Probability, Subjective and Objective

1 Two Kinds of Probability

Subjective Probability A person's subjective probability in p is the degree to which she is confident in p.

Example: Jones's subjective probability that it'll rain tomorrow is 0.3 because she is 30% confident that it'll rain tomorrow.

Objective Probability The objective probability of an event is meant to be a feature of the world that does not depend on the beliefs of any particular subject.

Example: the objective probability that a particle of ^{256}Sg will decay in the next 8.9 seconds is 50%.

2 How are they related?

The Objective-Subjective Connection The objective probability of A at time t is the subjective probability that a perfectly rational agent would assign to A, if she had perfect information about the world at times $\leq t$ and no information about the world at times $> t$.[1]

3 Subjective Probability

A **credence function** for subject S is a function that assigns to each proposition a real number between 0 and 1, representing S's degree of confindence in that proposition

What does it take for a credence function to be rational?

1. internal coherence;

[1] Here I am tacitly presupposing that a perfectly rational agent is always certain about the objective probabilities at t, given full information about how the world is before t. So, in particular, for each complete history of the world up to t, H_t, there is a specification P_t of the objective probabilities at t such that the agent treats H_t and H_tP_t as equivalent. (This assumption is potentially controversial but adds simplicity to our discussion.)

2. update by conditionalization;
3. Bayes' Law;
4. the Principle of Indifference.

3.1 Internal Coherence

For a credence function to be **internally coherent** is for it to constitute a probability function.

A **probability function**, $p(\ldots)$, is an assignment of real numbers between 0 and 1 to propositions that satisfies the following two coherence conditions:

Necessity $p(A) = 1$ whenever A is a necessary truth

Additivity $p(A \text{ or } B) = p(A) + p(B)$ whenever A and B are incompatible propositions

3.2 Update by Conditionalization

If S is rational, she will update here credences as follows upon learning that B:

$$p^{new}(A) = p^{old}(A|B)$$

where p^{old} is the function describing S's credences before she learned that B, and p^{new} is the function describing her credences after she learned that B.

3.3 Bayes' Law

$p(AB) = p(A) \cdot p(B|A)$

3.4 The Principle of Indifference

Here's what we'd *like* to have in place:

Principle of Indifference Consider a set of propositions and suppose one knows that exactly one of them is true. Suppose, moreover, that one has no more reason to believe any one of them than any other. Then, insofar as one is rational, one should assign equal credence to each proposition in the set.

Unfortunately, this principle leads to inconsistency as stated. For instance:

> A factory produces cubes with a side-length $l \leq 1$. What is the probability that $l \in (0, \frac{1}{2}]$?
>
> *Argument 1* (length):
>
> - There is just as much reason to think that $l \in (0, \frac{1}{2}]$ as there is to think that $l \in (\frac{1}{2}, 1]$.
> - By the Principle of Indifference, $p(l \in (0, \frac{1}{2}]) = p(l \in (\frac{1}{2}, 1])$.
> - So $p(l \in (0, \frac{1}{2}]) = \frac{1}{2}$.
>
> *Argument 2* (area):
>
> - There is just as much reason to think that $a \in (0, \frac{1}{2}]$ as there is to think that $a \in (\frac{1}{2}, 1]$.
> - By the Principle of Indifference, $p(a \in (0, \frac{1}{2}]) = p(a \in (\frac{1}{2}, 1])$.
> - So $p(a \in (0, \frac{1}{2}]) = \frac{1}{2}$.
>
> But wait! $l \in (0, \frac{1}{2}] \leftrightarrow a \in (0, \frac{1}{4}]$.

4 Objective Probability

By the Objective-Subjective Connection, our conclusions about rational subjective probability deliver tell us that the objective probabilities:

1. constitute a probability function;
2. update by an analogue of conditionalization;
3. satisfy Bayes' Law;
4. [satisfy a Principle of Indifference?].

5 Yes, but what *is* objective probability?

5.1 Frequentism

What is it for the objective probability of a coin's landing Heads[2] to be 50%?

- According to **frequentism**, it is for 50% of coin tosses to land Heads.
- According to **hypothetical frequentism**, it is for the following subjunctive conditional to be true: if sufficiently many coin tosses took place, 50% of them would land Heads.

5.2 The Law of Large Numbers

Upon reflection, frequentism is obviously incorrect. What is true is this:

> If the coin were tossed a sufficiently large number of times, then it would *with very high probability* land Heads approximately 50% of the time.

More generally and precisely:

Law of Large Numbers Suppose that events of type T have a probability of p of resulting in outcome O. Then, for any real numbers ϵ and δ larger than zero, there is an N such that the following will be true with a probability of at least $1 - \epsilon$:

> If $M>N$ events of type T occur, the proportion of them that result in outcome O will be $p \pm \delta$.

5.3 Rationalism

- According to **rationalism**, there is nothing more to objective probability than the Objective-Subjective Connection.
- A **localist** agrees with rationalism and adds that the the objective probabilities are only well-defined in certain special circumstances; in particular, circumstances in which there is an unproblematic way of deploying a Principle of Indifference.

[2] Think of a "coin toss" as the result of observing a particle of ^{256}Sg for 8.9 seconds. If the particle decays within that period, our "coin" is said to have landed Heads; otherwise it is said to have landed Tails.