Newsboy Model with Pricing

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Newsboy Problem

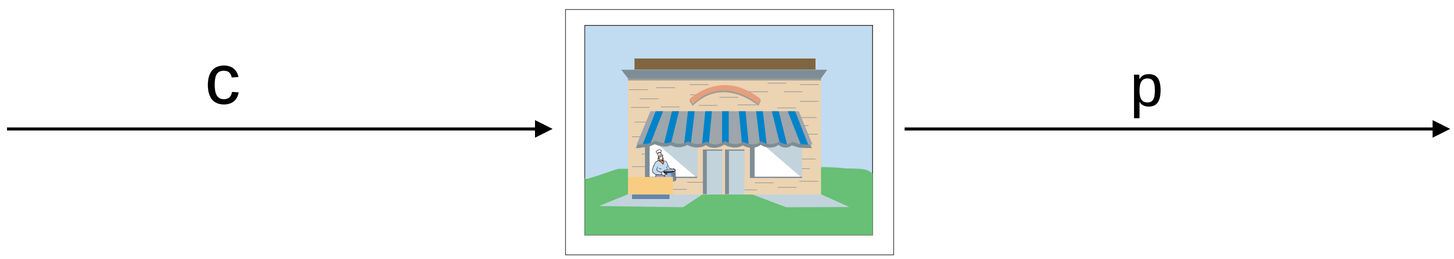

Image by MIT OpenCourseWare.

Demand $D = D(\epsilon)$

$F(y) = Prob\{D \leq y\}$

Optimal policy $F(y^*) = \frac{p-c}{p}$

What if...

(i) Price p is a decision variable, and

(ii) $D = D(p, \epsilon)$?

Price Depending Demand

Demand $D = D(p, \epsilon)$

$$F(y|p) = Prob(D \leq y|p)$$

Assume $\frac{\partial F(y|p)}{\partial p} \geq 0$

i.e., demand decreases stochastically with price

$$R(y,p) = -cy + pE[\min(y,D)]$$

$$R(y,p) = -cy + p\int_0^y (1 - F(x))dx$$

Additive Demand

$D(p,\epsilon) = g(p) + \epsilon$ where $g(p) = a - bp$

$a > 0,\ b > 0,\ E[\epsilon] = 0$

$$r(y,p) = \begin{cases} -cy + p[g(p) + \epsilon] & if \;\; g(p) + \epsilon \leq y \\ -cy + py & if \;\; g(p) + \epsilon > y \end{cases}$$

Let $z = y - g(p)$

$$r(z,p) = \begin{cases} -c[z + g(p)] + p[g(p) + \epsilon] & if \;\; \epsilon \leq z \\ -c[z + g(p)] + p[z + g(p)] & if \;\; \epsilon > z \end{cases}$$

$$r(z,p) = -cz + g(p)(p - c) + p\,\mathrm{min}(z,\epsilon)$$

Additive Demand (continued)

$$r(z,p) = -cz + g(p)(p-c) + p\,\mathrm{min}(z,\epsilon)$$

$$R(z,p) = E[r(z,p)]$$
$$= -cq + g(p)(p-c) + pE[\mathrm{min}(z,\epsilon)]$$

$$\frac{\partial R(z,p)}{\partial p} = g'(p)(p-c) + g(p) + E[\mathrm{min}(z,\epsilon)]$$
$$= a + bc - 2bp + E[\mathrm{min}(z,\epsilon)]$$

$$p^* = \frac{a+bc+E[min(z^*,\epsilon)]}{2b}$$

Deterministic Demand

$$R(p) = (p - c)g(p) = (p - c)(a - bp)$$

$$\frac{\partial R(p)}{\partial p} = a - 2bp + bc$$

“Riskless Price” or “Deterministic Price”

$$p^0 = \frac{a+bc}{2b} > \frac{a+bc+E[\min(z^*,\epsilon)]}{2b} = p^*$$

Multiplicative Demand

$D(p,\epsilon) = g(p)\epsilon$ where $g(p) = ap^{-b}$

$a > 0,\ b > 0,\ E[\epsilon] = 1$

$$r(y,p) = \begin{cases} -cy + pg(p)\epsilon & if\ \ g(p)\epsilon \le y \\ -cy + py & if\ \ g(p)\epsilon > y \end{cases}$$

Let $z = \frac{y}{g(p)}$

$$r(z,p) = \begin{cases} -cg(p)z + pg(p)\epsilon & if\ \ \epsilon \le z \\ -cg(p)z + pg(p)z & if\ \ \epsilon > z \end{cases}$$

$$r(z,p) = -czg(p) + pg(p)\min(z,\epsilon)$$

Multiplicative Demand (continued)

$$r(z,p) = -czg(p) + pg(p)\,\mathrm{min}(z,\epsilon)$$

$$\begin{aligned} R(z,p) &= E[r(z,p)] \\ &= -czg(p) + pg(p)E[\mathrm{min}(z,\epsilon)] \\ &= g(p)(-cz + p\Delta) \end{aligned}$$

$$\begin{aligned} \frac{\partial R(z,p)}{\partial p} &= g'(p)(-cz + p\Delta) + g(p)\Delta \\ &= a(-b)p^{-b-1}(-cz + p\Delta) + ap^{-b}\Delta \\ &= ap^{-b-1}(bcz - bp\Delta + p\Delta) \end{aligned}$$

$$p^* = \frac{bcz}{\Delta(b-1)} = \frac{bc}{b-1}\frac{z^*}{\Delta}$$

Deterministic Demand

$$R(p) = (p-c)g(p) = (p-c)ap^{-b}$$

$$\frac{\partial R(p)}{\partial p} = ap^{-b} - b(p-c)ap^{-b-1}$$
$$= ap^{-b-1}(p - bp + bc)$$

"Riskless Price" or "Deterministic Price"

$$p^0 = \frac{bc}{b-1} < \frac{bc}{b-1}\frac{z^*}{\Delta} = p^*$$

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