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Too Many Alternatives: density, symmetry and other predicaments
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Goals

1. To review the argument for density (and modularity) made in Fox and Hackl (in press)
2. To provide evidence that the core generalizations we argued for are attested in areas for which an account in terms of density is not available
3. To suggest that our account might nevertheless be right: density is a special case in which exhaustivity fails, but other cases are predicted as well
4. To characterize a broader generalization about the shape of alternatives that yields the effects we observed
5. To speculate about additional cases that might fall under the broader generalization.

1. The proposal in F&H

(1) The Universal Density of Measurements (UDM): Measurement Scales that are needed for Natural Language Semantics are *formally* dense (though contextually might be discrete).

(2) a. The Intuitive Claim: Scales of height, size, speed, and the like are dense.
b. The Radical Claim: All Scales are dense; cardinality in not a concept of NLS.

1.1. A constraint on Implicatures and *only*:

1.1.1. Density as an intuitive property of scales

THE BASIC EFFECT

(3) a. John weighs more than 150 pounds.
*Implicature: John weighs exactly S(150) pounds. Where S is the successor function
b. John weighs very little. *He only weighs more than 150.

UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

(4) a. You're required to weigh more than 300 pounds (if you want to participate in this fight).
Implicature: There is no degree greater than 150, d, s.t. you are required to weigh more than d pounds.
b. You're only required to weigh more than 150 pounds.

EXISTENTIAL MODALS DO NOT

(5) a. You're allowed to weigh more than 150 pounds (and still participate in this fight).
*Implicature: There is no degree greater than 150, d, s.t. you are allowed to weigh more than d pounds.
b. *You're only allowed to weigh more than 150 pounds.

1.1.2. Density as a formal property

THE BASIC EFFECT

(6) a. John has more than 3 children.
*Implicature: John has exactly 4 children.
b. John has very few children. *He only has more than THREE.

UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

(7) a. You're required to read more than 30 books.
Implicature: There is no degree greater than 30, d, s.t. you are required to read more than d books.
b. You're only required to read more than 30_F books.

EXISTENTIAL MODALS DO NOT

(8) a. You're allowed to smoke more than 30 cigarettes.
*Implicature: There is no degree greater than 30, d, s.t. you are allowed to smoke more than d cigarettes.
b. *You're only allowed to smoke more than 30_F cigarettes.

1.2. A Constraint on Questions and Definite Descriptions (Negative Islands)

1.2.1. Density as an intuitive property of scales

THE BASIC EFFECT

(9) a. *How much does John not weigh?
What is the successor of John's weight (the minimal amount d, such that he doesn't weigh d much)?
b. *I have *the amount of water that you don't*.
cf. I have the amount of water that you do.
I have an amount of water that you don't.

UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

(10) a. How much are you sure that this vessel won't weigh?
b. How much radiation are we not allowed to expose our workers to?
c. The amount of radiation that we are not allowed to expose our workers to is greater than we had thought.
d. The amount of money that you are sure that this stock will never sell for is quite high. (Are you sure that your estimation is correct.)

EXISTENTIAL MODALS DO NOT

(11) a. How much money are we not allowed to bring in to this country?
b. *How much money are we not required to bring in to this country?

(12) a. How much money are we requited not to bring in to this country?
b. * How much money are we allowed not to bring in to this country?

(13) a. How much radiation is the company not allowed to expose its workers to?
b. *How much food is the company not required to give its workers?[1]

(14) a. How much radiation is the company required not to expose its workers to?
b. *How much food is the company allowed not to give its workers?

1.2.2. Density as a formal property of scales

THE BASIC EFFECT

(15) *How many kids do you not have?

UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

(16) a. If you live in China, how many children are you not allowed to have?
b. How many days a week are you not allowed to work (according to union regulations)?
c. How many soldiers is it (absolutely) certain that the enemy doesn't have?

EXISTENTIAL MODALS DO NOT

(17) a. *If you live in Sweden, how many children are you not required to have?
b. *How many days a week are you not required to work (even according to the company's regulations)?
c. *How many soldiers is it possible that the enemy doesn't have?

(18) a. ???Combien John n'a-t-il pas lu de livres?
How many John n'has-he not read of books
b. ? Combien peux-tu me dire avec (absolue) certitude que John n'a pas lu de livres?
How many can-you me tell with (absolute) certainty that John has not read of books
(Benjamin Spector, pc)

(19) a. *Combien Jean n'a-t-il (pas) d'enfants?
How many John n'has-he not of children
b. ?Combien les chinois ne peuvent ils (pas) avoir d'enfants?
How many the chineese n'alloweed-them not have of-children?
(Valentine Hacquard, pc)

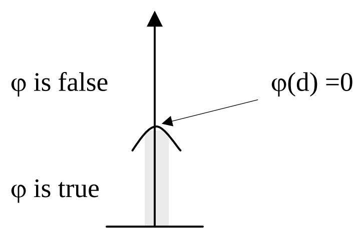
[1] Ignore the following irrelevant reading: What is the amount of food such that there is food in that amount and the company is not required to give that food to its workers. To avoid this problem:
(i) (When you enter the country) How much money are you not allowed to have.
(ii) *(When you enter the country) How much money are you not required to have.

1.3. The Generalization

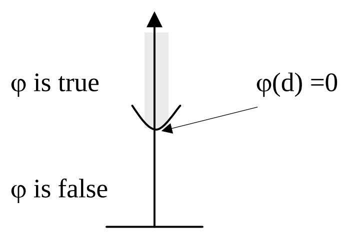
(20) We call $\varphi_{<d,st>}$ an *N-open property* if:
$\forall w,d[(\varphi(d)(w) = 1] \rightarrow \exists d'[(\varphi(d')(w) = 1 \ \& \ \varphi(d')$ asymmetrically entails $\varphi(d)]$.

(21) Examples of an upward monotone N-open property.

$\varphi = \lambda d$. John weighs more than d $\varphi = \lambda d$. John has more than d children



(22) Examples of a downward-monotone N-open property
$\varphi = \lambda d$. John doesn't weigh d pounds $\varphi = \lambda d$. John have d children



(23) Constraint on Interval Maximization (CIM): N-open properties cannot be maximized by $MAX_{inf.}$
$MAX_{inf}(\varphi_{<\alpha,st>})(w)$ = the $x \in D_\alpha$, s.t., $\varphi(x)(w) = 1$ and $\forall y(\varphi(y)(w)=1 \rightarrow \varphi(x)$ entails $\varphi(y)$

Lexical entries[2]

(24) a. $[[exh]](\varphi)(d)(w) \Leftrightarrow d = MAX_{inf}(\varphi)(w)$.
b. $[[only]](\varphi)(d)(w) \Leftrightarrow d = MAX_{inf}(\varphi)(w)$, when defined.
c. $[[?]](\varphi_{\alpha,st}) = \lambda w{:}\exists d[d= MAX_{inf}(\varphi)(w)].\{\varphi(x): x \in D_\alpha\}$
d. $[[the]](\varphi)(w) = MAX_{inf}(\varphi)(w)$ (von Fintel, Fox, Iatridou)

[2] The lexical entries for *only* and *exh* in (24) presuppose a movement theory of association with focus. In the next section, I presuppose a Roothian theory of association with focus: *only* and *exh* take a proposition p and a set of alternatives A (usually the focus value of the "prejacent"). The Roothian statements translate in an obvious way to (24)a and b as long as $[p=\varphi(d)] \ \& \ \forall q \in A[\exists d\ [q=\varphi(y)] \ \& \ \forall d\ [\exists q \in A\ [q=\varphi(y)]$. Under such circumstances, $only_{Roothian}(A)(p) = only(\varphi)(x)$.

(25) Basic Consequence: If φ expresses an N-open monotone property of degrees, then the following should be unacceptable.[3]

a. *exh (φ)(d)

b. *only (φ)(d)

c. *wh_d (φ)

d. *the (φ)

(26) Basic Logical facts.

a. A universal modal can close interval: If φ is an N-open monotone property of degrees, $[\lambda d.\Box\ \varphi\ (d)]$ is not.
Proof: Let the modal base for $\Box$ in w^0 be $\{w{:}\varphi(d^*)(w)=1\}$. It is now easy to see that $MAX_{inf}([\lambda d.\Box\ \varphi\ (d)])(w^0) = d^*$.

b. An existential modal cannot close interval: If φ is an N-open monotone property of degrees, so is $[\lambda d.\ \Diamond\ \varphi\ (d)]$ is not.
Proof: Assume otherwise, and let $MAX_{inf}([\lambda d.\ \Diamond\ \varphi\ (d)])(w^0) = d^*$.
$\Diamond\ \varphi(d^*)$ is true in w^0. Hence,
There is a world, w* in $MB_{w0}(\Diamond)$, s.t. $\varphi(d^*)(w^*) =1]$. Since φ is N-open
There is a degree d**, s.t. $[(\varphi(d^{**})(w^*) = 1\ \&\ \varphi(d^{**})$ asymmetrically entails $\varphi(d^*)]$,
Hence, $[\Diamond(\varphi\ (d^{**}))](w^0) =1$ and $d^*\neq MAX_{inf}([\lambda d.\ \Diamond\ \varphi\ (d)])(w^0)$.

(27) Consequence for universal modals: A universal modal can close an interval; hence even if φ is an N-open monotone property of degrees, the following should be acceptable

a. exh $([\lambda d.\Box\ \varphi\ (d)])(d')$

b. only $([\lambda d.\Box\ \varphi\ (d)])(d')$

c. wh_d $([\lambda d.\Box\ \varphi\ (d)])$

d. the $([\lambda d.\Box\ \varphi\ (d)])$

(28) Consequence for existential modals: An existential modal cannot close an interval. Hence, if φ is an N-open monotone property of degrees, then the following should be unacceptable

a. *exh $([\lambda d.\Diamond\varphi\ (d)])(d')$

b. *only $([\lambda d.\Diamond\varphi\ (d)])(d')$

c. *wh_d $([\lambda d.\Diamond\varphi\ (d)])$

d. *the $([\lambda d.\Diamond\ \varphi\ (d)])$

The fact that the CIM plays an explanatory role in accounting for the status of expressions in natural language can be taken as evidence that natural language has N-open properties, i.e. for The Intuitive Claim, (2)a.

The fact that the CIM seems to be at work in *all* degree constructions (even those that putatively make reference to cardinality) constitutes our argument for the universality of the UDM, i.e. for The Radical Claim, (2)b.

[3] In Fox (2006), I argued for a contradiction-free meaning of *exh* and *only* (weaker than that of Groenendijk and Stokhof). These arguments are not in conflict with F&H; in all relevant cases where exhaustification leads to contradictions under (24), it would be vacuous under my definition.

2. Evidence of a Missed Generalization

The same pattern observed in F&H is attested when a sentence p with an implicature ¬q is connected with q by disjunction. However, F&H's account does not extend to such disjunctions.[4]

THE BASIC EFFECT

(29) a. John has 3 or more children.
*Implicature: John has exactly 4 children.
b. John has very few children. *He only has 3 or MORE.

(30) John talked to Mary or Sue, or both.
*Implicature: John didn't talk to both Mary and Sue.

UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

(31) a. You're required to read 30 books or more.
Implicature: There is no degree greater than 30, d, s.t. you are required to read more than d books.
b. You're only required to read 30 books or MORE.
b'. You're only required to read 30 or MORE books.

(32) John is required to talk to Mary or Sue, both.
Implicature: John is not required to talk to both Mary and Sue.

EXISTENTIAL MODALS DO NOT

(33) a. You're allowed to smoke 30 cigarettes or more.
*Implicature: There is no degree greater than 30, d, s.t. you are allowed to smoke more than d cigarettes.
b. *You're only allowed to smoke more than 30_F cigarettes.

(34) John is allowed to talk to Mary or Sue, both.
*Implicature: John is not allowed to talk to both Mary and Sue.

[4] This pattern was pointed out to us by Gennaro Chierchia, Nathan Klindinst, Philippe Schlenker, and Bemjamin Spector.

3. Hurford's Generalization and Symmetric Altenatives[5]

Hurford's Generalization (HG): *A or B* is infelicitous when B entails A.[6]

(35) a. ??John is an American or a Californian.
b. ??I was born in France or Paris.

Hurford used this generalization to argue for a strong meaning for disjunction (ExOR):

(36) I will apply to Cornell or UMASS or to both.

One can extend this argument to other scalar items (cf. Gazdar 1979):

(37) a. I will read two books or three.
b. I will do some of the homework or all of it.

(36)' [Exh(I will apply to Cornell or_F UMASS)] or [I will apply to Cornell and UMASS]

$(p \nabla q) \vee (p \wedge q)$

Further argument for such a representation (developed together with G. Chierchia):

(38) It's either the case that each of the kids did some of the homework or that John did all of it and every other kid did just some of it

Important: In the first disjunct in (38) *Exh* would have to be below a universal quantifier for Hurford's constraint to be satisfied. This is a good result since the sentence is false/odd if there is a kid other than John who did all of the homework. The sentences thus provide independent evidence for Hurford's constraint.

Computing Implicatures for (36)':

ALT (36)' includes at least the following and excluding the stronger members leads to a contradiction:

$$(p \nabla q) \vee (p \wedge q) \quad \begin{matrix} (p \wedge q) \\ \\ (p \nabla q) \end{matrix}$$

[5] Proposed in Spector 2005, with an extension to part of the core paradigm mentioned above (see sections 4 and 5 below), as well as to sentences with *at least* (however with no reliance on Hurford's constraints). I made the proposal for the Hurford cases in my (2004) class on implicatures, but I argued for a totally different account for sentences involving *at least,* which is closely tied to superlative morphology (and to arguments by Nouwen 2004 that *at least* has a modal component).

[6] See also Simons, M. (2000). Issues in the Semantics and Pragmatics of Disjunction. New York and London, Garland Pub.

$\text{Exh}[(p\nabla q) \vee (p\wedge q)] = [(p\nabla q) \vee (p\wedge q)]\ \&\ \neg[(p\wedge q)]\ \&\ \neg[(p\nabla q)] = \varnothing$

UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

John is required to talk to Mary or Sue, both.
Implicature: John is not required to talk to both Mary and Sue.

$\Box\ [(p\nabla q) \vee (p\wedge q)]$

$\text{Exh}[(32)] = \Box[(p\nabla q) \vee (p\wedge q)]\ \&\ \neg\Box[(p\wedge q)]\ \&\ \neg\Box[(p\nabla q)] \neq \varnothing$

EXISTENTIAL MODALS DO NOT

(34) John is allowed to talk to Mary or Sue, both.
*Implicature: John is not allowed to talk to both Mary and Sue.

$\Diamond\ [(p\nabla q) \vee (p\wedge q)]$

$\text{Exh}[(34)] = \Diamond\ [(p\nabla q) \vee (p\wedge q)]\ \&\ \neg\Diamond\ [(p\wedge q)]\ \&\ \neg\Diamond\ [(p\nabla q)] = \varnothing$

4. Spector 2005

Extend the Hurford cases to comparatives. (See also Russell in press.)

(39) Alt(*more than 3*) = Alt(*4 or more*) = {*more than 4, exactly 4*}

Questions/Problems:

1. Why should we have the alternatives in (39)?
2. Would we still need density to account for the constraints on questions and definite descriptions? Are we missing a generalization here?
3. Empirical arguments against an identical account of *more than n* and *n+1 or more*

(34) a. You are allowed to smoke more than 5 cigarettes. I am luckier, I am allowed to smoke more than 6. More specifically, exactly 7.
b. You are allowed to smoke 6 or more cigarettes. I am luckier, I am allowed to smoke 7 or more. #More specifically, exactly 7.

The oddness of (34)b is the result of "free choice permission" which follows on many accounts from the shape of the alternatives (Kratzer and Shimoyama, Alonso-Ovalle, Fox, Klindinst).

(40) a. You're only required to read more than THREE books. I.e., you're not required to read more than 4. To be more precise, you're required to read exactly 4 (i.e, you're not allowed to read 5)

b. You're only required to read 4 or MORE books. I.e., you're not required to read more than 4. #To be more precise, you're required to read exactly 4 (i.e, you're not allowed to read 5)

Tentative Conclusion: The source of the similar behavior for the cases discussed by F&H, and for the Hurford cases is at least partially different. In all cases an exhaustive operator leads to a contradiction. The reason for the contradiction, however, could different. In the Hurford case, contradiction results from a symmetric set of alternatives. In the other case, it's the result of the UDM. Al least with questions and definite descriptions the UDM is (right now) the only account.

5. A Broader Generalization

Are we missing a real generalization.

Not necessarily!

Independent evidence for a broader generalization:

(41) Every student in this department has to have 3 professors on his committee.
a. #Mary is the only professor on John's committee.
b. Mary is the only professor that John is certain to have on his committee.
c. #Mary is the only professor that John might have on his committee.

a. #Which professor did John have on his committee?
b. Which professor is John certain to have on his committee?
c. ??Which professor might John have on his committee?

Neither Density nor Symmetry is involved in (41). Still, the same generalization seems to hold. Clearly, we need something broader.

Let p be a proposition and A a set of propositions (possibly infinite). We will say that p is merely Locally exhaustifiable given A [*MLE(p, A)*] if every proposition in A can be denied consistently with p, but the denial of all alternatives is inconsistent with p.

$\forall q \in A$[$p \& \neg q$ is consistent], but $p \& \bigcap\{\neg q: q \in A\}$ is inconsistent.

(42) [*MLE(p, A)*] $\Leftrightarrow$ $[\forall q \in A[(p \& \neg q) \neq \emptyset]$, but $[p \& \bigcap\{\neg q: q \in A\}] = \emptyset$.[7]

If MLE(p, A)
EX(A)(p) is inconsistent, where EX(A)(p) $\Leftrightarrow [p \ \& \ \forall q \in A[q \rightarrow (p \Rightarrow q)]$

Let p be a proposition and φ a property of type $\langle \alpha, st \rangle$, s.t.

[7] Taking into account the phenomena discussed in Fox (2006), we would need to strengthen the definition as follows: MLE(p,A) $\Leftrightarrow [\forall q \in A[(p \& \neg q) \neq \emptyset]$, but $[\neg \exists q \in A$, s.t. $q \in IE(p,A)]$.

(a) MLE(p, A); (b) $\forall x_\alpha$[MLE ($\varphi_{\langle\alpha, st\rangle,}$ (x), φ(ALT))], where φ(ALT) = {φ(y): y $\in D_\alpha$}

THE BASIC EFFECT

Fact #1

Only(A)(p) and Ex(A)(p) are both contradictory.

Prediction #1

a. We should expect *Only S* to be *unacceptable* if p is the denotation of S and A is the focus value of S.
b. We should expect *S* to *lack* a scalar implicature if p is its denotation and A is the set (of denotations) of the relevant alternatives.

c. We should expect *wh*φ to be an *unacceptable* question.
d. We should expect *the*φ to be an *unacceptable* definite description.

UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

Fact #2

EX($\Box$A)($\Box$p) is consistent (where $\Box$A = {$\Box$p: p$\in$A)})

Proof: by constructing the following modal base

{w: p(w)=1 & $\exists$q$\in$A(q(w)=0)}

Prediction #2

a. We should expect *Only* $\Box$*S* to be *acceptable* if $\Box$p is the denotation of S and $\Box$A is the focus value of S.
b. We should expect $\Box$*S to have a scalar implicature* if p is its denotation and $\Box$A is the set (of denotations) of the relevant alternatives.
c. We should expect *wh*$\lambda x \Box \varphi(x)$ to be an *acceptable* question.
d. We should expect *the*$\lambda x \Box \varphi(x)$ to be an *acceptable* question.

EXISTENTIAL MODALS DO NOT

Fact #3

EX($\Diamond$A)($\Diamond$p) is inconsistent (where $\Diamond$A = {$\Diamond$p: p$\in$A)})

Proof: Assume otherwise, and let MB be the modal base which satisfies EX($\Diamond$A)($\Diamond$p). Since $\Diamond$p is true, there is w$\in$MB, s.t. p(w)=1, w_p. For each q$\in$A, such that p $\not\subset$ q, q(w_p)=0 since [$\neg\Diamond$q](w)=1. But this means that all members of A could be denied consistently, contrary to assumption.

Prediction #3

a. We should expect *Only* $\Diamond S$ to be *unacceptable* if ◊p is the denotation of S and ◊A is the focus value of S.
b. We should expect $\Diamond S$ to *lack* a scalar implicatures if ◊p is its denotation and ◊A is the set (of denotations) of the relevant alternatives.

c. We should expect $wh \lambda x \Diamond \varphi(x)$ to be an *unacceptable* question.
d. We should expect $the \lambda x \Diamond \varphi(x)$ to be an *unacceptable* question.

6. Other Weak Islands

Some “weak islands” obey the generalization that we identified, others do not:

(43) a. *(I know) how you did not [behave t].
b. (I know) how you are not allowed to [behave t].
c. *(I know) how you are not required to [behave t].

(44) a. *(I know) why you did not [behave well t].
b. *(I know) why you are not allowed to [behave well t].
c. *(I know) how you are not required to [behave well t].

(45) Hypothesis: All Questions that obey negative islands with obviation by universal quantification (above negation or existential quantification below negation) share the following property: $\forall x_\alpha$[MLE ($\varphi_{\langle\alpha, st\rangle,}$(x), {$\varphi$(y): y$\in D_\alpha$}], where φ is the question property.

Speculation about (43) (*far from an account*): The domain of manners is not closed under sum formation (Szabolcsi and Zwarts).

Possible prediction (see Fodor 1972, Gajewski 2005, and Szabolcsi and Haddican 2004):

(46) a. He didn’t talk to John and Bill. (neg > &; & > neg)
b. He didn’t behave sensibly and kindly. (neg > &; (?)& > neg)

(47) a. He doesn’t believe that I talked to John and Bill.
(neg > &; & > neg)
b. He doesn’t believe that I behaved sensibly and kindly.
(neg > &; (*)& > neg)

(48) a. You will be punished twice because he doesn’t believe that you did the reading and the homework.
b. (#)You will be doubly fined because he doesn’t believe that you drove slowly and carefully.

If it could somehow be a logical fact that that there is more than one manner, s.t. John did not behave in that manner, (43) would fall under the generalization outlined in section 5.

7. Open Questions