Subject 24.242. HW Sample answers.

1. Write a register program that calculates (x + y).

   1. **If Register 2 is 0, go to 5.**
   2. **Subtract 1 from Register 2, unless it's 0.**
   3. **Add 1 to register 1.**
   4. **Go to 1.**
   5. **STOP.**

2. Show that a set is Δ if and only if its characteristic function is Σ. (The *characteristic function* χ_S of a set S is given by stipulating that $\chi_S(n) = 1$ if $n \in S$, and it's equal to 0 if $n \notin S$.

   (⇒) If the set S is Δ, then there are bounded formulas $\varphi(x,y)$ and $\psi(x,y)$ such that $S = \{x: (\exists y)\varphi(x,y)$, and its complement is $\{x: (\exists y)\psi(x,y)\}$. Then χ_S is equal to $\{<x,z>: (\exists y)((\varphi([x],y) \wedge z = s0) \vee (\psi(x,y) \wedge z = 0))$.

   (⇐) Suppose χ_S is Δ; say it's $\{<x,y>: (\exists z)\theta([x],[y],z)$. The S is equal to $\{x: (\exists z)\theta([x],s0,z)$, and its complement is $\{x: (\exists z)\theta([x],0,z)\}$.