

ESD.86

Descriptive Statistics and Statistical Graphics

Dan Frey

Associate Professor of Mechanical Engineering and Engineering Systems



Plan for Session

- Descriptive statistics
- Visual perception / cognition
- Tufte's paper
 - Statistical graphics
 - Statistical thinking

Confidence Intervals

- Assuming a given distribution and a sample size n and a given value of the parameter θ the 95% confidence interval from U to V is s.t. the estimate of the parameter $\hat{\theta}$

$$\Pr(U < \hat{\theta} < V | \theta) = 95\%$$

- The confidence interval depends on the confidence level, the sample variance, and the sample size

Concept Question

- Here is a sample of data
- This Matlab code computes a 95% confidence interval for the mean
- Then it runs a t-test on the data and the hypothesis that it was from a normal distribution with a mean greater than or equal to the lower confidence bound
- What p-value will the test return?

```
sample=[1.2 3.1 4.5 2.3 0.8 7.2];  
[h,p,ci] = ttest(sample);  
[h2,p2] =ttest(sample,ci(1),0.05,'left')
```

- 1) exactly 0.95
- 2) exactly 0.975
- 3) more than 0.975
- 4) less than 0.95
- 5) none of the above

Concept Question

- This Matlab code repeatedly generates simulated "data" (5 samples) from a normally distributed population with a known mean and computes a 95% confidence interval
- How often will the 95% confidence interval include the mean of the distribution from which the samples came?

```
pm=1;  
for i=1:1000  
    sample=random('Normal',pm,1,1,5);  
    [h,p,ci] = ttest(sample);  
    in_int(i)=(ci(1)<=pm)*(ci(2)>=pm);  
end  
mean(in_int)
```

- 1) 95% of the time
- 2) a little less than 95%
- 3) a little more than 95%
- 4) Not enough information

Bayesian Credible Interval

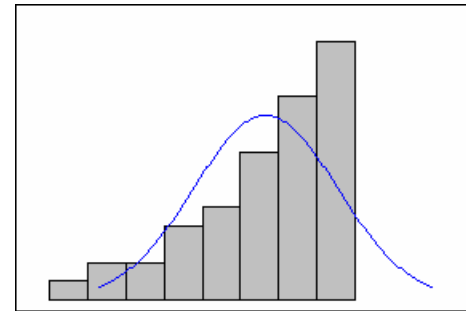
- In Bayesian statistics, a credible interval is a posterior probability interval...
- For example, a statement such as "following the experiment, a 90% credible interval for the parameter t is 35-45" means that the posterior probability that t lies in the interval from 35 to 45 is 0.9.

Measures of Dispersion

- Population Variance $VAR(X) = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$
- Sample variance $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$
 - an unbiased estimate of $\sigma^2 = E((x - E(x))^2)$
- n^{th} central moment $E((x - E(x))^n)$
- n^{th} moment about m $E((x - m)^n)$

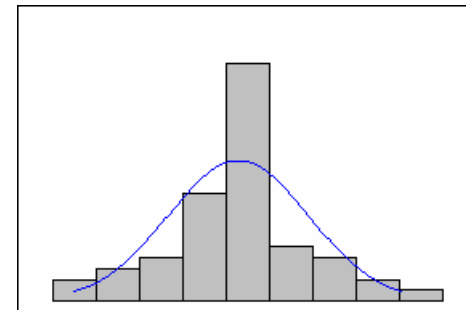
Skewness and Kurtosis

- Skewness $E((x - E(x))^3)$



positively skewed distribution

- Kurtosis $E((x - E(x))^4)$



positive kurtosis

Correlation Coefficient

- Sample

$$r = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{(n-1)S_X S_Y}$$

$$S_X^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

- Which is an estimate of

$$\frac{E((x - E(x))(y - E(y)))}{\sigma_x \sigma_y}$$

But What Does it Mean?

*Pick a Box!
Are they all the same?*

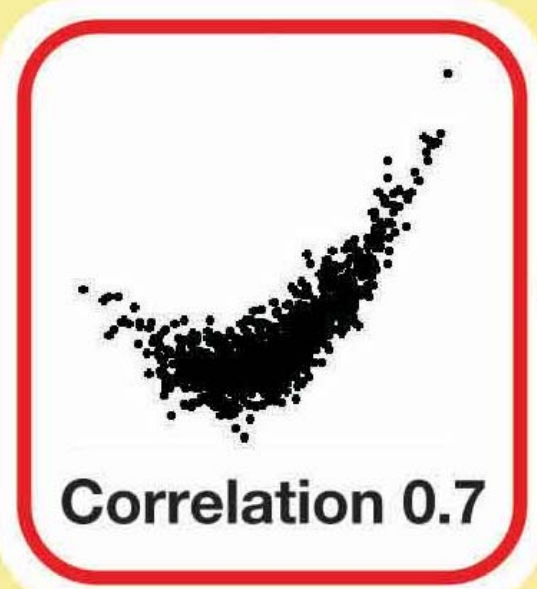
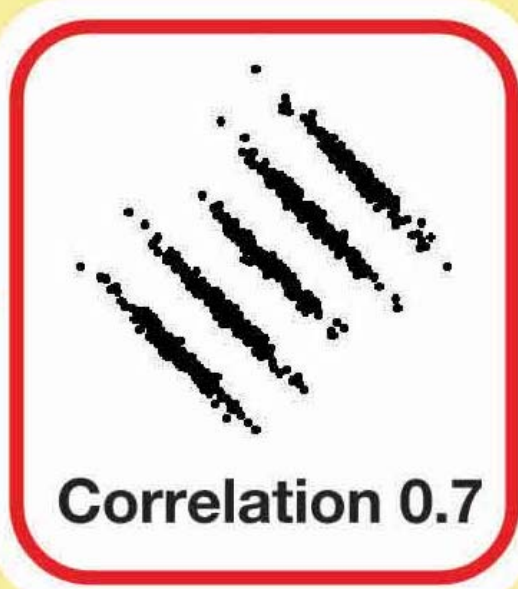
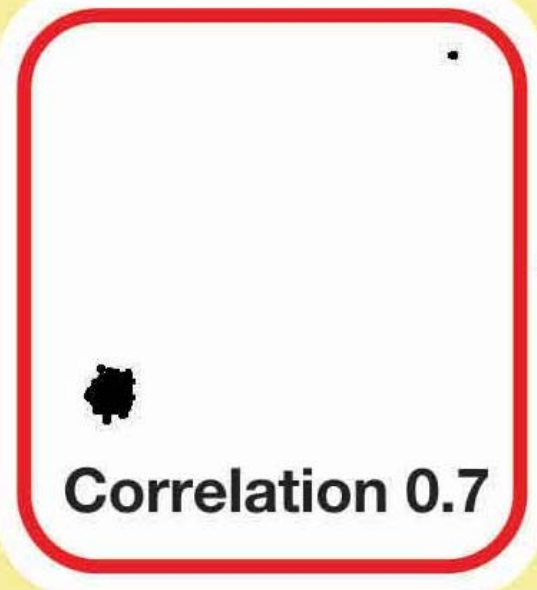
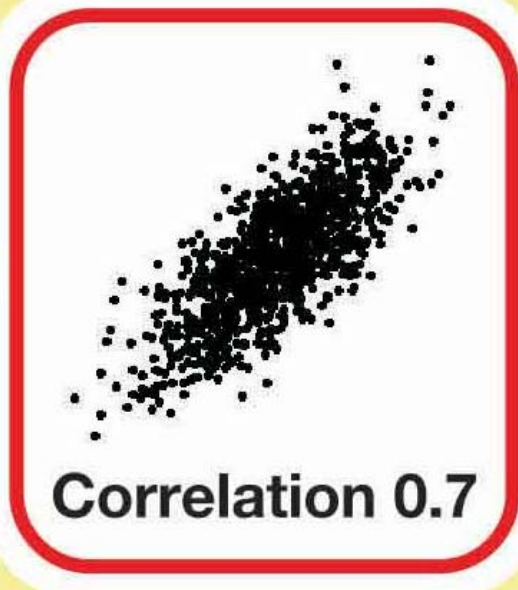


SECTION ON
STATISTICAL
GRAPHICS



AMERICAN
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ASSOCIATION

<http://www.amstat-online.org/sections/graphics/>



The Human Brain

- The human brain is tremendously complex
- It possesses many highly specialized component parts each associated with specific tasks
- The functioning of the human brain gives rise to
 - Consciousness
 - Intelligence (ability to learn, plan...)
 - Emotions
 - Decision making
- “The mind ... accomplishes remarkable feats no engineer can duplicate...”

Pinker, Steven, 1997, *How the Mind Works*, Norton & Co, New York.

Evolution and the Brain

- “The mind is a system of organs of computation, designed by natural selection to solve the kinds of problems our ancestors faced...”

Image removed due to copyright restrictions.
Diagram showing three stages in skull & brain evolution.

Three Brains in One

- Reptilian Complex
 - digestion, reproduction, circulation, breathing, "fight or flight" response
- Limbic System
 - houses primary centers of emotion
 - hippocampus -- important aspects of long term memory
- Neocortex
 - processing senses
 - logic
 - language
 - motor control

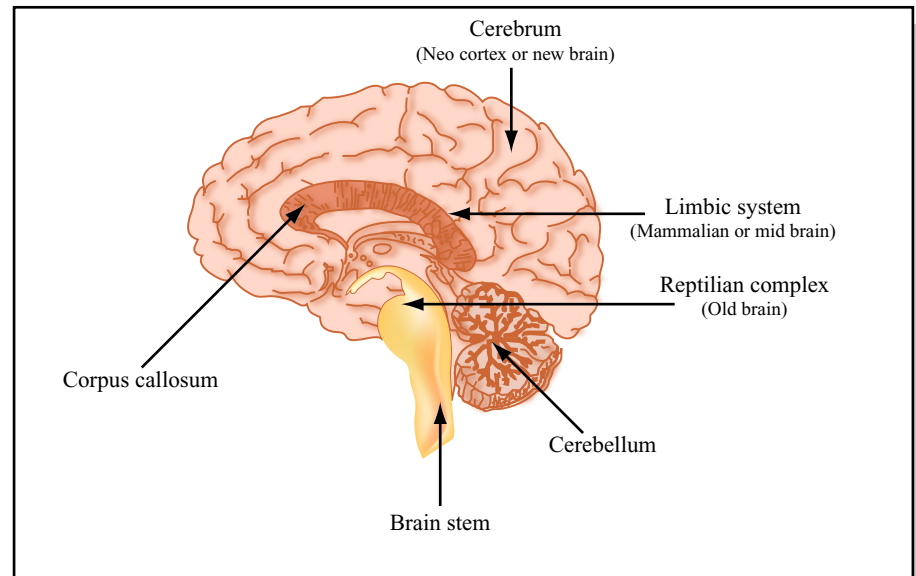
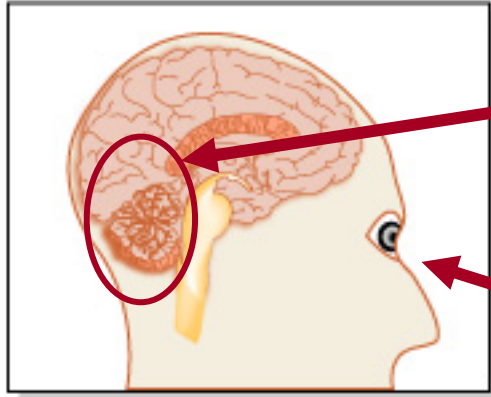


Figure by MIT OCW.

Neocortex

- 2mm thick
- 6 layers
- About the size of a newspaper (unfolded)
 - So it has to be wrinkled up to fit in the skull
- About 3×10^{10} neurons
- About 1000 synapses per neuron
- Each neuron capable of 200 cycles / sec
 - 5 million times slower than a computer

Visual Cortex



About $\frac{1}{3}$ of the cerebral cortex.

Retina – a $2\frac{1}{2}$ -D data stream
~ a million nerve fibers

Figure by MIT OCW.

The fiber pathways are two-way... They carry as much information down from higher conceptual areas as from lower sensory areas...

Pinker, Steven, 1997, *How the Mind Works*, Norton & Co, New York.

Visual Cognition

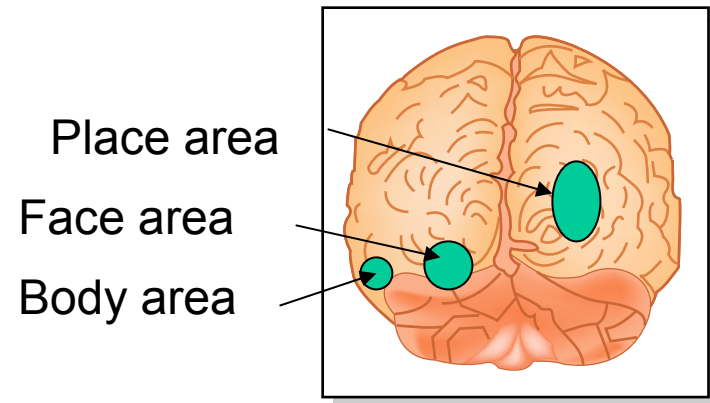


Figure by MIT OCW.

- Humans are much better than computers at classifying objects based on complex, noisy, ambiguous images
- Certain classes of things have specialized areas
- The same areas involved in “seeing” a face or place are active in thinking about it

Kanwisher, Nancy, 2003, *Imaging Visual Cognition*.

Face Recognition

- “saccade” = a fast motion of the eye (~3 per second)
- Your eye can be tracked as it looks at faces
- Your eyes are apparently very active in seeing a face
- The activity depends on familiarity

Images of eye tracking during face recognition removed due to copyright restrictions.

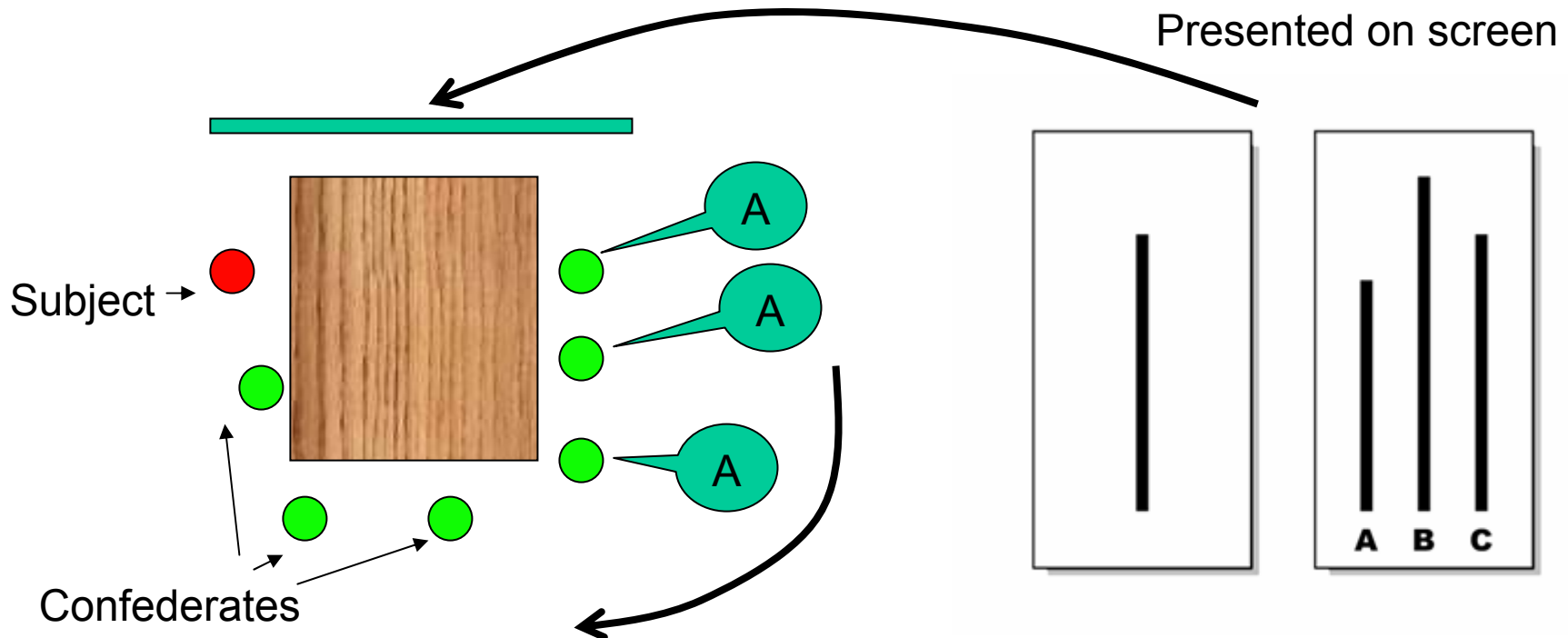
Change Blindness

- “...focused attention is needed to detect change”*
- You cannot assume that information flowing into your brain will be attended to; it may be ignored completely
- Is it also true of graduate students?
- Is it true of a professors?
- Is it possible to do good research when key information fails to be acknowledged?

*Rensink, R. A., 2002, “Change Detection,”
Annual Review of Psychology, **53**:4245-277.

Asch Conformity Experiments

- Subjects asked to participate in a "vision test"
- In reality, all but one of the participants were confederates of the experimenter
- A high proportion (~33%) conformed to the erroneous majority view
- Many subjects showed extreme discomfort, but conform anyway



A Hypothesis

- Facts about the brain provide some insight into problems in:
 - statistical inference
 - research methods
 - decision making
- Cognitive sciences and psychology also provide insights into the remedies

Edward R. Tufte's Paper

- "Visual and Statistical Thinking: Displays of Evidence for Making Decisions"
- What are its key points?
 - History
 - Research practices
 - Statistics *per se*

The General Argument

1. (Tufte) "An essential analytic task in making decisions based on evidence is to understand how things work – mechanism, trade-offs, process and dynamics, cause and effect. That is, intervention-thinking and policy-thinking demand causality-thinking."
2. (Tufte) "Making decisions based on evidence requires the appropriate display of that evidence. Good displays of that data help to reveal knowledge relevant to understanding mechanism, process and dynamics, cause and effect. That is, displays of statistical data should directly serve the analytic task at hand."
3. (Frey) Replace the instances of the term "making decisions" with "doing high quality ESD research." The sentences above are still correct yet have different implications.

The Cholera Epidemic in London, 1854

Map removed due to
copyright restrictions.

- Dot plot of cholera instances suggests a spatial pattern
- Centered on a water pump

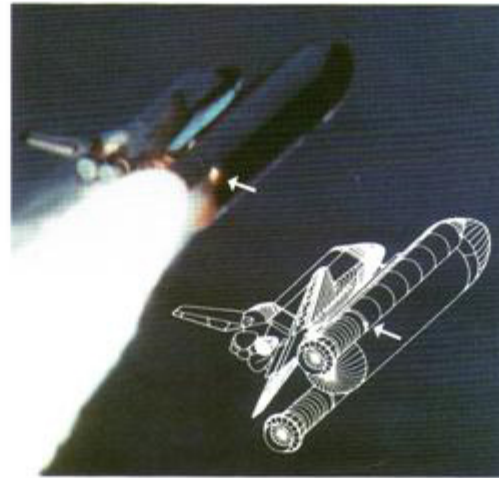
E.W. Gilbert, "Pioneer Maps of Health and Disease in England," *Geographical Journal*, 124 (1958), 172-183.

What about the Exceptions?

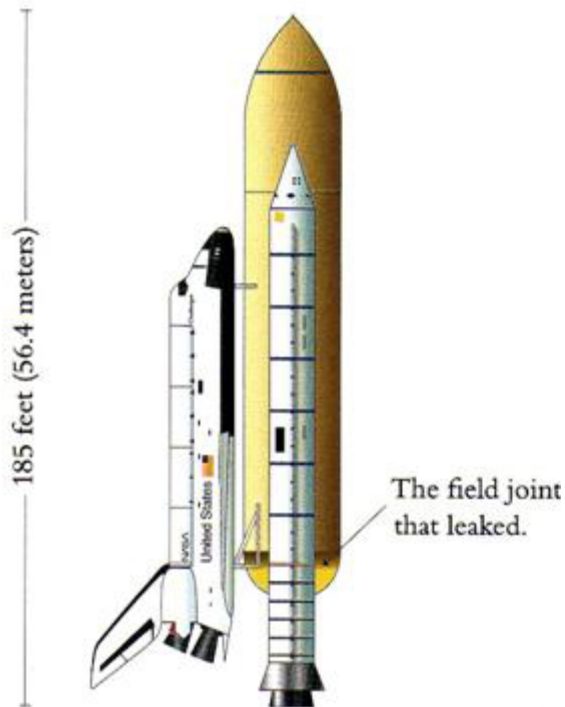
Map removed due copyright restrictions.
Source: Tufte, Edward. Chapter 2 in *Visual Explanations: Images and Quantities, Evidence and Narrative*. Cheshire, CT: Graphics Press, 1997.

- Very few instances in the work house and brewery despite proximity to the suspected pump
- Also some instances far from the pump

The Challenger Decision



Source: NASA



Courtesy of NASA.

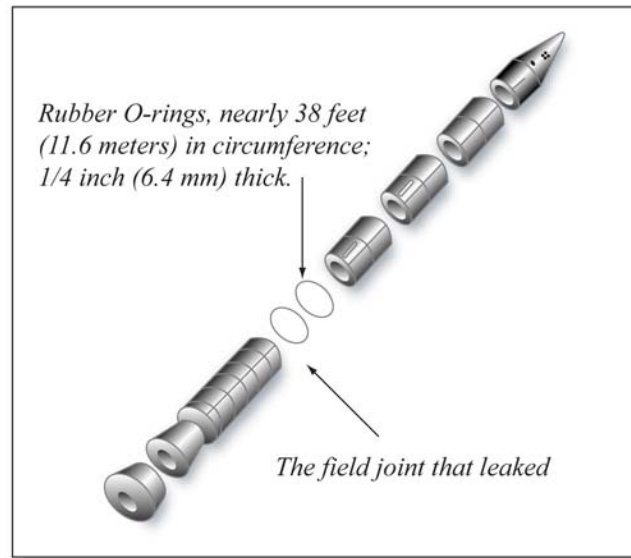


Figure by MIT OCW.

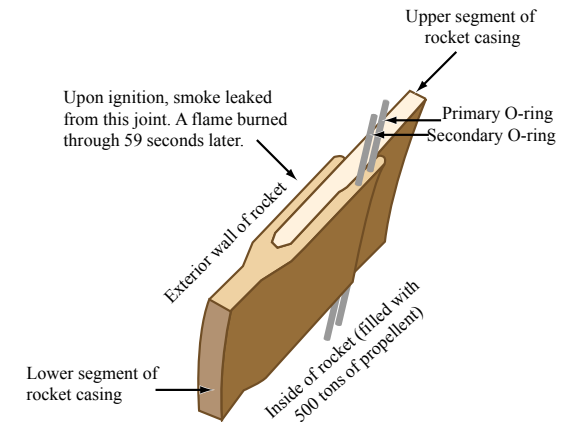


Figure by MIT OCW.

Confusion about Evidence, Burden of Proof, and Significance

report from engineers:

VP decision:

Source: NASA

MTI ASSESSMENT OF TEMPERATURE CONCERN ON SRM-25 (51L) LAUNCH

- 0 CALCULATIONS SHOW THAT SRM-25 O-RINGS WILL BE 20° COLDER THAN SRM-15 O-RINGS
- 0 TEMPERATURE DATA NOT CONCLUSIVE ON PREDICTING PRIMARY O-RING BLOW-BY
- 0 ENGINEERING ASSESSMENT IS THAT:
 - 0 COLDER O-RINGS WILL HAVE INCREASED EFFECTIVE DUROMETER ("HARDER")
 - 0 "HARDER" O-RINGS WILL TAKE LONGER TO "SEAT"
 - 0 MORE GAS MAY PASS PRIMARY O-RING BEFORE THE PRIMARY SEAL SEATS (RELATIVE TO SRM-15)
 - 0 DEMONSTRATED SEALING THRESHOLD IS 3 TIMES GREATER THAN 0.038" EROSION EXPERIENCED ON SRM-15
 - 0 IF THE PRIMARY SEAL DOES NOT SEAT, THE SECONDARY SEAL WILL SEAT
 - 0 PRESSURE WILL GET TO SECONDARY SEAL BEFORE THE METAL PARTS ROTATE
 - 0 O-RING PRESSURE LEAK CHECK PLACES SECONDARY SEAL IN OUTBOARD POSITION WHICH MINIMIZES SEALING TIME
- 0 MTI RECOMMENDS STS-51L LAUNCH PROCEED ON 28 JANUARY 1986
 - 0 SRM-25 WILL NOT BE SIGNIFICANTLY DIFFERENT FROM SRM-15


JOE C. KILMINSTER, VICE PRESIDENT
SPACE BOOSTER PROGRAMS

CONCLUSIONS:

- 0 TEMPERATURE OF O-RING IS NOT ONLY PARAMETER CONTROLLING BLOW-BY

SRM 15 WITH BLOW-BY HAD AN O-RING TEMP AT 53°F
SRM 22 WITH BLOW-BY HAD AN O-RING TEMP AT 75°F
FOUR DEVELOPMENT MOTORS WITH NO BLOW-BY WERE TESTED AT O-RING TEMP OF 47° TO 52 °F

DEVELOPMENT MOTORS HAD PUTTY PACKING WHICH RESULTED IN BETTER PERFORMANCE
- 0 AT ABOUT 50°F BLOW-BY COULD BE EXPERIENCED IN CASE JOINTS
- 0 TEMP FOR SRM 25 ON 1-28-86 LAUNCH WILL BE 29°F 9 AM
38°F 2 PM
- 0 HAVE NO DATA THAT WOULD INDICATE SRM 25 IS DIFFERENT THAN SRM 15 OTHER THAN TEMP

RECOMMENDATIONS:

- 0 O-RING TEMP MUST BE ≥ 53 °F AT LAUNCH

DEVELOPMENT MOTORS AT 47° TO 52°F WITH PUTTY PACKING HAD NO BLOW-BY
SRM 15 (THE BEST SIMULATION) WORKED AT 53 °F
- 0 PROJECT AMBIENT CONDITIONS (TEMP & WIND) TO DETERMINE LAUNCH TIME

Source: NASA

Absence of evidence \neq
evidence of absence

Would this Have Been Convincing?

Image removed due to copyright restrictions.

Plot of O-ring damage index vs. Temperature at time of launch.

Source: Tufte, Edward. *Visual Explanations: Images and Quantities, Evidence and Narrative*. Cheshire, CT: Graphics Press, February 1997.

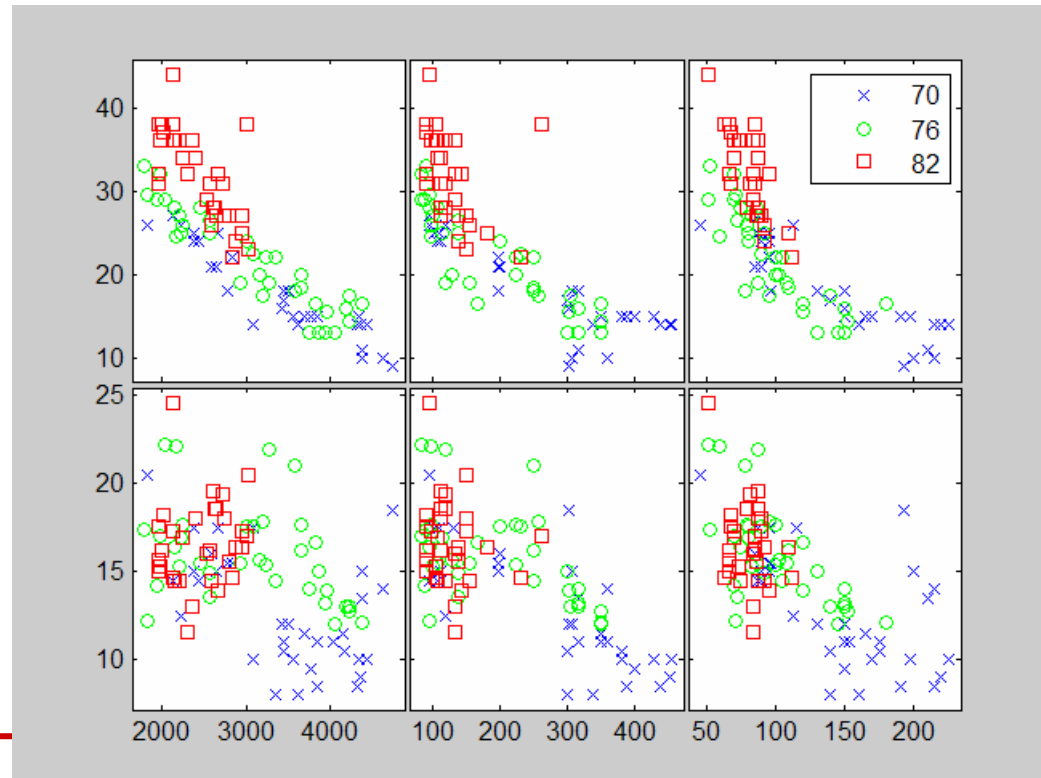
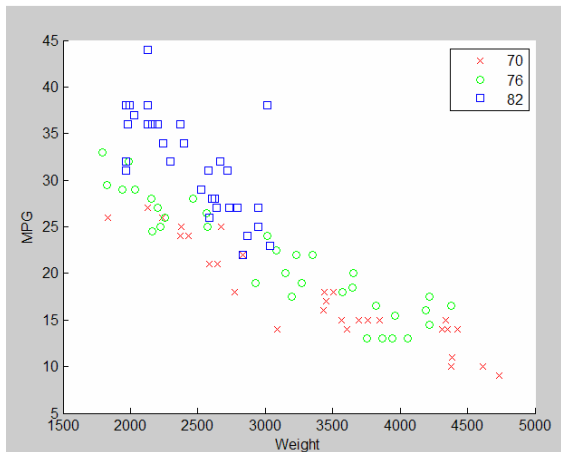
- Continuous measure of degradation
- Plotted versus the suspected variable
- Shows the range over which the prediction is needed
- The lack of data in that range is the most striking feature

What about Feynman's Demonstration?

- It did show the phenomenon at issue
- But it also confounds many factors
- It doesn't show consistency with most of the data (non-failures)

Photo and diagrams removed due to copyright restrictions.

Scatter Plots



```
load carsmall
```

```
gscatter(Weight,MPG,Model_Year,"','xos')
```

```
figure(2)
```

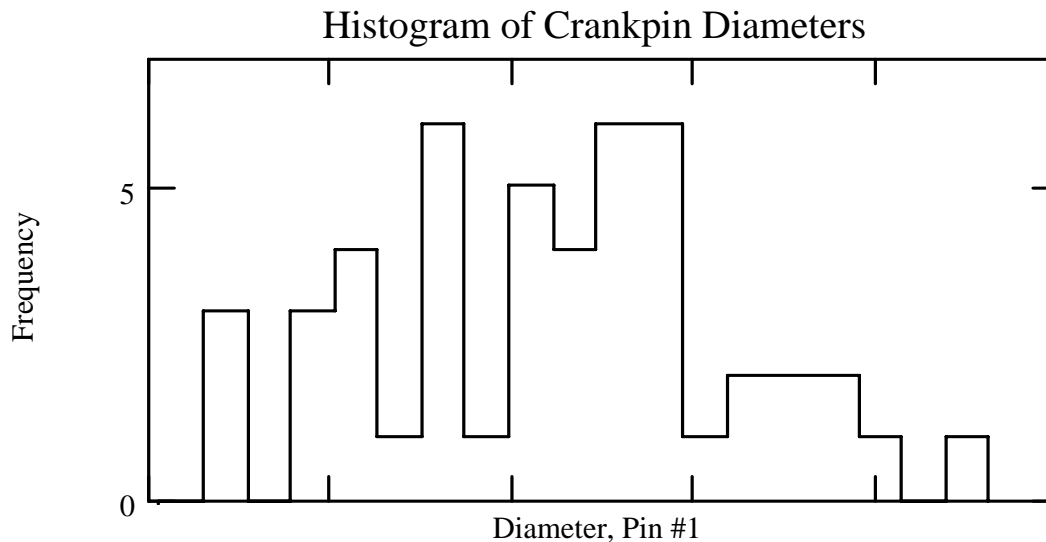
```
xvars = [Weight Displacement Horsepower];
```

```
yvars = [MPG Acceleration];
```

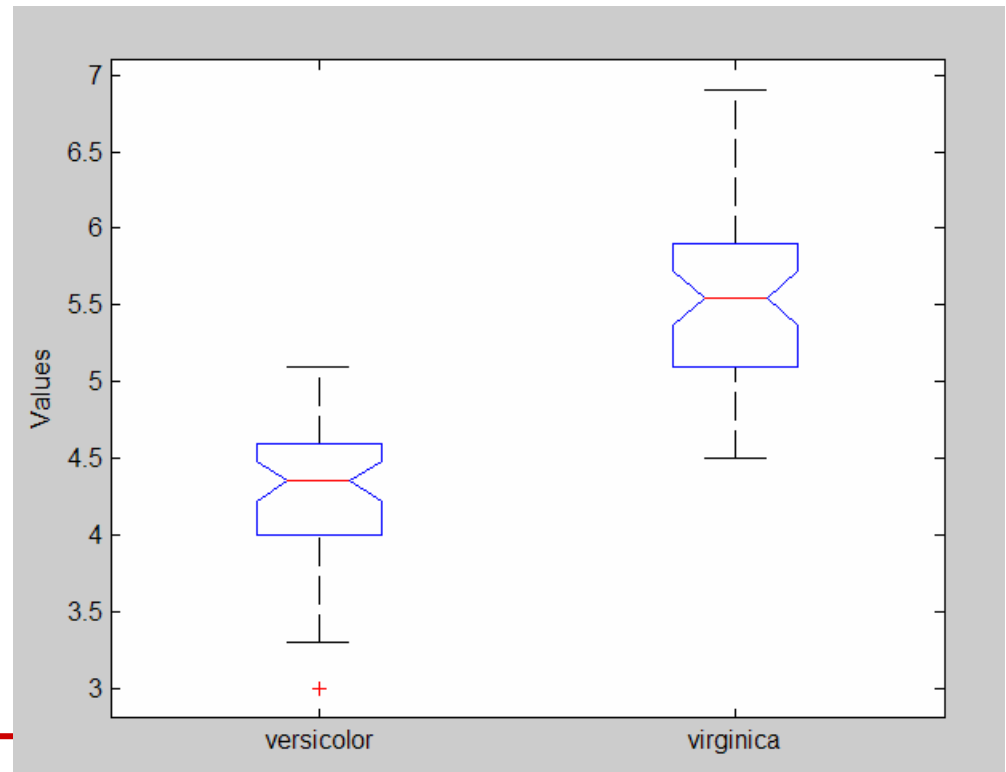
```
gplotmatrix(xvars,yvars,Model_Year,"','xos')
```


Histograms

- A graph of continuous data
- Approximates a pdf in the limit of large n

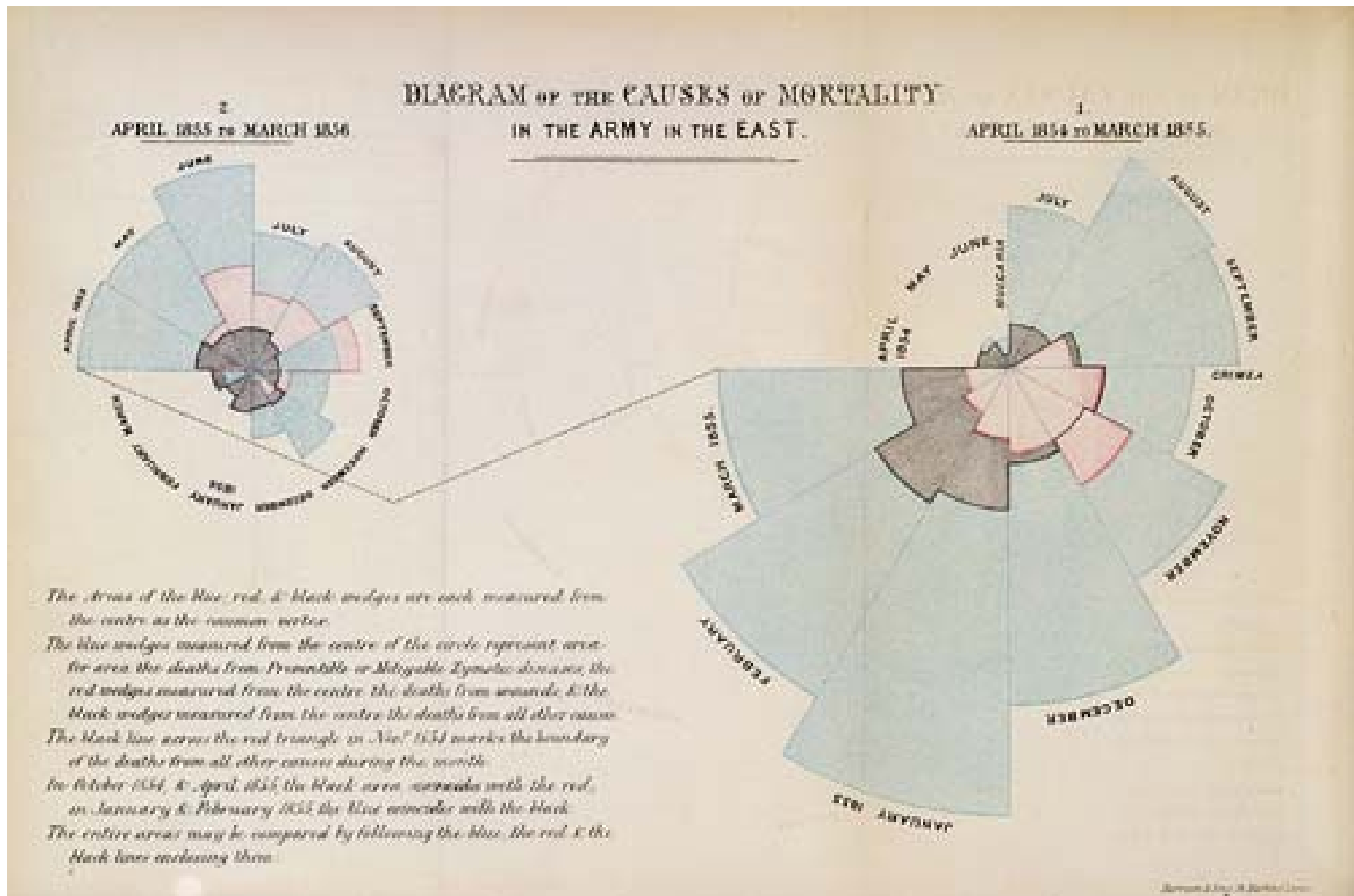


Box and Whisker Plot



```
load fisheriris
s1 = meas(51:100,3);
s2 = meas(101:150,3);
boxplot([s1
s2], 'notch', 'on', 'labels', {'versicolor', 'virginica'})
```

Florence Nightingale's Polar-Area Diagrams



Source: Nightingale, F. "Notes on Matters Affecting the Health, Efficiency and Hospital Administration of the British Army." 1858.

Next Steps

- Wednesday, 11 April
 - Session on Regression (no pre-read)
- Friday, 13 April
 - Session to support the term project
 - Be prepared to stand up and talk for 5 minutes about your ideas and your progress
- 16-17 April, No classes (Patriot's Day)
- Wednesday 18 April, ANOVA