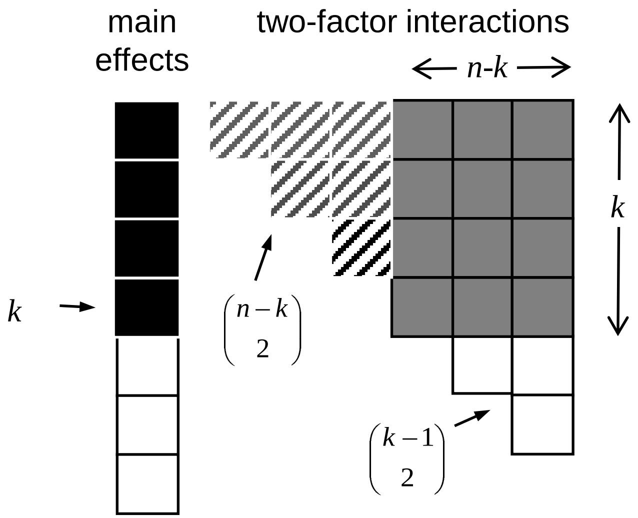
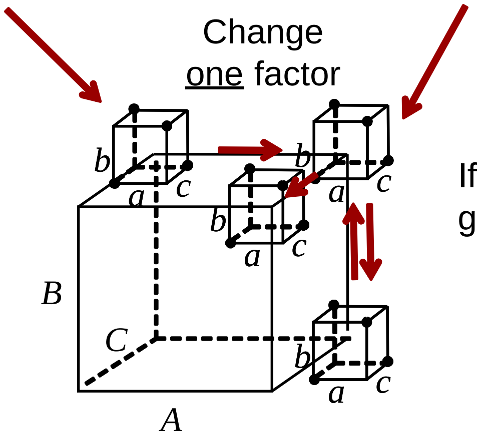
ESD.77 – Multidisciplinary System Design Optimization

Robust Design





Run a resolution III on noise factors

Change <u>one</u> factor

Again, run a resolution III on noise factors. If there is an improvement, in transmitted variance, retain the change

If the response gets worse, go back to the previous state

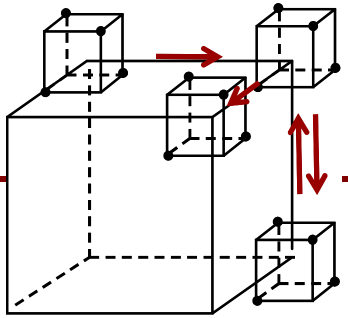
Stop after you've changed every factor once

Dan Frey

Associate Professor of Mechanical Engineering and Engineering Systems

Research Overview

Concept Design

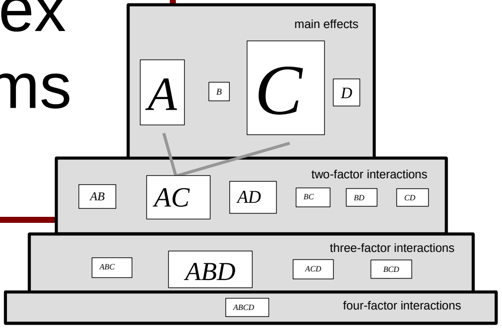


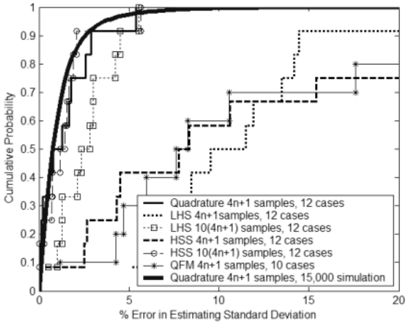
Outreach to K-12

Adaptive Experimentation and Robust Design

$$\Pr\left(\beta_{12}x_1^*x_2^* > 0 \middle| \beta_{12} > \beta_{ij}\right) \cong \frac{1}{\pi}\binom{n}{2}\int_0^{\infty}\int_{-x_2}^{\infty}\frac{\left[erf\left(\frac{1}{\sqrt{2}}\frac{x_1}{\sigma_{INT}}\right)\right]^{\binom{n}{2}-1} e^{\frac{-x_1^2}{2\sigma_{INT}^2}+\frac{-x_2^2}{2\left(\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2\right)}}}{\sigma_{INT}\sqrt{\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2}}dx_2dx_1$$

Complex Systems





Methodology Validation

Outline

- Introduction
  - History
  - Motivation
- Recent research
  - Adaptive experimentation
  - Robust design

“An experiment is simply a question put to nature … The chief requirement is simplicity: only one question should be asked at a time.”

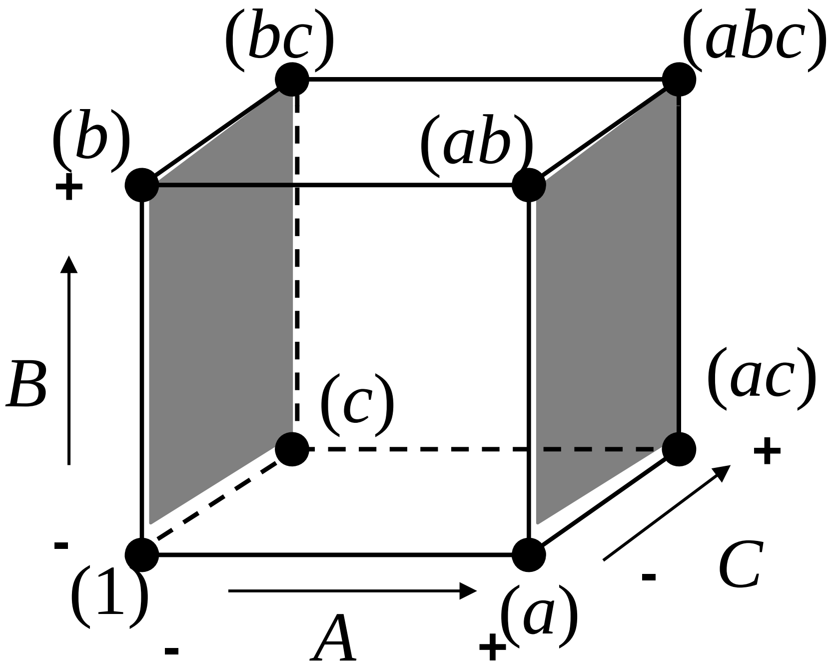
Russell, E. J., 1926, “Field experiments: How they are made and what they are,” *Journal of the Ministry of Agriculture* **32**:989-1001.

“To call in the statistician after the experiment is done may be no more than asking him to perform a post-mortem examination: he may be able to say what the experiment died of.”

- Fisher, R. A., Indian Statistical Congress, Sankhya, 1938.

Estimation of Factor Effects

Say the independent experimental error of observations (a), (ab), et cetera is σ_{ε}.



We define the main effect estimate A to be

$$A \equiv \frac{1}{4}\left[(abc)+(ab)+(ac)+(a)-(b)-(c)-(bc)-(1)\right]$$

The standard deviation of the estimate is

$$\sigma_A = \frac{1}{4}\sqrt{8}\sigma_{\varepsilon} = \frac{1}{2}\sqrt{2}\sigma_{\varepsilon}$$

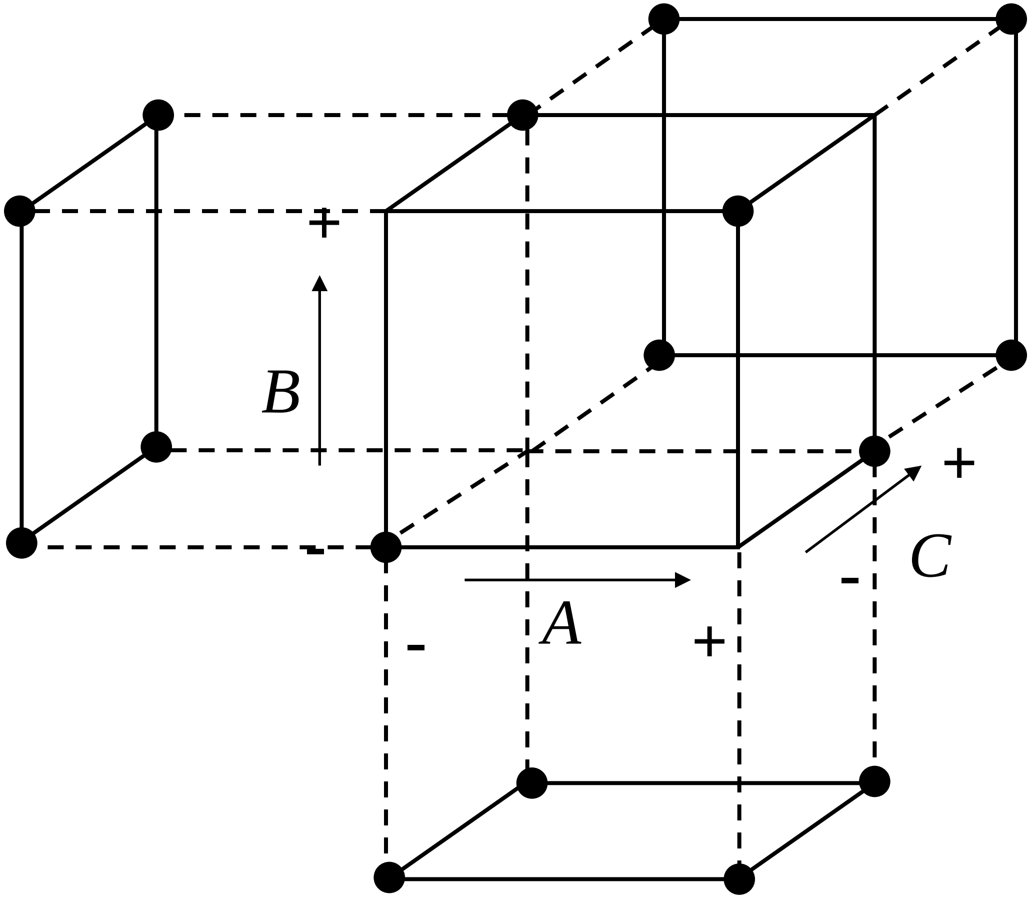
A factor of two improvement in efficiency as compared to “single question methods”

Fractional Factorial Experiments

“It will sometimes be advantageous deliberately to sacrifice all possibility of obtaining information on some points, these being confidently believed to be unimportant … These comparisons to be sacrificed will be deliberately confounded with certain elements of the soil heterogeneity… Some additional care should, however, be taken…”

Fisher, R. A., 1926, “The Arrangement of Field Experiments,” *Journal of the Ministry of Agriculture of Great Britain*, 33: 503-513.

Fractional Factorial Experiments



$$2_{III}^{3-1}$$

Fractional Factorial Experiments

Trial	A	B	C	D	E	F	G	FG=-A
1	-1	-1	-1	-1	-1	-1	-1	+1
2	-1	-1	-1	+1	+1	+1	+1	+1
3	-1	+1	+1	-1	-1	+1	+1	+1
4	-1	+1	+1	+1	+1	-1	-1	+1
5	+1	-1	+1	-1	+1	-1	+1	-1
6	+1	-1	+1	+1	-1	+1	-1	-1
7	+1	+1	-1	-1	+1	+1	-1	-1
8	+1	+1	-1	+1	-1	-1	+1	-1

2^{7-4} Design (aka “orthogonal array”)

Every factor is at each level an equal number of times (balance).

High replication numbers provide precision in effect estimation.

Resolution III.

Robust Parameter Design

Robust Parameter Design … is a statistical / engineering methodology that aims at reducing the performance variation of a system (i.e. a product or process) by choosing the setting of its control factors to make it less sensitive to noise variation.

Wu, C. F. J. and M. Hamada, 2000, *Experiments: Planning, Analysis, and Parameter Design Optimization*, John Wiley & Sons, NY.

Cross (or Product) Arrays

Noise Factors 2^{3-1}_{III}

Control Factors

2^{7-4}_{III}

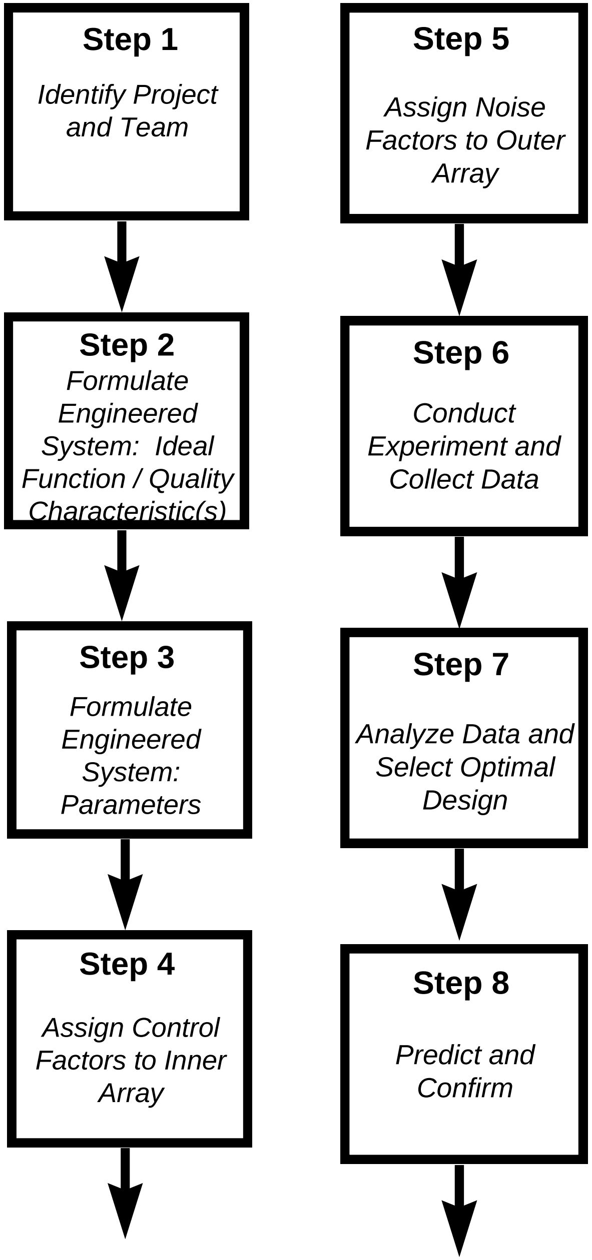
								a	-1	-1	+1	+1
								b	-1	+1	-1	+1
								c	-1	+1	+1	-1
	A	**B**	**C**	**D**	**E**	**F**	**G**					
1	-1	-1	-1	-1	-1	-1	-1					
2	-1	-1	-1	+1	+1	+1	+1					
3	-1	+1	+1	-1	-1	+1	+1					
4	-1	+1	+1	+1	+1	-1	-1					
5	+1	-1	+1	-1	+1	-1	+1					
6	+1	-1	+1	+1	-1	+1	-1					
7	+1	+1	-1	-1	+1	+1	-1					
8	+1	+1	-1	+1	-1	-1	+1					

$2^{7-4}_{III} \times 2^{3-1}_{III}$

Taguchi, G., 1976, *System of Experimental Design*.

Step 4 Summary:

- Determine control factor levels
- Calculate the DOF
- Determine if there are any interactions
- Select the appropriate orthogonal array



Step 1
Identify Project and Team

Step 2
Formulate Engineered System: Ideal Function / Quality Characteristic(s)

Step 3
Formulate Engineered System: Parameters

Step 4
Assign Control Factors to Inner Array

Step 5
Assign Noise Factors to Outer Array

Step 6
Conduct Experiment and Collect Data

Step 7
Analyze Data and Select Optimal Design

Step 8
Predict and Confirm

Majority View on "One at a Time"

One way of thinking of the great advances of the science of experimentation in this century is as **the final demise of the "one factor at a time" method**, although it should be said that there are still organizations which have never heard of factorial experimentation and use up many man hours wandering a crooked path.

Logothetis, N., and Wynn, H.P., 1994, *Quality Through Design: Experimental Design, Off-line Quality Control and Taguchi's Contributions*, Clarendon Press, Oxford.

Minority Views on "One at a Time"

"…the factorial design has certain deficiencies … It devotes observations to exploring regions that may be of no interest…These deficiencies … suggest that an efficient design for the present purpose ought to be sequential; that is, ought to adjust the experimental program at each stage in light of the results of prior stages."

Friedman, Milton, and L. J. Savage, 1947, "Planning Experiments Seeking Maxima", in *Techniques of Statistical Analysis*, pp. 365-372.

"Some scientists do their experimental work in single steps. They hope to learn something from each run … they see and react to data more rapidly …If he has in fact found out a good deal by his methods, it must be true that the effects are at least three or four times his average random error per trial."

Cuthbert Daniel, 1973, "One-at-a-Time Plans", *Journal of the American Statistical Association*, vol. 68, no. 342, pp. 353-360.

My Observations of Industry

- Farming equipent company has reliability problems
- Large blocks of robustness experiments had been planned at outset of the design work
- More than 50% were not finished
- Reasons given
  - Unforseen changes
  - Resource pressure
  - Satisficing

“Well, in the third experiment, we found a solution that met all our needs, so we cancelled the rest of the experiments and moved on to other tasks…”

More Observations of Industry

- Time for design (concept to market) is going down
- Fewer physical experiments are being conducted
- Greater reliance on computation / CAE
- Poor answers in computer modeling are common
  - Right model → Inaccurate answer
  - Right model → No answer whatsoever
  - Not-so right model → Inaccurate answer
    - Unmodeled effects
    - Bugs in coding the model

Human Subjects Experiment

- Hypothesis: Engineers using a flawed simulation are more likely to detect the flaw while using OFAT than while using a more complex design.
- Method: Between-subjects experiment with human subjects (engineers) performing parameter design with OFAT vs. designed experiment.

Treatment: Design Space Sampling Method

Trial	A	B	C	D	E	F	G
1	-	-	-	-	-	-	-
2	+	-	-	-	-	-	-
3		+	-	-	-	-	-
4			+	-	-	-	-
5				+	-	-	-
6					+	-	-
7						+	-
8							+

filled in as required to adapt

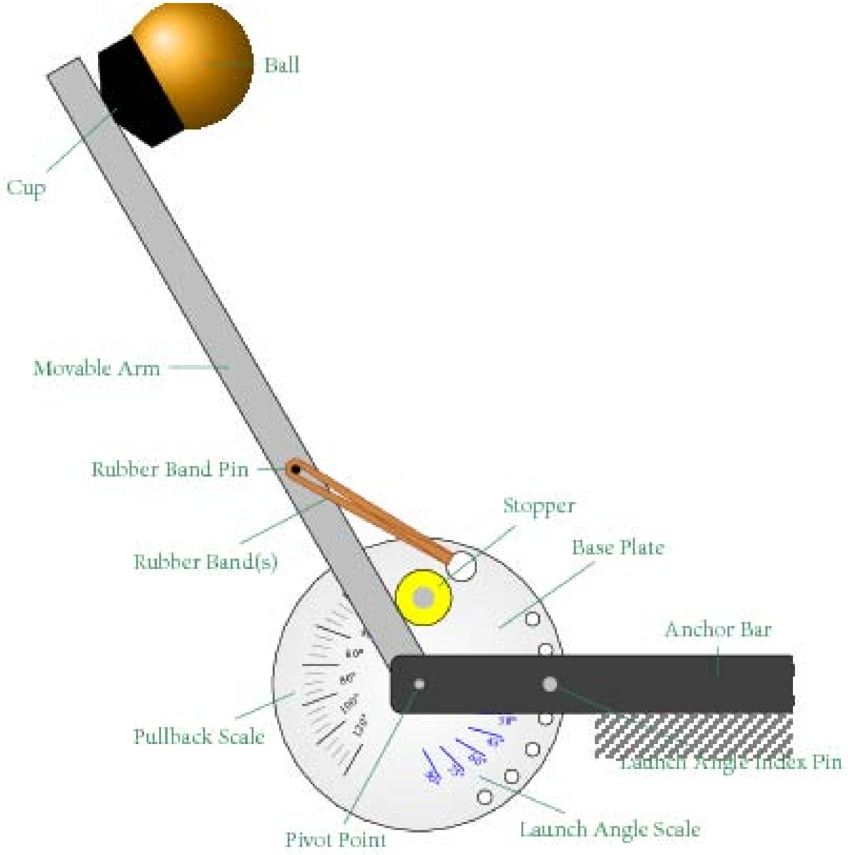
- Adaptive OFAT
  - One factor changes in each trial

Trial	A	B	C	D	E	F	G
1	-	-	-	-	-	-	-
2	+	-	+	-	-	+	+
3	+	+	+	-	+	-	-
4	-	+	-	-	+	+	+
5	+	-	-	+	+	-	+
6	-	-	+	+	+	+	-
7	-	+	+	+	-	-	+
8	+	+	-	+	-	+	-

- Plackett-Burman L8
  - Four factors change between any two trials

Using a 2^7 system avoids possible confounding factor of number of trials.
Increasing number of factors likely means increasing discrepancy in detection.
Larger effect sizes require fewer test subjects for given Type I and II errors.

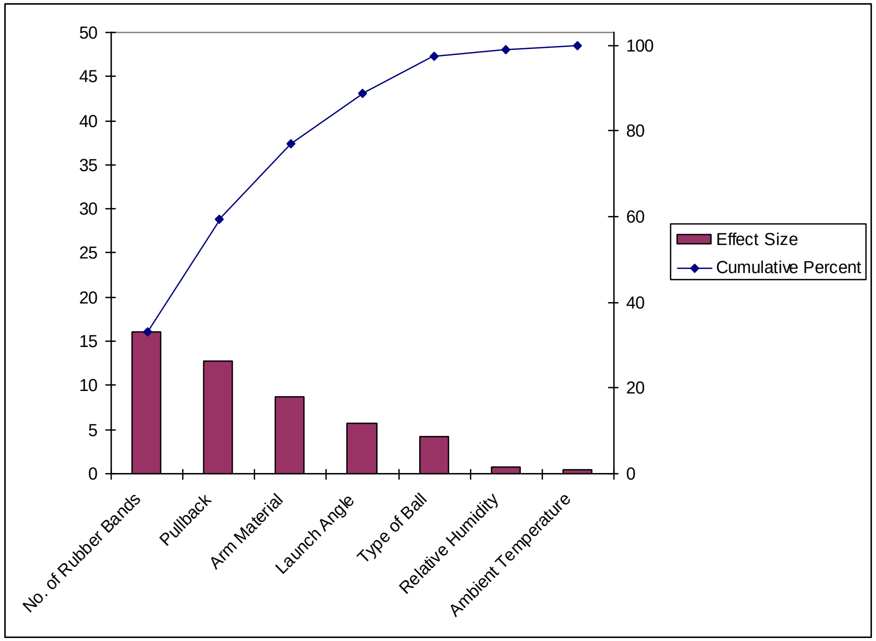
Parameter Design of Catapult to Hit Target



- Modeled on XPult^{TM}
- Commonly used in DOE demonstrations
- Extended to 2^7 system by introducing
  - Arm material
  - Air relative humidity
  - Air temperature

Control Factors and Settings

Control Factor	Nominal Setting	Alternate Setting
Relative Humidity	25%	75%
Pullback	30 degrees	40 degrees
Type of Ball	Orange (Large-ball TT)	White (regulation TT)
Arm Material	Magnesium	Aluminum
Launch Angle	60 degrees	45 degrees
Rubber Bands	3	2
Ambient Temperature	72 F	32 F



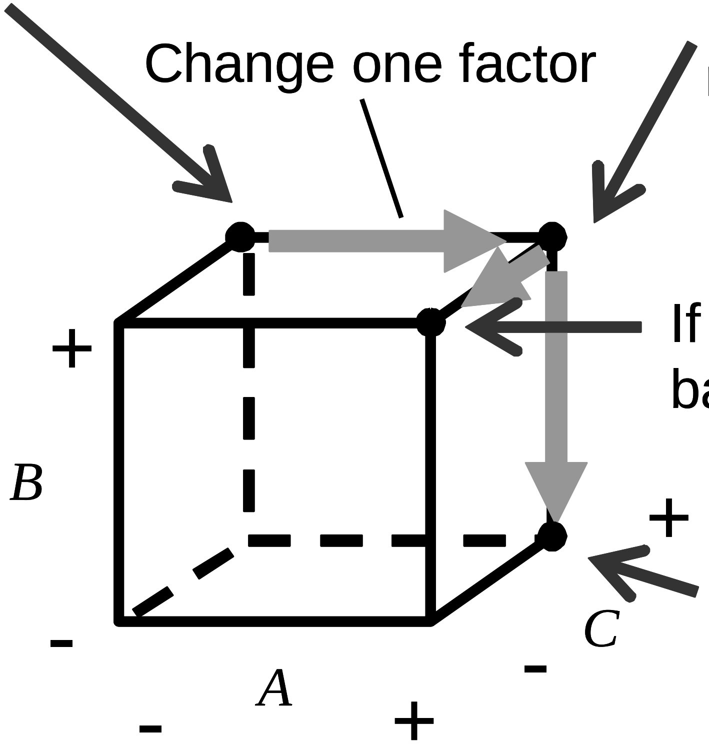
- Control factor tied directly to simulation mistake
- Arm material selected for its moderate effect size
- Computer simulation equations are “correct”, but intentional mistake is that arm material properties are reversed
- Control factor ordering is not random, to prevent variance due to learning effect
- “Bad” control factor placed in 4th column in both designs

Results of Human Subjects Experiment

- Pilot with N = 8
- Study with N = 55 (1 withdrawal)
- External validity high
  - 50 full time engineers and 5 engineering students
  - experience ranged from 6 mo. to 40+ yr.
- Outcome measured by subject debriefing at end

Method	Detected	Not detected	Detection Rate (95% CI)
OFAT	14	13	(0.3195,0.7133)
PBL8	1	26	(0.0009,0.1897)

Adaptive OFAT Experimentation



Do an experiment

Change one factor

If there is an improvement, retain the change

If the response gets worse, go back to the previous state

Stop after you've changed every factor

Frey, D. D., F. Engelhardt, and E. Greitzer, 2003, "A Role for One Factor at a Time Experimentation in Parameter Design", *Research in Engineering Design* 14(2): 65-74.

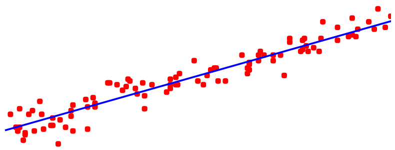
Empirical Evaluation of Adaptive OFAT Experimentation

- Meta-analysis of 66 responses from published, full factorial data sets
- When experimental error is <25% of the combined factor effects OR interactions are >25% of the combined factor effects, adaptive OFAT provides more improvement on average than fractional factorial DOE.

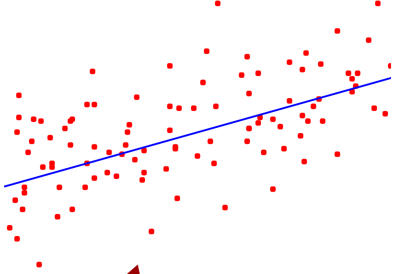
Frey, D. D., F. Engelhardt, and E. Greitzer, 2003, "A Role for One Factor at a Time Experimentation in Parameter Design", *Research in Engineering Design* 14(2): 65-74.

Detailed Results

$\sigma = 0.1\sqrt{MS_{FE}}$



$\sigma = 0.4\sqrt{MS_{FE}}$



OFAT/FF

Gray if OFAT>FF

		Strength of Experimental Error										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Interaction Strength	**Mild**	100/99	99/98	98/98	96/96	94/94	89/92	86/88	81/86	77/82	73/79	69/75
	Moderate	96/90	95/90	93/89	90/88	86/86	83/84	80/81	76/81	72/77	69/74	64/70
	Strong	86/67	85/64	82/62	79/63	77/63	72/64	71/63	67/61	64/58	62/55	56/50
	Dominant	80/39	79/36	77/34	75/37	72/37	70/35	69/35	64/34	63/31	61/35	59/35

A Mathematical Model of Adaptive OFAT

initial observation → $O_0 = y\left(\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_n\right)$

observation with first factor toggled → $O_1 = y\left(-\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_n\right)$

first factor set → $x_1^* = \tilde{x}_1 sign\left\{O_0 - O_1\right\}$

repeat for all remaining factors

$$\text{for } i = 2 \ldots n$$

$$O_i = y\left(x_1^*, \ldots x_{i-1}^*, -\tilde{x}_i, \tilde{x}_{i+1}, \ldots \tilde{x}_n\right)$$

$$x_i^* = \tilde{x}_i sign\left\{\max\left(O_0, O_1, \ldots O_{i-1}\right) - O_i\right\}$$

process ends after *n*+1 observations with $E\left[y\left(x_1^*, x_2^*, \ldots x_n^*\right)\right]$

Frey, D. D., and H. Wang, 2006, “Adaptive One-Factor-at-a-Time Experimentation and Expected Value of Improvement”, *Technometrics* 48(3):418-31.

A Mathematical Model of a Population of Engineering Systems

$$y(x_1, x_2, \ldots x_n) = \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{ij} x_i x_j + \varepsilon_k$$

system response

$$\beta_i \sim \mathrm{N}\left(0, \sigma_{ME}{}^2\right)$$

main effects

$$\beta_{ij} \sim \mathrm{N}\left(0, \sigma_{INT}{}^2\right)$$

two-factor interactions

$$\varepsilon_k \sim \mathrm{N}\left(0, \sigma_{\varepsilon}{}^2\right)$$

experimental error

$y_{\max} \equiv$ the largest response within the space of discrete, coded, two-level factors $x_i \in \{-1, +1\}$

Model adapted from Chipman, H., M. Hamada, and C. F. J. Wu, 2001, "A Bayesian Variable Selection Approach for Analyzing Designed Experiments with Complex Aliasing", *Technometrics* 39(4)372-381.

Probability of Exploiting an Effect

- The i^{th} main effect is said to be "exploited" if

$$\boxed{\beta_i x_i^* > 0}$$

- The two-factor interaction between the i^{th} and j^{th} factors is said to be "exploited" if $\boxed{\beta_{ij} x_i^* x_j^* > 0}$

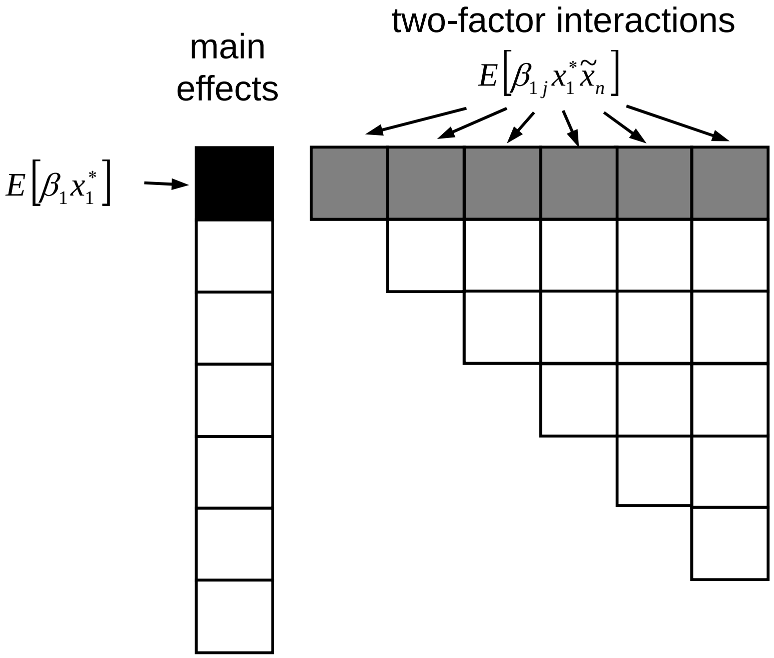
- The probabilities and conditional probabilities of exploiting effects provide insight into the mechanisms by which a method provides improvements

The Expected Value of the Response after the *First* Step

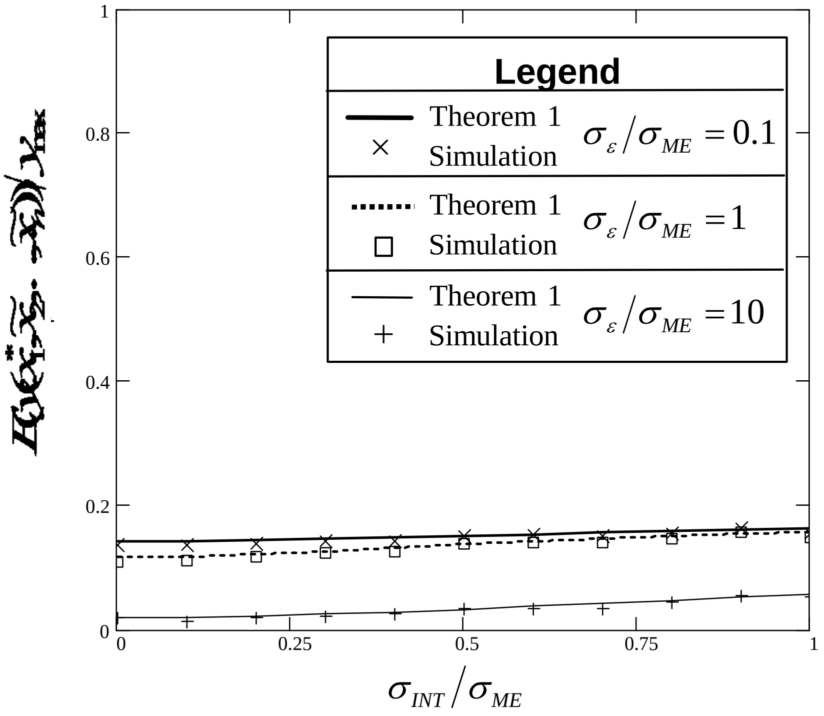
$$E(y(x_1^*,\tilde{x}_2,\ldots,\tilde{x}_n)) = E\left[\beta_1 x_1^*\right] + (n-1)E\left[\beta_{1j} x_1^* \tilde{x}_j\right]$$

$$E\left[\beta_1 x_1^*\right] = \sqrt{\frac{2}{\pi}}\,\frac{\sigma_{ME}^2}{\sqrt{\sigma_{ME}^2 + (n-1)\sigma_{INT}^2 + \frac{1}{2}\sigma_\varepsilon^2}}$$

$$E\left[\beta_{1j} x_1^* \tilde{x}_n\right] = \sqrt{\frac{2}{\pi}}\,\frac{\sigma_{INT}^2}{\sqrt{\sigma_{ME}^2 + (n-1)\sigma_{INT}^2 + \frac{1}{2}\sigma_\varepsilon^2}}$$



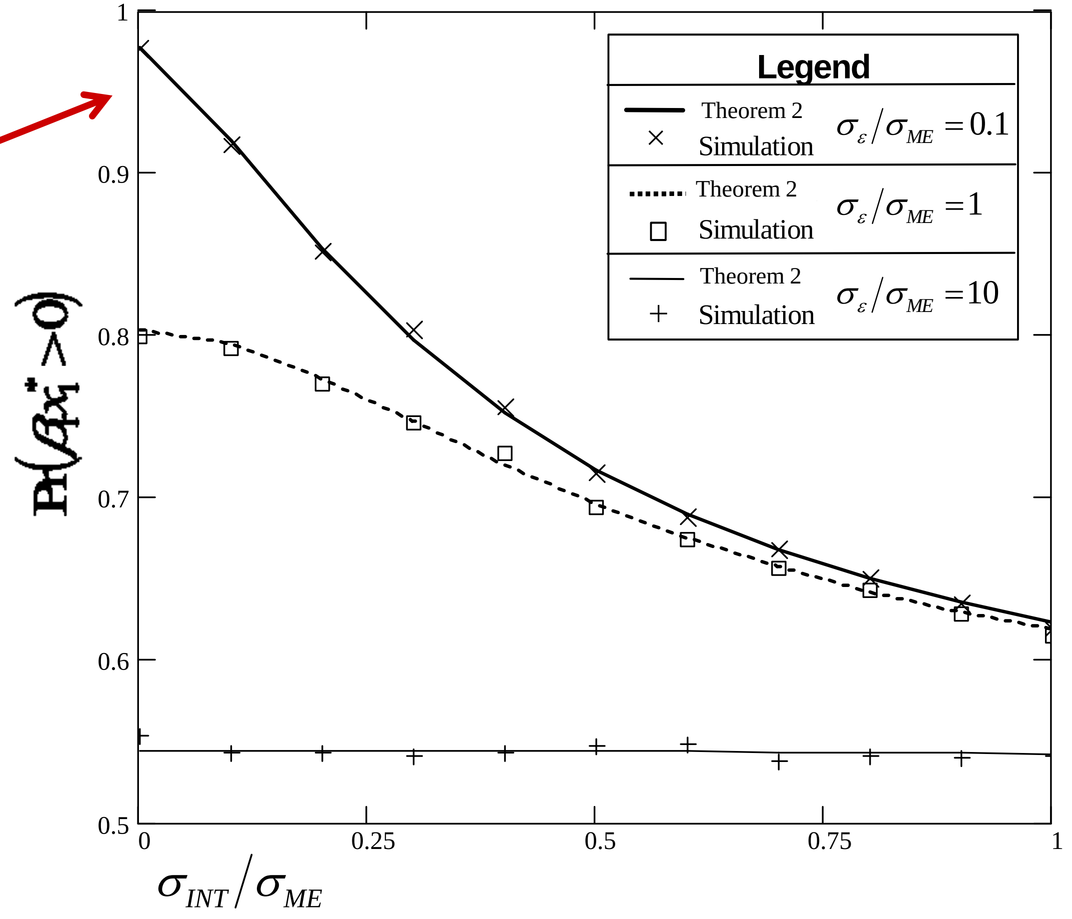
n=7



Probability of Exploiting the First Main Effect

$$\Pr\left(\beta_1 x_1^* > 0\right) = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}\frac{\sigma_{ME}}{\sqrt{\sigma_{ME}{}^2 + (n-1)\sigma_{INT}{}^2 + \frac{1}{2}\sigma_\varepsilon{}^2}}$$

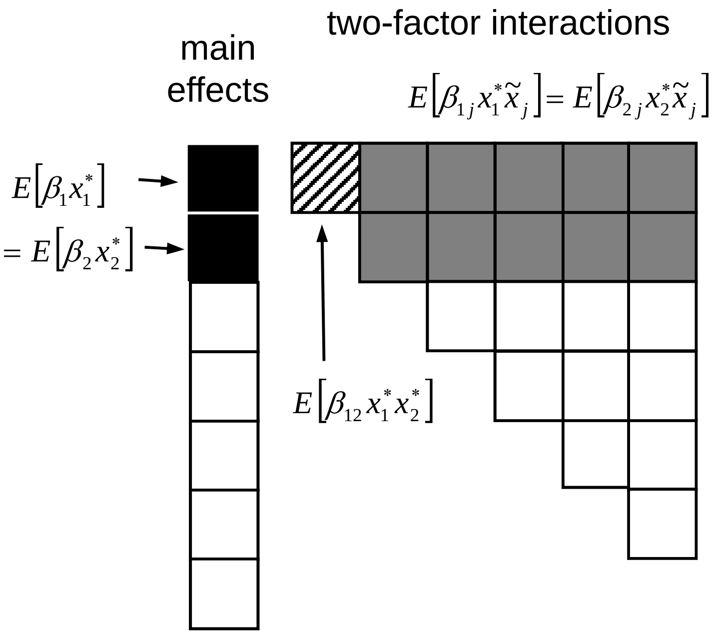
If interactions are small and error is not too large, OFAT will tend to exploit main effects

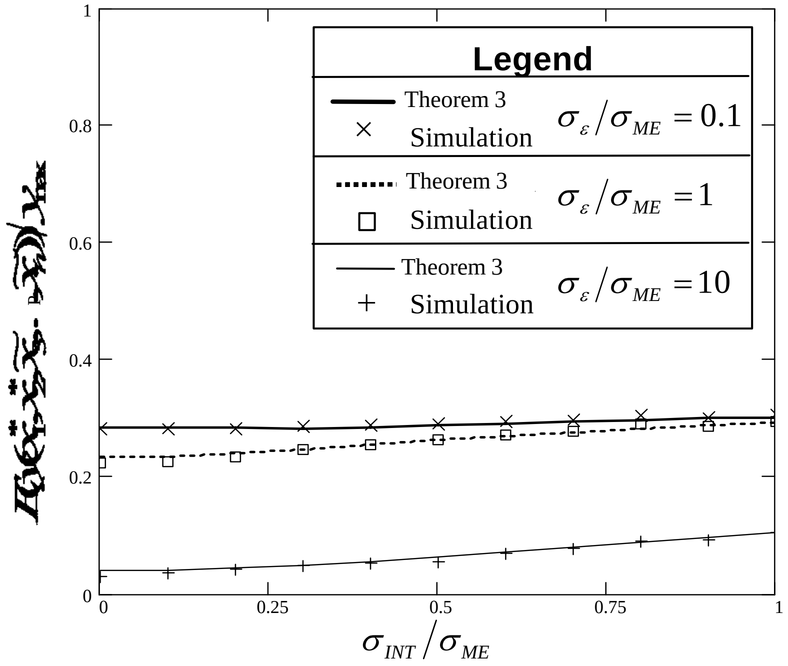


The Expected Value of the Response After the *Second* Step

$$E(y(x_1^*,x_2^*,\tilde{x}_3,\ldots,\tilde{x}_n)) = 2E\left[\beta_1 x_1^*\right] + 2(n-2)E\left[\beta_{1j} x_1^*\right] + E\left[\beta_{12} x_1^* x_2^*\right]$$

$$E\left[\beta_{12} x_1^* x_2^*\right] = \sqrt{\frac{2}{\pi}}\left[\frac{\sigma_{INT}^2}{\sqrt{\sigma_{ME}^2 + (n-1)\sigma_{INT}^2 + \frac{\sigma_\varepsilon^2}{2}}}\right]$$



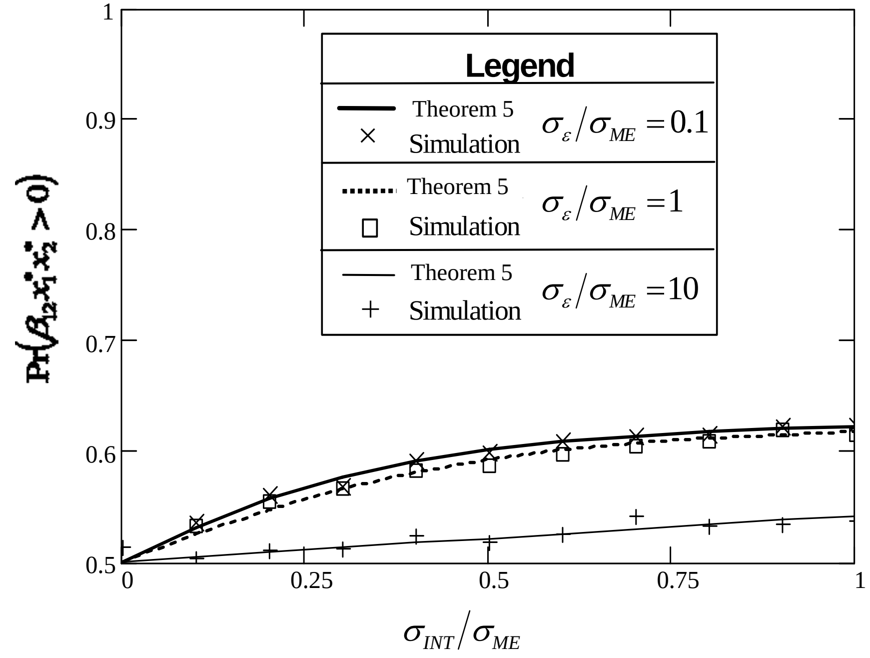


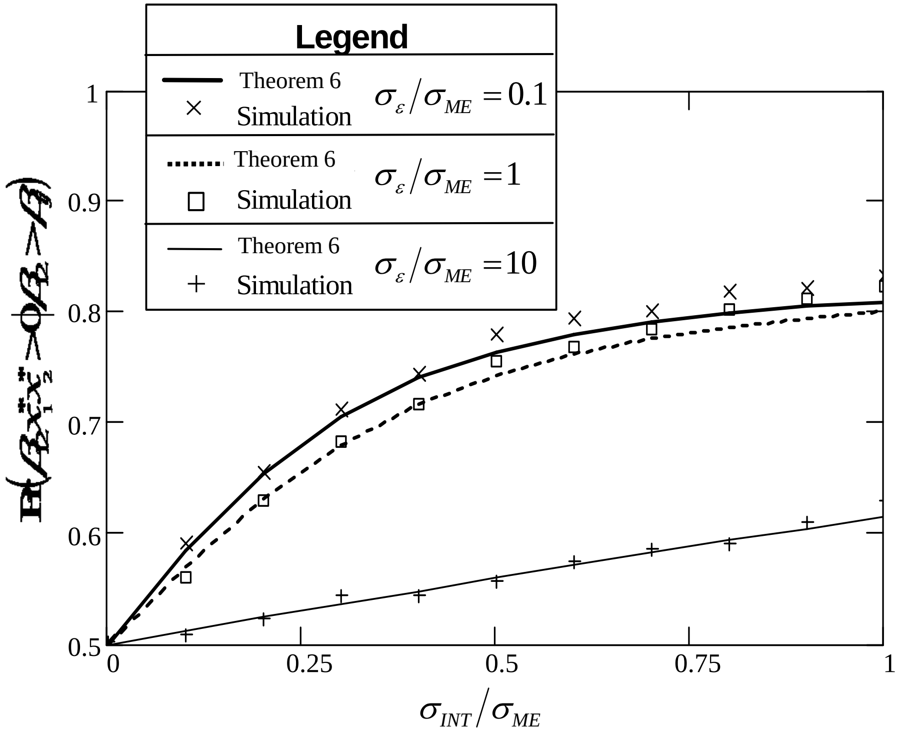
Probability of Exploiting the First Interaction

$$\Pr\left(\beta_{12}x_1^*x_2^*>0\right)=\frac{1}{2}+\frac{1}{\pi}\tan^{-1}\frac{\sigma_{INT}}{\sqrt{\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2}}$$

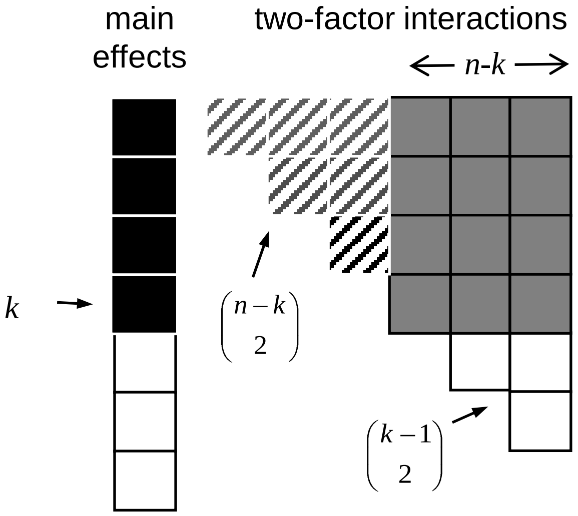
$$\Pr\left(\beta_{12}x_1^*x_2^*>0\middle|\beta_{12}>\beta_{ij}\right)>$$

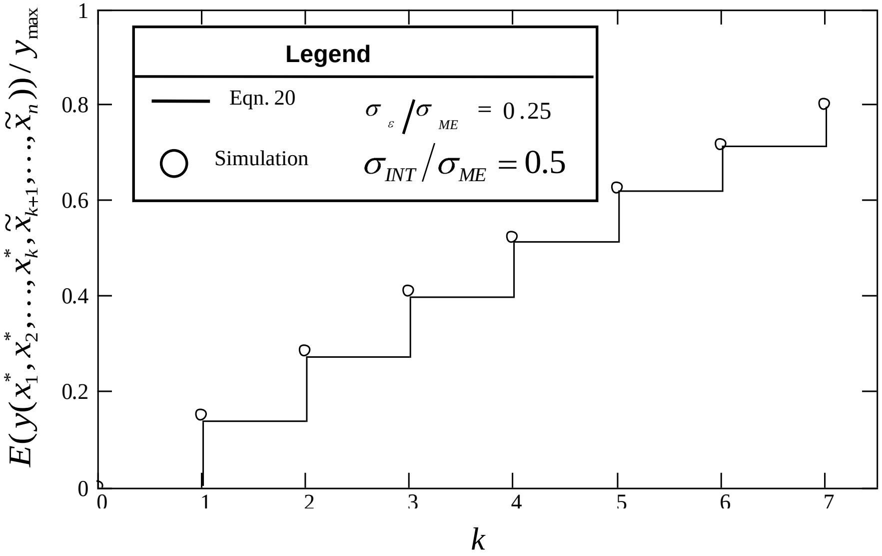
$$\frac{1}{\pi}\binom{n}{2}\int_0^\infty\int_{-x_2}^\infty\frac{\left[erf\left(\frac{1}{\sqrt{2}}\frac{x_1}{\sigma_{INT}}\right)\right]^{\binom{n}{2}-1}e^{\frac{-x_1^2}{2\sigma_{INT}^2}+\frac{-x_2^2}{2\left(\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2\right)}}}{\sigma_{INT}\sqrt{\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2}}dx_2dx_1$$





And it Continues

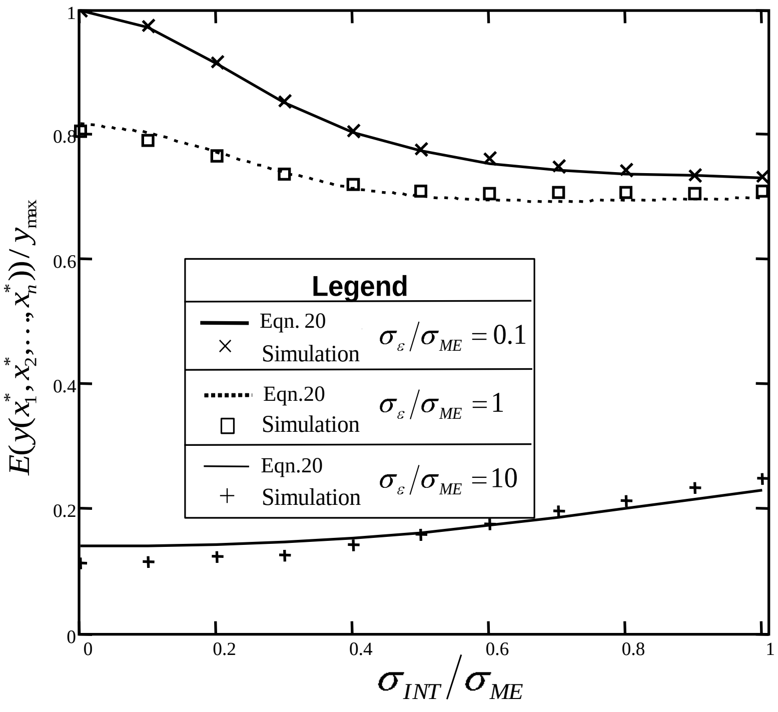




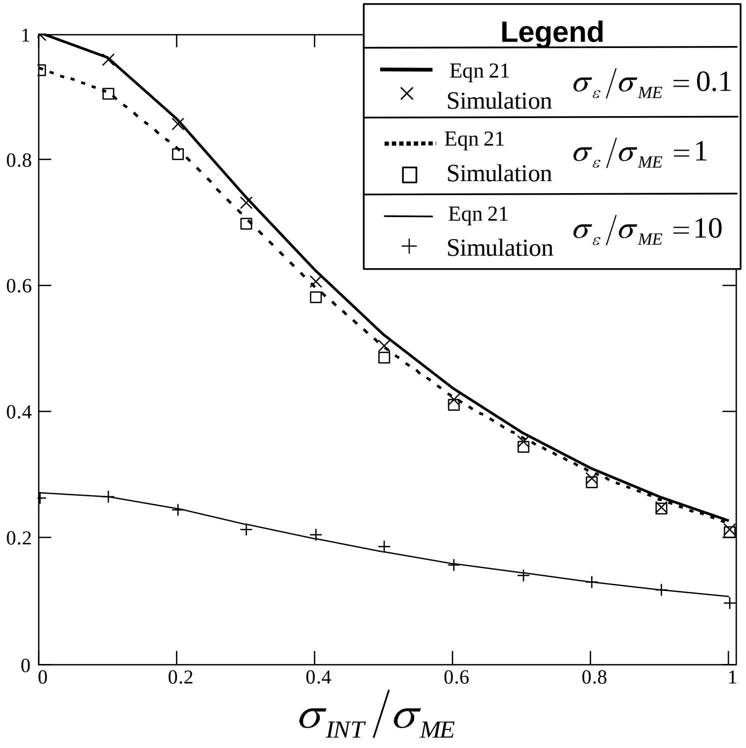
$$\Pr\ \beta_{ij}x_i^*x_j^* > 0\ \geq \Pr\ \beta_{12}x_1^*x_2^* > 0$$

We can prove that the probability of exploiting interactions is sustained. Further we can now prove exploitation probability is a function of *j* only and increases monotonically.

Final Outcome

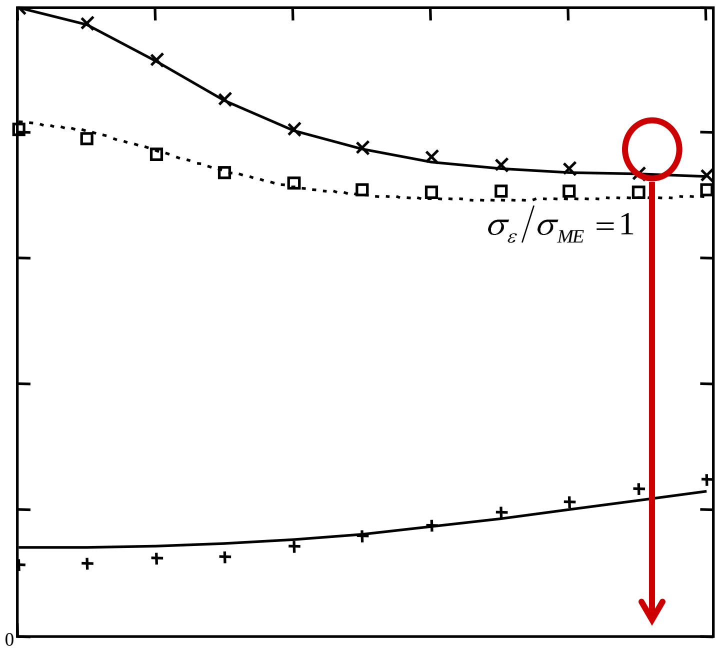


Adaptive OFAT



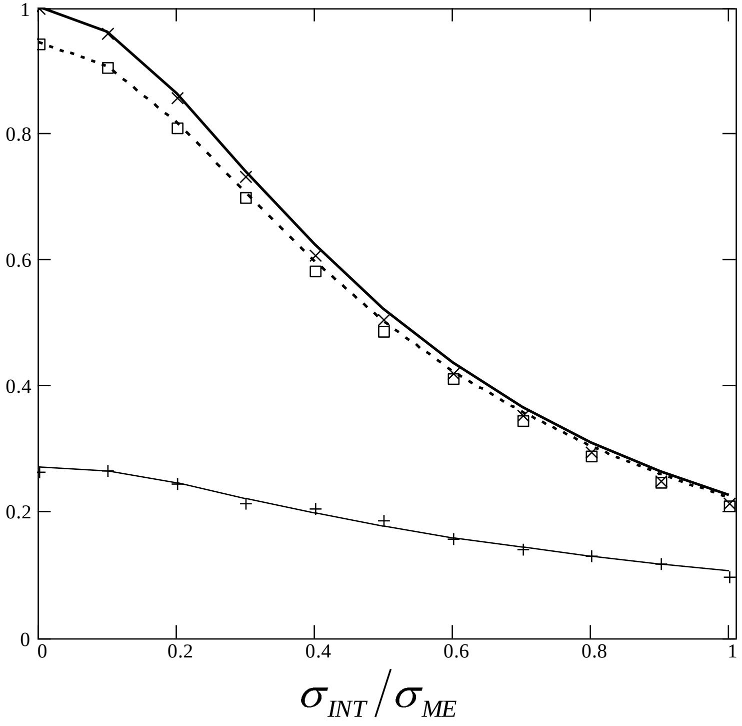
Resolution III Design

Final Outcome



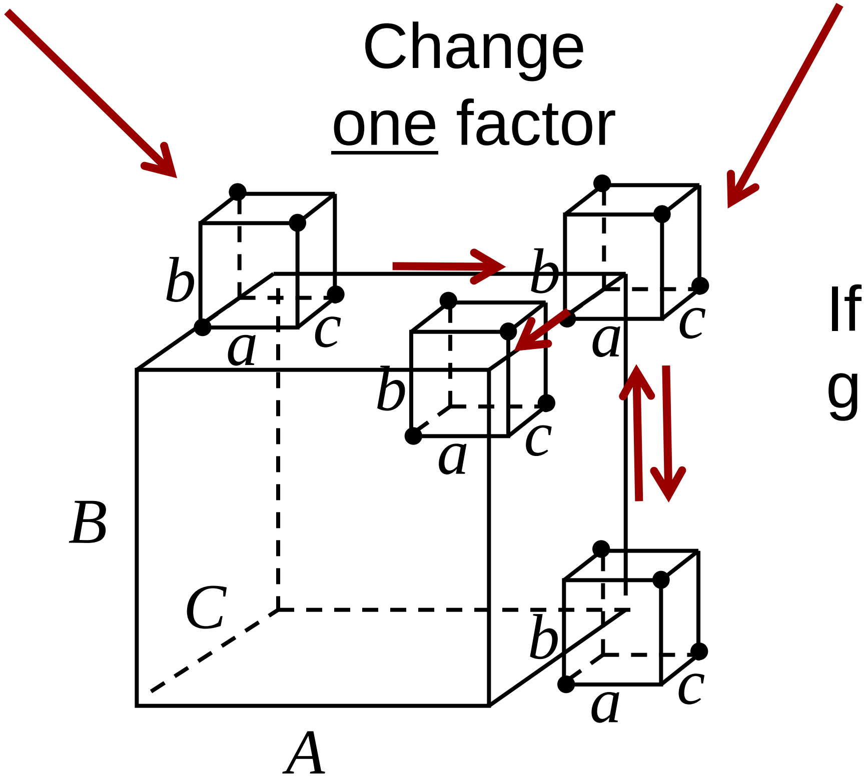
~0.25

Adaptive OFAT



Resolution III Design

Adaptive "One Factor at a Time" for Robust Design



Run a resolution III on noise factors

Change <u>one</u> factor

Again, run a resolution III on noise factors. If there is an improvement, in transmitted variance, retain the change

If the response gets worse, go back to the previous state

Stop after you've changed every factor once

Frey, D. D., and N. Sudarsanam, 2007, "An Adaptive One-factor-at-a-time Method for Robust Parameter Design: Comparison with Crossed Arrays via Case Studies," accepted to *ASME Journal of Mechanical Design*.

Sheet Metal Spinning

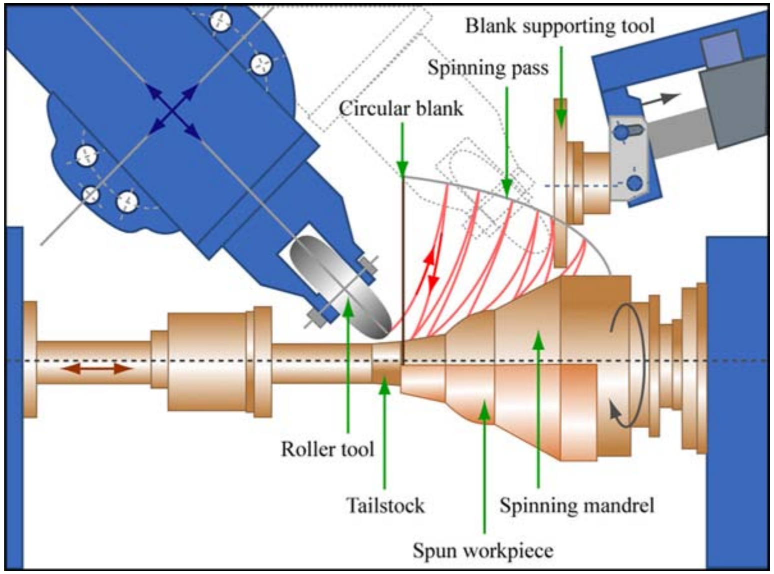


Image by MIT OpenCourseWare.

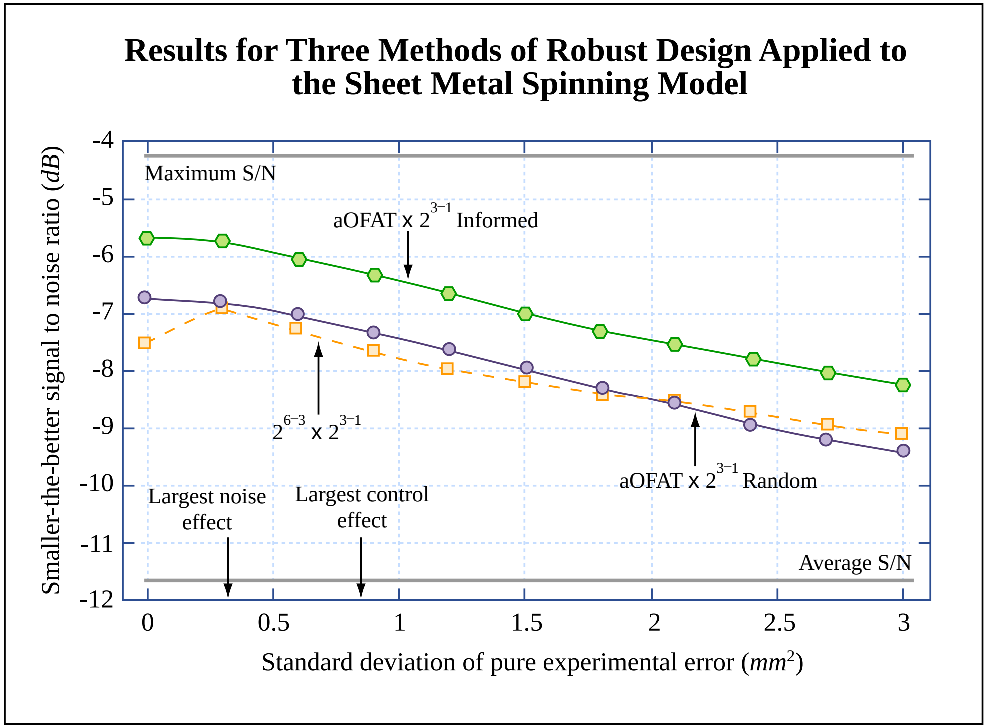
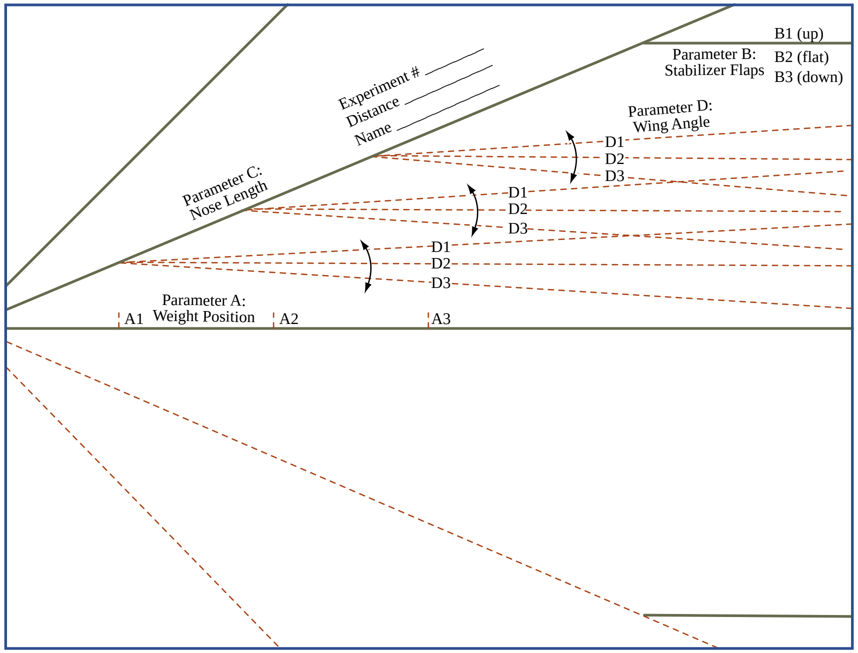


Image by MIT OpenCourseWare.

Paper Airplane

MIT Design of Experiments Exercise v2.0



Expt. #	Weight. A	Stabiliz. B	Nose C	Wing D
1	A1	B1	C1	D1
2	A1	B2	C2	D2
3	A1	B3	C3	D3
4	A2	B1	C2	D3
5	A2	B2	C3	D1
6	A2	B3	C1	D2
7	A3	B1	C3	D2
8	A3	B2	C1	D3
9	A3	B3	C2	D1

Results for Three Methods of Robust Design Applied to the Paper Airplane Physical Experiment

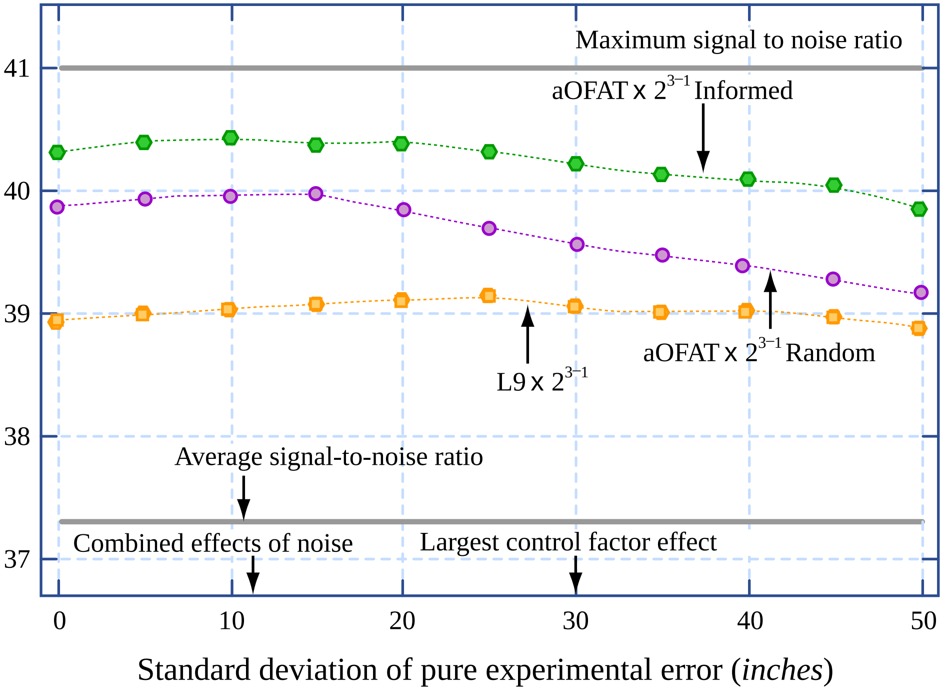
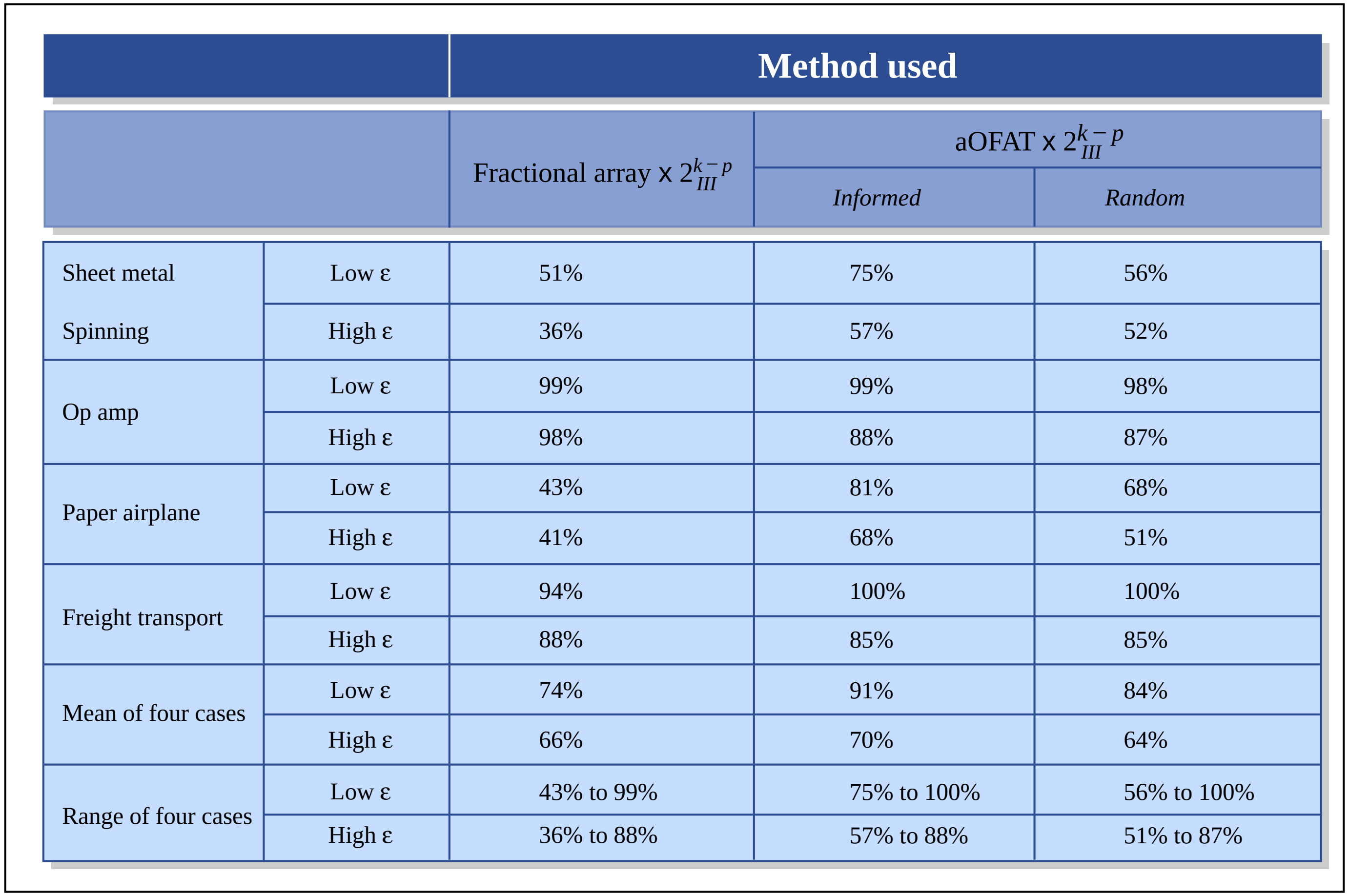


Image by MIT OpenCourseWare.

Results Across Four Case studies

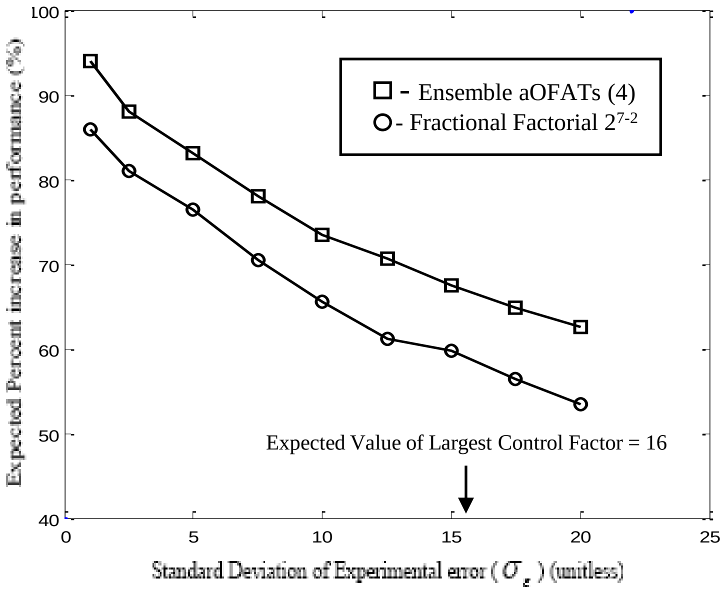


		Method used		
		Fractional array x 2_{III}^{k-p}	aOFAT x 2_{III}^{k-p}	
			Informed	*Random*
Sheet metal Spinning	Low ε	51%	75%	56%
	High ε	36%	57%	52%
Op amp	Low ε	99%	99%	98%
	High ε	98%	88%	87%
Paper airplane	Low ε	43%	81%	68%
	High ε	41%	68%	51%
Freight transport	Low ε	94%	100%	100%
	High ε	88%	85%	85%
Mean of four cases	Low ε	74%	91%	84%
	High ε	66%	70%	64%
Range of four cases	Low ε	43% to 99%	75% to 100%	56% to 100%
	High ε	36% to 88%	57% to 88%	51% to 87%

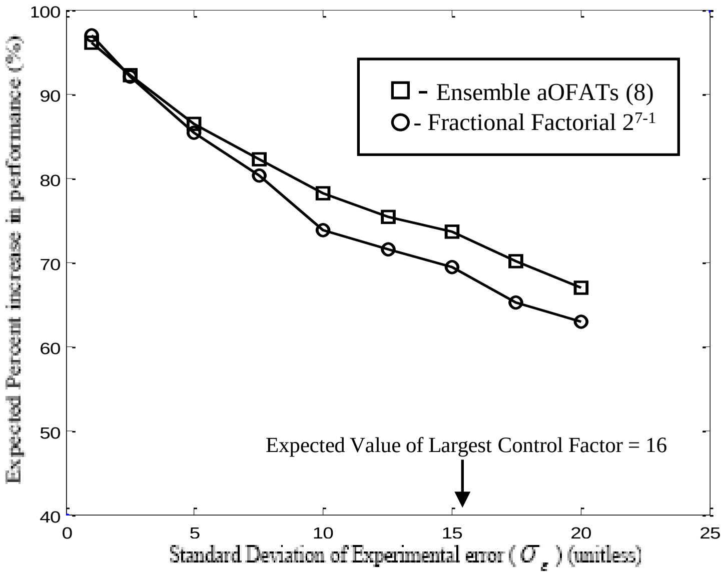
Image by MIT OpenCourseWare.

Frey, D. D., N. and Sudarsanam, 2006, “An Adaptive One-factor-at-a-time Method for Robust Parameter Design: Comparison with Crossed Arrays via Case Studies,” accepted to *ASME Journal of Mechanical Design*.

Ensembles of *a*OFATs



Comparing an Ensemble of 4 *a*OFATs with a 2^{7-2} Fractional Factorial array using the HPM



Comparing an Ensemble of 8 *a*OFATs with a 2^{7-1} Fractional Factorial array using the HPM

Conclusions

- A new model and theorems show that
  - Adaptive OFAT plans exploit two-factor interactions especially when they are large
  - Adaptive OFAT plans provide around 80% of the benefits achievable via parameter design
- Adaptive OFAT can be “crossed” with factorial designs which proves to be highly effective

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