique) mother}}$
S_p (=[x likes x's mother]$_{\text{x has a (unique) mother}}$)

$\forall w \forall a[\text{Rel}(S_p, \varphi, w, a) \leftrightarrow a = John]$

Hence (36) presupposes that John has a unique mother.

10.2. Quantification

φ: Every(NP)(λx ($S(x)_{p(x)}$)

Claim: $\forall w \in C \forall a \in D_e$:
$\text{Rel}(S(x)_{p(x)}, \varphi, w, a) \leftrightarrow$
$a \in [\![NP]\!]^w\ \&\ \neg\exists b \neq a: b \in [\![NP]\!]^w\ \&\ [\![p(x)]\!]^{w,x\to b} = 1\ \&\ [\![S(x)_{p(x)}]\!]^{w,x\to b} = 0$

Hence *Every(NP)(λx ($S(x)_{p(x)}$)* presupposes that p holds of every member of the denotation of NP (the domain) or that there is one member of the domain of which p is true and S is false.

I.e., if the sentence is not false, then p must hold of every member of the domain.

φ: Some(NP)(λx ($S(x)_{p(x)}$)

Claim: $\forall w \in C \forall a \in D_e$:
$\text{Rel}(S(x)_{p(x)}, \varphi, w, a) \leftrightarrow$
$a \in [\![NP]\!]^w$ and $\neg\exists x \neq a: x \in [\![NP]\!]^w\ \&\ [\![p(x)]\!]^{w,1\to x} = 1\ \&\ [\![S(x)_{p(x)}]\!]^{w,1\to x} = 1$

Hence *Some(NP)(λx (S(x)$_{p(x)}$)* presupposes that p holds of every member of the NP domain or that there is one member of the domain of which p holds and $[\![\lambda x S(x)_{p(x)}]\!]$ holds as well.

11. Incremental Version

(37) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound-variable in $S(x)_{p(x)}$ (i.e. a variable free in S_p and bound in φ).

φ is assertable in C only if
$\forall w \in C\ \forall a \in D_\alpha\ [Rel_{inc}(S\ (x)_{p(x)}, \varphi, w, a) \rightarrow [\![p(x)]\!]^{w,x\rightarrow a} = 1]$

(38) a. $Rel_{inc}(S, \varphi(S), w, a) \Leftrightarrow_{def}$
$\exists \varphi' \in$ GOOD-FINAL(S, φ) s.t., $Rel(S, \varphi'(S), w, a)$

More Radical Incrementalization

(39) $Rel_{r\text{-}inc}(S, \varphi(S), w, a) \Leftrightarrow_{def} \exists S'$ s.t. $Rel_{inc}(S', \varphi(S'), w, a)$

(40) φ is assertable in C only if
$\forall w \in C\ \forall a \in D_\alpha\ [Rel_{r\text{-}inc}(S\ (x)_{p(x)}, \varphi, w, a) \rightarrow [\![p(x)]\!]^{w,x\rightarrow a} = 1]$

More constituents will be r-incrementally relevant than those that are incrementally relevant (which are in turn more than those that are globally relevant). Hence, the more we incrementalize the stronger the presuppositions.

In particular, (40) will give us the Heim/Schlenker predictions (see appendix C).

12. Proviso and Formal Alternatives

Schlenker's (2010) solution to the proviso problem: the set of possible strengthening of the presupposition of a sentence φ come from various forms of radical incrementalization, in particular by treating all sorts of constituents that do not follow the relevant presupposition trigger, as if they followed the trigger.

Since we get the classical (Heim/Schlenker) predictions by considering substitutions of the nuclear scope (which does not follow the trigger), we understand why the Heim/Schlenker presuppositions are possible strengthenings of the trivalent presuppositions.

13. Generalizing to an extensional system with any number of free variables

The global version

(41) Let $\varphi(S([x_i])_{p([xi])})$ be a sentence that dominates $S([x_i])_{p([xi])}$ where $x_1 \ldots x_n$ are all the to-be-bound-variable in $S([x_i])_{p([xi])}$.

φ is assertable in C only if
$\forall w \in C\ \forall [a_i]\ \mathrm{Rel}(S([x_i])_{p([xi])}, \varphi, w, [a_i]) \rightarrow [\![p([xi])]\!]^{w,[xi] \rightarrow [ai]} =1)$

(42) $\mathrm{Rel}(S([x_i])_{p([xi])}, \varphi, w, [a_i]) \Leftrightarrow_{def}$

$\exists\langle T_{[ai]}, F_{[ai]}\rangle$ s.t. $\langle T_{[ai]}, F_{[ai]}\rangle$ is an $[a_i]$-DE of $S([x_i])_{p([xi])}$ and
$[\![\varphi(T_{[ai]}))]\!]^{w,g} \neq [\![\varphi(F_{[ai]}))]\!]^{w,g}$

(43) $\langle T_{[ai]}, F_{[ai]}\rangle$ is an a-DE of $S([x_i])_{p([xi])} \Leftrightarrow_{def}$
$\forall w$
a. $\forall x \neq [ai]: [\![T_{[ai]}]\!]^{w, [xi] \rightarrow [ai]} = [\![F_{[ai]}]\!]^{w, [xi] \rightarrow [ai]}$
b. $\forall x \neq [ai]: [\![p([x_i])]\!]^{w, [xi] \rightarrow [ai]} = 1 \rightarrow [\![T_{[ai]}]\!]^{w, [xi] \rightarrow [ai]} = [\![S([x_i])]\!]^{w, [xi] \rightarrow [ai]}$
c. $T_{[ai]}([a_i])=1$ and $F_{[ai]}([a_i])=0$

Equivalently:
(44) $\langle T_{[ai]}, F_{[ai]}\rangle$ is an a-DE of $S([x_i])_{p([xi])}$ if $\exists\psi$
a. $\forall x \neq [ai]: [\![p([x_i])]\!]^{w, [xi] \rightarrow [ai]} = 1 \rightarrow [\![\psi]\!]^{w, [xi] \rightarrow [ai]} = [\![S([x_i])]\!]^{w, [xi] \rightarrow [ai]}$
b. $T_{[ai]} = ([x_i]=[a_i] \vee \psi)$
c. $F_{[ai]} = ([x_i]\neq[a_i] \wedge \psi)$

The incremental version
As above

14. Generalizing to an intensional system with any number of free variables

The lazy thing to do at this stage it to assume that world variables are always represented in the syntax and to hope that this reduces to what we have in section 13.

Do we get the right predictions?

In particular, how do intensional operators project?

The predictions here seem different from what is stated by Kartunnen (1973, 1974) and Heim (1992). So I have serious homework to do.

Possibly relevant:

(45) a. I think it's possible that John has a job. But it's also possible that his job pays very little.
b. I think it's possible that John has a job. But I'm not certain his job pays that much.
c. #I think it's possible that John has a job. But I'm certain his job pays very little.

(46) a. I think it's possible that John has a job. And it's possible that his wife has a job, as well.
b. I think it's possible that John has a job. But I'm not certain his wife has a job, as well.
c. #I think it's possible that John has a job. And I'm certain his wife has a job as well.

15. Problem from Infinite Domains

(47) An infinite number of boys drove their car to school.
$[_{\varphi}$ An infinite number of boys $[_{S(x)}$ x drove x's car to school$]_{\text{x has a unique car}}]$

$\forall w \in C\ \neg\exists a \in D_e\ (\mathrm{Rel}(S(x), \varphi, w, a))$.
Hence the sentence should presuppose nothing.

Revision:

(48) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound-variable in $S(x)_{p(x)}$

φ is assertable in C only if
$\forall w \in C\ \forall A \subseteq D_\alpha\ S(x)_{p(x)}\ \ \mathrm{Rel}_{\text{SUB-SET}}\ (S(x), \varphi, w, A) \rightarrow \exists A' \subseteq A(\forall a \in A' [\![p(x)]\!]^{w,x\rightarrow a} = 1)$[5]

Equivalently: φ is assertable in C only if
$\forall w \in C\ \forall A \subseteq D_\alpha\ S(x)_{p(x)}\ \ \mathrm{Rel}_{\text{SUB-SET}}\ (S(x), \varphi, w, A) \rightarrow \exists a \in A([\![p(x)]\!]^{w,x\rightarrow a} = 1)$

(49) $\mathrm{Rel}_{\text{SUB-SET}}\ (S(x), \varphi, w, A)\ \Leftrightarrow_{\text{def}}$
$\exists T_A, F_A$
a. $\langle T_A, F_A\rangle$ is an A-differing-extension of $S(x)_{p(x)}$ (an A-DE of $S(x)_{p(x)}$)
b. $[\![\varphi(T_A)]\!]^{w,g} \neq [\![\varphi(F_A)]\!]^{w,g}$

(50) $\langle T_A, F_A\rangle$ is an A-DE of $S(x)_{p(x)}$ if
$\forall w\ \forall a \in A [\![T_A]\!]^{w,x\rightarrow a} = 1\ \&\ [\![F_a]\!]^{w,x\rightarrow a} = 0\ \&$
$\forall \alpha \notin A\ [([\![T_a]\!]^{w,x\rightarrow \alpha} = [\![F_a]\!]^{w,\ x\rightarrow \alpha})\ \&$
$[([\![p(x)]\!]^{w,x\rightarrow \alpha} = 1) \rightarrow ([\![T_a]\!]^{w,x\rightarrow \alpha} = [\![S]\!]^{w,\ x\rightarrow \alpha})]]$

Note: this assertability condition is stronger than what we had previously since:

a. $\forall S, \varphi, w, a[\mathrm{Rel}(S(x), \varphi, w, a) \rightarrow \mathrm{Rel}_{\text{SUB-SET}}\ (S(x), \varphi, w, \{a\})]$
b. If $|A| = \infty\ \exists S, \varphi, w\ [\mathrm{Rel}_{\text{SUB-SET}}\ (S(x), \varphi, w, A)\ \& \forall a \in A \neg \mathrm{Rel}\ (S(x), \varphi, w, a)]$

[5] '$\mathrm{Rel}_{\text{SUB-SET}}(S(x), \varphi(S(x)), w, A)$' should be read as the value of S(x) is relevant for the value of φ in w for some subset of A.

Appendices

A. More General Statements (for the propositional case of section 9)

Compositionality of Relevance (R-compositionality): Let φ(S(A)) be a sentence that dominates S which, in turn, dominates A.

a. If A is (inc-)relevant for the value of S in w, and S is (inc-)relevant for the value of φ in w, then A is (inc-)relevant for the value of φ in w.
b. If A is not (inc-)relevant for the value of S in w, A is not (inc-)relevant for the value of φ in w.
c. If S is not (inc-)relevant for the value of φ in w, A is not (inc-)relevant for the value of φ in w.

Proof: trivial.

Terminology:

If a sentence φ obeys the incremental assertability condition in (30) in every context that entails p and fails to obey the condition in every context that does not entail p, we will say that φ *presupposes p*. It will turn that for every sentence φ, there is a unique proposition that φ presupposes. Hence we can write *Presup(φ)* for this unique presupposition.

In the proofs below, we assume for simplicity that (30) is an iff condition. (It is easy to restate the proofs without this assumption.)

A.1. Negation

Claim: Presup(¬φ) = Presup(φ)

Proof:

Let C be a context that does not entail Presup(φ)	
Let w∈C be a world s.t. Presup(φ)(w)=0.	
φ is not assertable in any C, s.t. w∈C.	by definition
∃S_p dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0.	by (30)
S_p is inc-relevant for ¬φ in w.	φ is always relevant for ¬φ + R-compositionality
Hence ¬φ is unassertable in C.	

Let C be a context that does entail Presup(φ)	
∀w∈C: Presup(φ)(w)=1.	
φ is assertable in C.	by definition

$\neg\exists w\in C$, S_p dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0. by (30)
$\neg\exists w\in C$, S_p dominated by $\neg\varphi$, s.t. S_p is inc-relevant for $\neg\varphi$ in w and p(w)=0.
R-compositionality

Hence $\neg\varphi$ is assertable in C.
Hence: Presup($\neg\varphi$) = Presup(φ)

A.2. disjunction

Presup($\varphi\vee\psi$) = Presup(φ)$\wedge$($\neg\varphi\rightarrow$Presup(ψ))

Proof:
Let C be a context that does not entail Presup(φ)$\wedge$($\neg\varphi\rightarrow$Presup(ψ)).
Let $w\in C$ be a world in which Presup(φ)$\wedge$($\neg\varphi\rightarrow$Presup(ψ)) is false.

First Possibility -- Presup(φ) is false in w:

$\exists S_p$ dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0. by (30)
S_p is incrementally relevant for $\varphi \vee \psi$ in w. choose contradiction for ψ

Hence $\varphi\vee\psi$ is not assertable in C by (30).

Second Possitivlity -- ($\neg\varphi\rightarrow$Presup(ψ)) is false in :,

$\neg\varphi$ is true in w and Presup(ψ) is false in w. by (30)
Since $\neg\varphi$ is true in w, ψ is relevant for the truth value of $\varphi \vee \psi$
and the rest is just as above

Hence $\varphi\vee\psi$ is not assertable in C

Under both possibilities $\varphi\vee\psi$ is unassertable in C.

Let C be a context that entails Presup(φ)$\wedge$($\neg\varphi\rightarrow$Presup(ψ)).

$\forall w\in C$: Presup(φ)(w)=1.
φ is assertable in C. by definition
$\neg\exists w\in C$, S_p dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0. by (30)
$\neg\exists w\in C$, S_p dominated by φ, s.t. S_p is inc-relevant for $\varphi\vee\psi$ in w and p(w)=0.
R-compositionality

$\forall w\in C$
if φ(w) = 1, ψ is irrelevant for the value of $\varphi\vee\psi$, and so is any S_p dominated by ψ
R-compositionality
if φ(w) = 0, then Presup(ψ)(w)=1 C $\Rightarrow$ $\neg\varphi\rightarrow$Presup(ψ)
So, there will be no S_p dominated by ψ, which is both inc. relevant for ψ and p(w)=0.
Hence:
$\neg\exists w\in C$, S_p dominated by ψ, s.t. S_p is inc-relevant for $\varphi\vee\psi$ in w and p(w)=0.
R-compositionality

Hence
$\neg\exists w\in C$, S_p dominated by $\varphi\vee\psi$, s.t. S_p is inc-relevant for $\varphi\vee\psi$ in w and p(w)=0.
Hence, $\varphi\vee\psi$ is assertable in C.

Hence: $\text{Presup}(\varphi\vee\psi) = \text{Presup}(\varphi)\wedge(\neg\varphi\rightarrow\text{Presup}(\psi))$

A.3…

B. Missing Proofs from section 10

B.1. Binding by an expression of type e

(51) φ: John λx [x likes x's mother]$_{\text{x has a (unique) mother}}$
$S(x)_{p(x)}$ (=[x likes x's mother]$_{\text{x has a (unique) mother}}$)

$\forall w\forall a[\text{Rel}(S(x)_{p(x)}, \varphi, w, a) \leftrightarrow a=John]$

Proof (trivial):
$\text{Rel}(S(x)_{p(x)}, \varphi, w, a)$ $\leftrightarrow$ by definition of relevance

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![\text{John } \lambda x T_a]\!]^w \neq [\![\text{John } \lambda x F_a]\!]$ $\leftrightarrow$ by lambda conversion

$\exists\langle T_a,F_a\rangle\ldots[\![T_a]\!]^{w,x\rightarrow John} \neq [\![F_a]\!]^{w,x\rightarrow John}$ $\leftrightarrow$ by definition of a-DE

$a = John$

Hence (36) presupposes that John has a unique mother.

B.2. Quantification

φ: Every(NP)(λx (S(x)$_{p(x)}$)

Claim:

$\forall w\in C\forall a\in D_e$:

$\text{Rel}(S(x)_{p(x)}, \varphi, w, a) \leftrightarrow$

$a \in [\![NP]\!]^w \,\&\, \neg\exists b\neq a{:}b\in[\![NP]\!]^w \,\&\, [\![p(x)]\!]^{w,\, x\rightarrow b} = 1 \,\&\, [\![S(x)_{p(x)}]\!]^{w,\, x\rightarrow b} = 0$

Proof:
$\text{Rel}(S(x)_{p(x)}, \varphi, w, a)$ $\leftrightarrow$ by definition of relevance

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![\text{every NP } 1T_a]\!]^w \neq [\![\text{every NP } 1F_a]\!]^w$ $\quad\leftrightarrow\quad$ lambda conversion + the observation that $F_a \subset T_a$

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![NP]\!]^w \subseteq [\![T_a]\!]^{w,x\to a} \wedge \neg([\![NP]\!]^w \subseteq [\![F_a]\!]^{w,x\to a})$ $\quad\leftrightarrow\quad$ $[\![T_a]\!]^{w,x\to a} \setminus [F_a]\!]^{w,x\to a} = \{a\}$

$a \in [\![NP]\!]^w \ \& \ \exists\langle T_a,F_a\rangle\, \forall b\neq a[\ b \in [\![NP]\!]^w \to (x \in [\![T_a]\!]^{w,x\to b})]$
$\leftrightarrow$ by definition of a-DE

$a \in [\![NP]\!]^w \ \& \ \forall b\neq a[\ b \in [\![NP]\!]^w \to ([\![p(x)]\!]^{w,\, x\to b}=0 \text{ or } [\![S(x)_{p(x)}]\!]^{w,\, x\to b}=1]]$

$\leftrightarrow$ replace $\forall$ with $\neg\exists\neg$
and let negation migrate rightwards

$a \in [\![NP]\!]^w \ \& \ \neg\exists b\neq a{:}b \in [\![NP]\!]^w \ \& \ [\![p]\!]^{w,\, x\to b}=1 \ \& \ [\![S(x)_{p(x)}]\!]^{w,\, x\to b}=0$

Hence *Every(NP)(λx (S(x)$_{p(x)}$)* presupposes that p holds of every member of the denotation of NP (the domain) or that there is one member of the domain of which p is true and S is false.

I.e., if the sentence is not false, then p must hold of every member of the domain.

φ: Some(NP)(λx (S(x)$_{p(x)}$)

Claim:

$\forall w \in C\, \forall a \in D_e$: $Rel(S(x)_{p(x)}, \varphi, w, a)$ $\quad\leftrightarrow$

$a \in [\![NP]\!]^w$ and
$\neg\exists x\neq a{:}x \in [\![NP]\!]^w \ \& \ [\![p(x)]\!]^{w,\, 1\to x}=1 \ \& \ [\![S(x)_{p(x)}]\!]^{w,\, 1\to x}=1$

Proof:
$Rel(S(x)_{p(x)}, \varphi, w, a)$ $\quad\leftrightarrow\quad$ by definition of relevance

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![\text{some NP } \lambda x T_a]\!]^w \neq [\![\text{some NP } \lambda x F_a]\!]^w$ $\quad\leftrightarrow\quad$ lambda conversion + the observation that $F_a \subset T_a$

$\varnothing \neq [\![NP]\!]^w \cap [\![T_a]\!]^{w,x\to a} \ \& \ \varnothing = [\![NP]\!]^w \cap [\![F_a]\!]^{w,x\to a}$ $\quad\leftrightarrow\quad$ $[\![T_a]\!]^{w,x\to a} \setminus [F_a]\!]^{w,x\to a} = \{a\}$

$a \in [\![NP]\!]^w \ \& \ \exists\langle T_a,F_a\rangle\, \forall b\neq a[\ b \in [\![NP]\!]^w \to (b \notin [\![T_a]\!]^{w,x\to b})]$
$\leftrightarrow$ by definition of a-DE

$a \in [\![NP]\!]^w \ \& \ \forall b\neq a[\ b \in [\![NP]\!]^w \to ([\![p(b)]\!]^{w,\, x\to b}=0 \text{ or } [\![S(x)_{p(x)}]\!]^{w,\, x\to b}=0]]$

$\leftrightarrow$ replace $\forall$ with $\neg\exists\neg$
and let negation migrate rightwards

$a \in [\![NP]\!]^w \,\&\, \neg\exists b{\neq}a{:}b{\in}[\![NP]\!]^w \,\&\, [\![p]\!]^{w,\, x\to b}{=}1 \,\&\, [\![S(x)_{p(x)}\,]\!]^{w,\, x\to b} = 1$

Hence *Some(NP)(*λx *(S(x)*$_{p(x)}$*)* presupposes that p holds of every member of the NP domain or that there is one member of the domain of which p holds and $[\![\lambda x S(x)_{p(x)}]\!]$ holds as well.

C. Understanding the consequences of r-incrementalization

To get the Heim/Schlenker Generalization, we will strengthen the assertability condition by weakening our global notion of relevance to what we call potential-relevance (Rel_p). It will be easy to see that what we said in section 11 is correct: the incrementalization of Rel_p will be equivalent to the r-incrementalization of our earlier notion Rel.

(52) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound variable in $S(x)_{p(x)}$ (i.e. a variable free in S_p and bound in φ).

φ is assertable in C only if
$\forall w{\in}C\ \forall a{\in}D_\alpha\ (Rel_p(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \ \to [\![p(x)]\!]^{w,x\to a} =1)$

(53) $Rel_p(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \Leftrightarrow_{def}$
$\exists T_a, F_a$
a. $[\![T_a]\!]^{w,x\to a} =1 \ \&\ [\![F_a]\!]^{w,x\to a} =0 \ \&\ \forall\alpha{\neq}a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,\, x\to\alpha})$ and
b. $[\![\varphi(T_a)]\!]^{w,g} \neq [\![\varphi(F_a)]\!]^{w,g}$

Equivalently:
(53)' $Rel_p(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \Leftrightarrow_{def}$
$\exists\psi\exists T_a\exists F_a$
a. $T_a=[x=a \vee \psi]$ and $F_a=[x{\neq}a \wedge \psi]$
b. $[\![\varphi(T_a)]\!]^w \neq [\![\varphi(F_a)]\!]^w$

C.1. Binding by an expression of type e

(54) φ: John λx [x likes x's mother]$_{\text{x has a (unique) mother}}$
$S(x)_{p(x)}$ (=[x likes x's mother]$_{\text{x has a (unique) mother}}$)

For every w:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \leftrightarrow a{=}John.$

Proof (trivial):

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \quad \leftrightarrow$ by definition of p-relevance

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a} =1\ \&\ [\![F_a]\!]^{w,x\to a} =0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&$
$[\![John\ \lambda x T_a]\!]^{w} \neq [\![John\ \lambda x F_a]\!] \leftrightarrow$ by lambda conversion

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a} =1\ \&\ [\![F_a]\!]^{w,x\to a} =0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&$
$[\![T_a]\!]^{w,x\to John} \neq [\![F_a]\!]^{w,x\to John} \quad \Leftrightarrow$

$a = John$

Hence (54) presupposes that John has a unique mother.

C.2. Quantification

φ: Every(NP)(x (S(x)$_{p(x)}$)

Claim:

$\forall w\in C \forall a\in D_e$:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \Leftrightarrow a \in [\![NP]\!]^{w}$

Proof:
$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \quad \leftrightarrow$ by definition of p-relevance

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a} =1\ \&\ [\![F_a]\!]^{w,x\to a} =0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&$
$[\![every\ NP\ \lambda x T_a]\!]^{w} \neq [\![every\ NP\ \lambda x F_a]\!]^{w} \quad \leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a} =1\ \&\ [\![F_a]\!]^{w,x\to a} =0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,x\to\alpha})\ \&$
$[\![NP]\!]^{w} \subseteq [\![T_a]\!]^{w,x\to a} \wedge \neg([\![NP]\!]^{w} \subseteq [\![F_a]\!]^{w,x\to a}) \quad \leftrightarrow$

$a \in [\![NP]\!]^{w}$

Hence *Every(NP)(λx(S(x)$_{p(x)}$)* presupposes that p holds of every member of the denotation of NP

φ: Some(NP)(λx (S(x)$_{p(x)}$)

Claim:

$\forall w\in C \forall a\in D_e$:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \leftrightarrow a \in [\![NP]\!]^{w}$

Proof:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w)$ $\leftrightarrow$ by definition of p-relevance

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a} = 1\ \&\ [\![F_a]\!]^{w,x\to a} = 0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,\ x\to\alpha})\ \&$
$[\![\text{some NP } \lambda x T_a]\!]^w \neq [\![\text{some NP } \lambda x F_a]\!]^w$ $\leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\exists T_a,F_a\ [\![T_a]\!]^{w,x\to a} = 1\ \&\ [\![F_a]\!]^{w,x\to a} = 0\ \&\ \forall\alpha\neq a\ ([\![T_a]\!]^{w,x\to\alpha} = [\![F_a]\!]^{w,\ x\to\alpha})\ \&$
$[\![NP]\!]^w \cap [\![T_a]\!]^{w,x\to a} \neq \emptyset$ and $[\![NP]\!]^w \cap [\![F_a]\!]^{w,x\to a} = \emptyset$ $\leftrightarrow$

$a \in [\![NP]\!]^w$

Hence *Some(NP)(λx(S(x)*$_{p(x)}$*))* presupposes that p holds of every member of the NP domain.

D. More General Statement (for a language with variables)

Hopefully some other time