Harvard-MIT Division of Health Sciences and Technology
HST.750: Modeling Issues in Speech and Hearing, Spring 2006
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Problem/Discussion Set for "Analytic vs Computational"

1. **Newton in a Tapered Tube.** After P&A and S&Z went to press, disagreement was discovered in the literature about how to describe fluid motion in a tapered tube. Consider the motion of a slab of fluid of width Δx in a tapered tube of variable cross-section $S(x)$ as illustrated in Fig. 1.

Δx

$p(x,t)$ = pressure
$S(x)$ = cross-sectional area
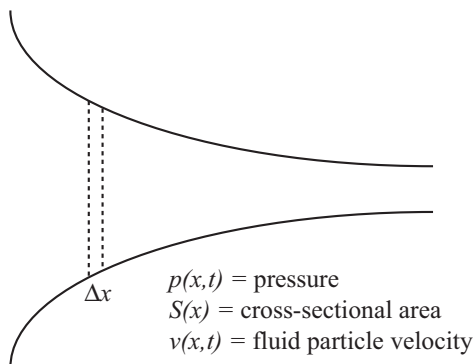$v(x,t)$ = fluid particle velocity

Figure 1: Charming little schematic of a fluid-filled tapered tube.

Applying Newton's 2nd Law yields the relation

$$\underbrace{S(x)p(x,t) - S(x+\Delta x)p(x+\Delta x,t)}_{\text{Net force on slab}} = \underbrace{\rho \Delta x\, S(x+\tfrac{1}{2}\Delta x)}_{\text{mass of slab}}\, \underbrace{\dot{v}(x+\tfrac{1}{2}\Delta x,t)}_{\text{acceleration}}\,, \qquad (1)$$

where $p(x,t)$ is the pressure (assumed uniform in any cross section), ρ is the fluid density, $v(x,t)$ is the fluid particle velocity, and the dot indicates a time derivative. Take the limit $\Delta x \to 0$, define the fluid volume velocity $u \equiv Sv$, and assume an harmonic time dependence at angular frequency ω [e.g., let $p(x,t) = P(x,\omega)e^{i\omega t}$ and $u(x,t) = U(x,\omega)e^{i\omega t}$]. Then

$$\frac{d(SP)}{dx} = -i\omega\rho U \qquad \text{(e.g., Fletcher; de Boer; Koshigoe et al.).[1]} \qquad (2)$$

But S&Z and P&A both seem to use

$$\frac{dP}{dx} = -i\omega\frac{\rho}{S}U\,. \qquad (3)$$

(a) Verify that S&Z and P&A use Eq. (3) or its equivalent.

(b) Verify that the two equations are equivalent only if $S = $ constant. Which, if either, of the two equations is correct for the tapered tube?

(c) What effect does the error—if that's what it is—have on S&Z's and P&A's conclusions about tapering?

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[1] References:

Fletcher, H. (1953). *Speech and Hearing in Communication.* ASA Edition (1995), J. B. Allen, ed. Woodbury, NY: Acoustical Society of America. (Page 243, Eq. 14–18).

de Boer, E. (1980). "Auditory physics. Physical principles in hearing theory. I," *Phys. Rep.* **62**:88-174. (Page 140, Eq. 6–d).

Koshigoe, S., W.K. Kwok, and A. Tubis (1983). "Effects of perilymph viscosity on low-frequency intracochlear pressures and the cochlear input impedance of the cat," *J. Acoust. Soc. Am.*, **74**:486–492. (Page 487: Eqs. 3 & 4).

2. **Erroneous(?) Formula for the Cochlear Input Impedance.** As LN&P discuss (Sec. 3a, p. 125, 2nd column), it is sometimes argued that the cochlear input impedance has the approximate value

$$Z_c \approx \rho c / A \ , \tag{4}$$

corresponding to the wave impedance of an acoustic plane wave propagating without reflection in an infinite tube (where ρ = density of perilymph, c = speed of sound in perilymph, and A = area of stapes footplate). They argue, however, that there are compelling *theoretical* and *experimental* reasons for rejecting this view. Briefly summarize these reasons. Can you nonetheless find a way of *reinterpreting* the formula that makes it correct?

3. **Circularity?** S&Z begin §I-F on page 1278 by assuming that the wavelength changes slowly. From this, they derive Eq. (13) for the cochlear input impedance. Combining Eq. (13) with the LN&P data, S&Z obtain Eq. (21), which they then interpret as indicating that the wavelength changes slowly! As if to justify this apparently circular argument, S&Z write at the end of §I-H that Eq. (21) provides "*ex post facto* justification for the WKB expansion used to obtain Eq. (13)." Is their reasoning legit, or is the entire argument circular? Why or why not? If the argument *is* circular, can it be straightened? If so, how?

4. **Apical Reflections.** In Fig. 7, P&A investigate the effect of tapering and note that "the amplitude of the apical reflection ... increases for stimulus frequencies below $f_{CF}^{cl}(x_L)$." They explain that this is due to "an increase in impedance mismatch between the helicotrema and the scalae" resulting in "a larger reflection coefficient at the apical end of the cochlea". P&A reiterate this point in their final paragraph: "In the realistic scalae area case, the magnitude of the [apical] reflections [is] much greater due to an increase in impedance mismatch between the scalae and the helicotrema."

   (a) How convincing do you find their argument for the increase in amplitude of the standing waves? For example, what impedances do P&A use for the scalae and for the helicotrema in Fig. 7? How does the predicted impedance mismatch (reflection coefficient) depend on tapering (i.e., on s_1)?

   (b) If you aren't yet convinced, what might P&A have done to make their case more compelling? Or maybe you can provide an alternative explanation for their findings?

5. **Sanity Checks.** LN&P use mechanical analogues; P&A use numerical techniques; S&Z use approximate analytic methods. What "sanity checks" do each of them use to determine whether their results are reasonable (i.e., not simply the result of some technical error)? Are there additional things you think they might/should have done?

6. **Modeling Strategies.** LN&P, P&A, and S&Z take three very different approaches to modeling essentially the same data.

   (a) Characterize these different approaches. Your answer should go beyond simply labeling them as "computational" or "analytic"; these distinctions are only part of the story.

   (b) To what extent do the papers reach similar conclusions? Where do they disagree? How do you resolve any discrepancies?

   (c) What are the strengths and weaknesses of the various approaches?

   (d) Discuss the relative pros and cons of "computational" vs "analytic" models. Illustrate your remarks with examples from the three papers.

7. **Justification for Approach.** LN&P use lumped mechanical analogues and both P&A and S&Z use a linear, one-dimensional transmission-line model. How do the authors justify their use of these models? Discuss other simplifying assumptions made by the authors. Are the assumptions justified? What limits do these assumptions place on the "region of validity" of each model?

8. **Selling It.** How do the different papers "sell" their work to the reader? For example, how do the different authors motivate their work? Do they convince you that a model of the cochlear input impedance is important or interesting? How are the different models and interpretations of the data influenced by these goals? Can you think of ways that the "packaging" might have been improved to enhance the accessibility and/or impact[2] of the work?

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[2]According to the Science Citation Index, as of Feb. 2003 P&A and S&Z have been cited a total of 10 and 7 times, respectively, not counting innumerable citations by the authors themselves.