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**6.976**

***High Speed Communication Circuits and Systems***

***Lecture 16***

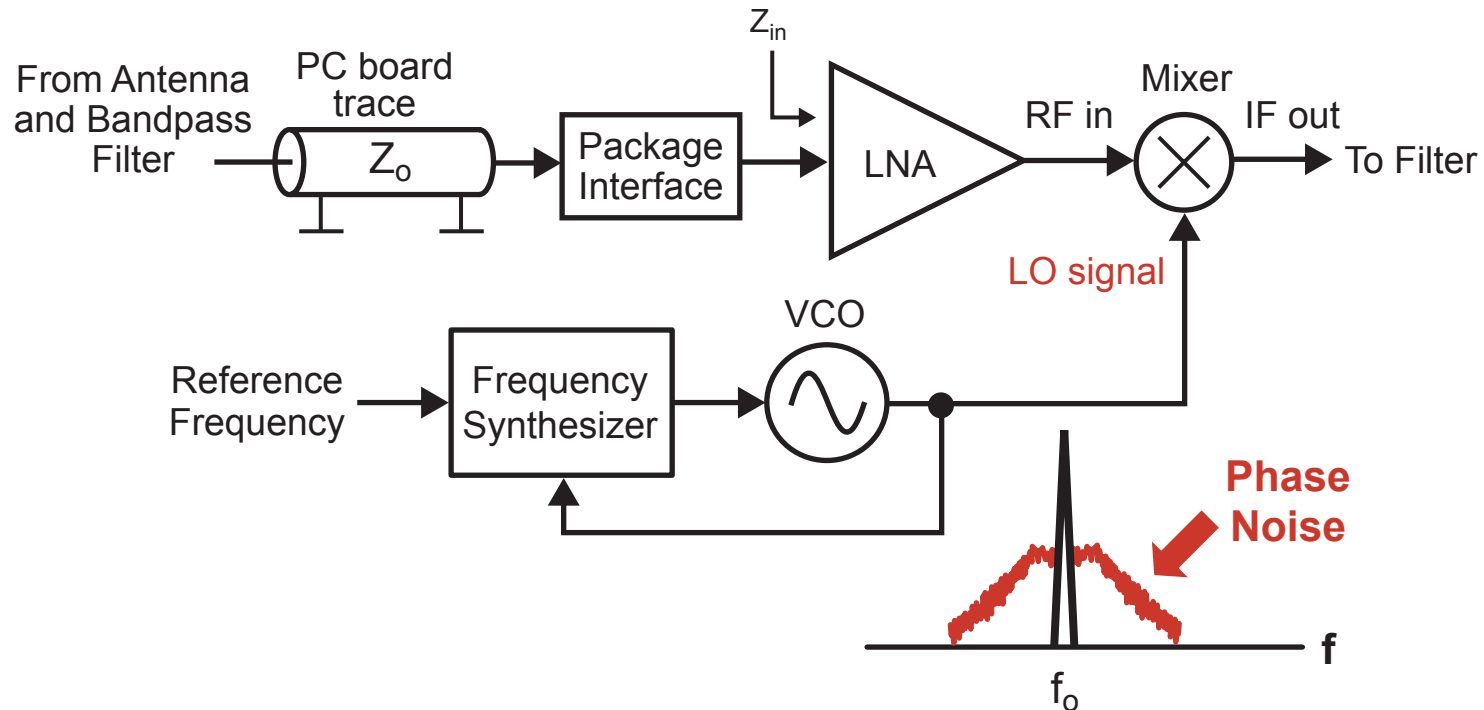
***Noise in Integer-N Frequency Synthesizers***

**Michael Perrott**

**Massachusetts Institute of Technology**

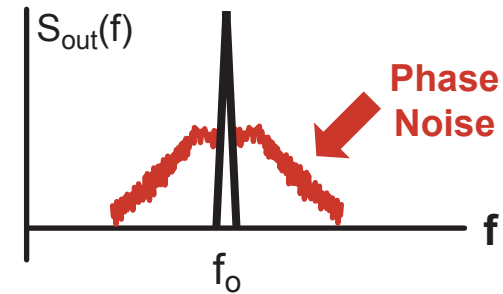
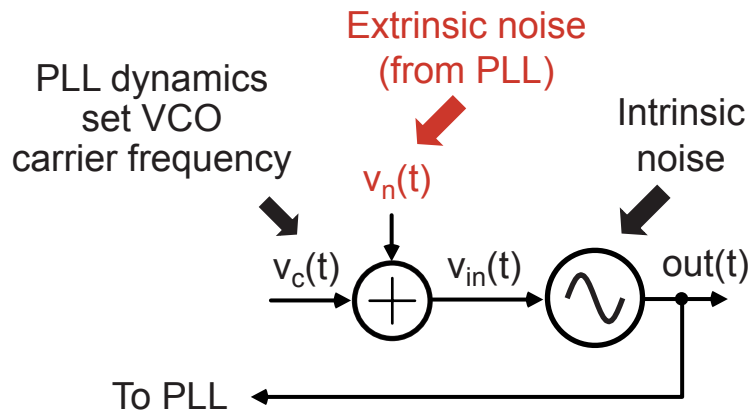
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# Frequency Synthesizer Noise in Wireless Systems



- Synthesizer noise has a negative impact on system
  - Receiver – lower sensitivity, poorer blocking performance
  - Transmitter – increased spectral emissions (output spectrum must meet a mask requirement)
- Noise is characterized in frequency domain

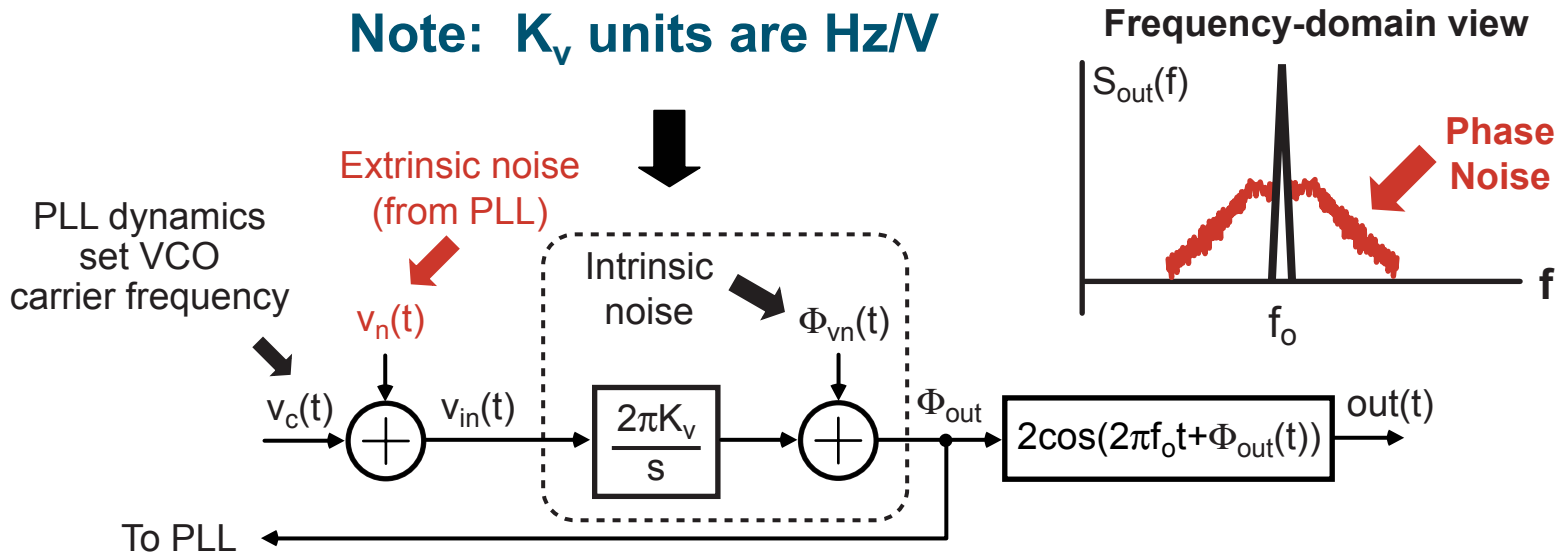
# Noise Modeling for Frequency Synthesizers



- **PLL has an impact on VCO noise in two ways**
  - Adds extrinsic noise from various PLL circuits
  - Highpass filters VCO noise through PLL feedback dynamics
- **Focus on modeling the above based on phase deviations**
  - Simpler than dealing directly with PLL sine wave output

# Phase Deviation Model for Noise Analysis

Note:  $K_v$  units are Hz/V



- **Model the impact of noise on instantaneous phase**
  - Relationship between PLL output and instantaneous phase

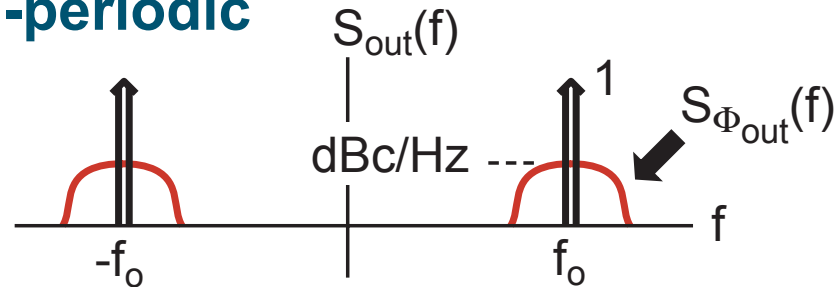
$$out(t) = 2 \cos(2\pi f_o t + \Phi_{out}(t))$$

- Output spectrum (from Lecture 12)

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$

# Phase Noise Versus Spurious Noise

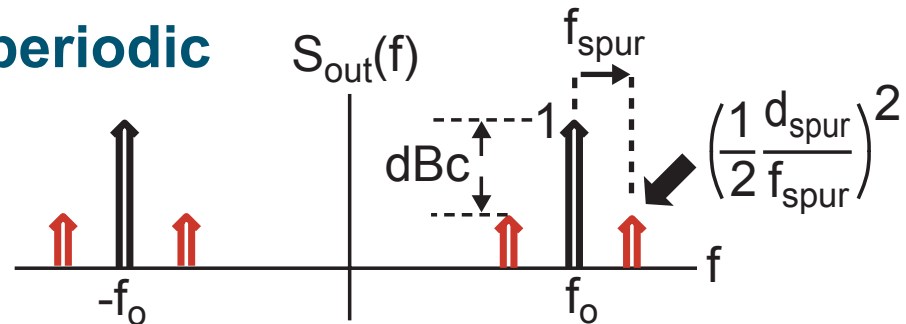
- Phase noise is non-periodic



- Described as a spectral density relative to carrier power

$$L(f) = 10 \log(S_{\Phi_{out}}(f)) \text{ dBc/Hz}$$

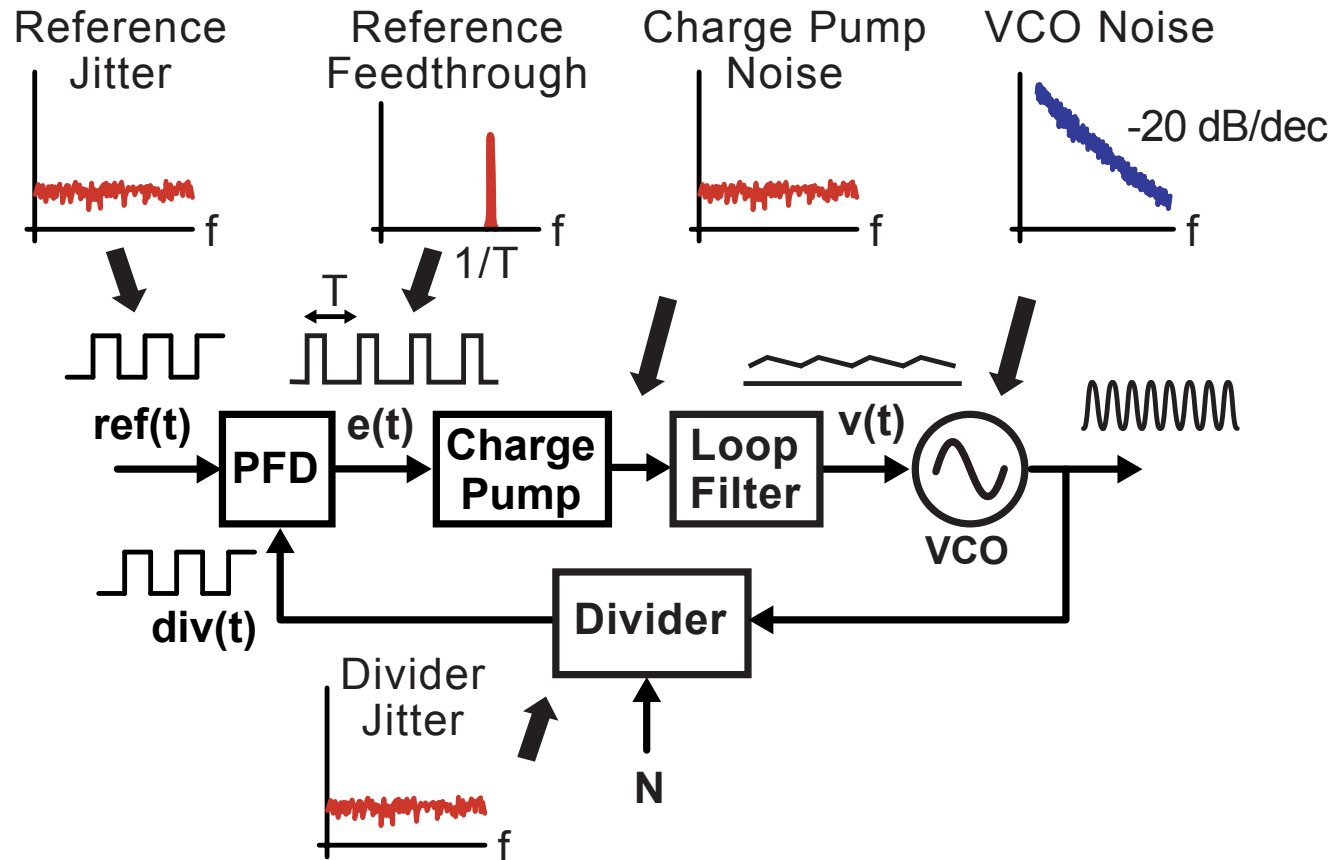
- Spurious noise is periodic



- Described as tone power relative to carrier power

$$20 \log \left( \frac{d_{spur}}{2f_{spur}} \right) \text{ dBc}$$

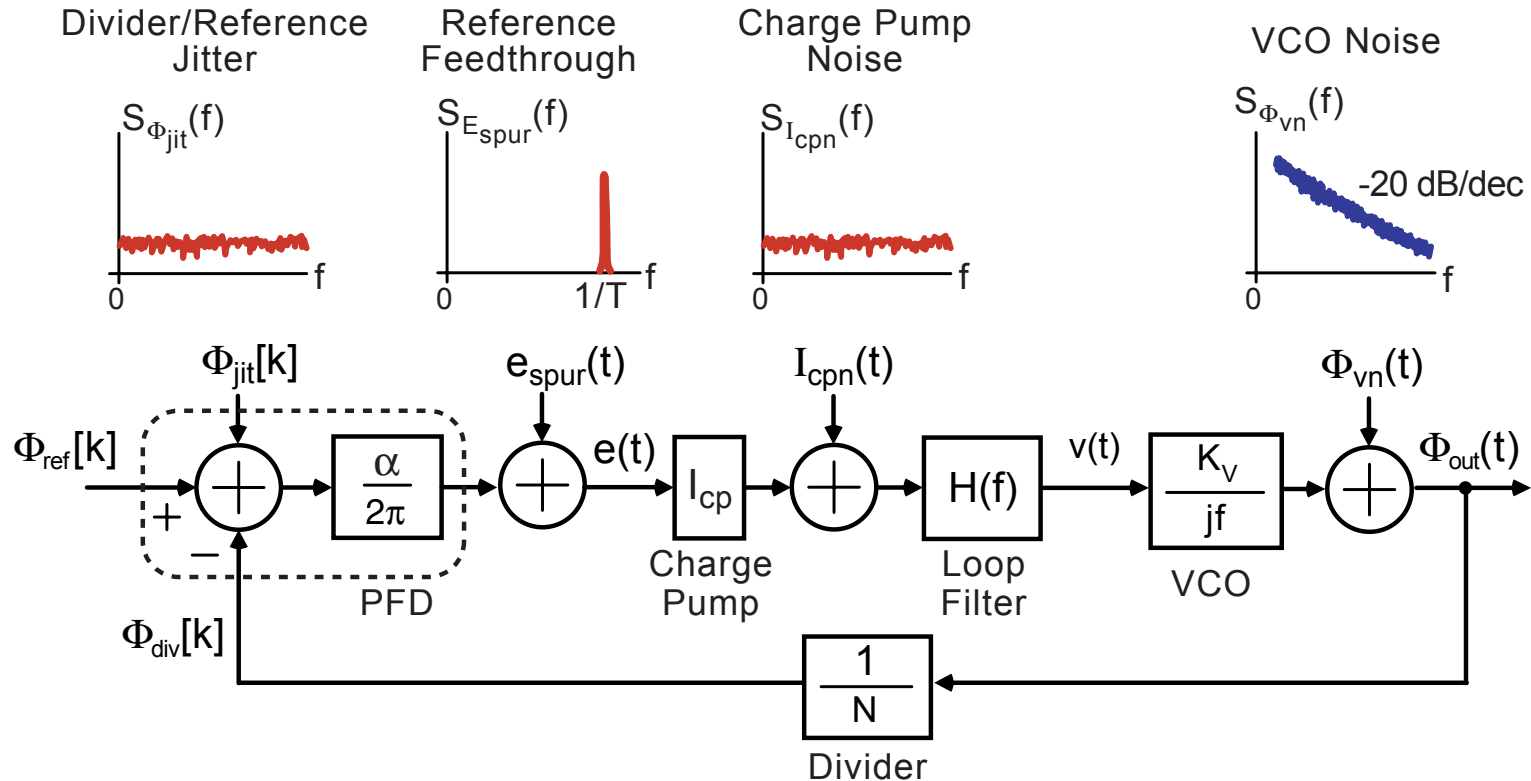
# Sources of Noise in Frequency Synthesizers



## ■ Extrinsic noise sources to VCO

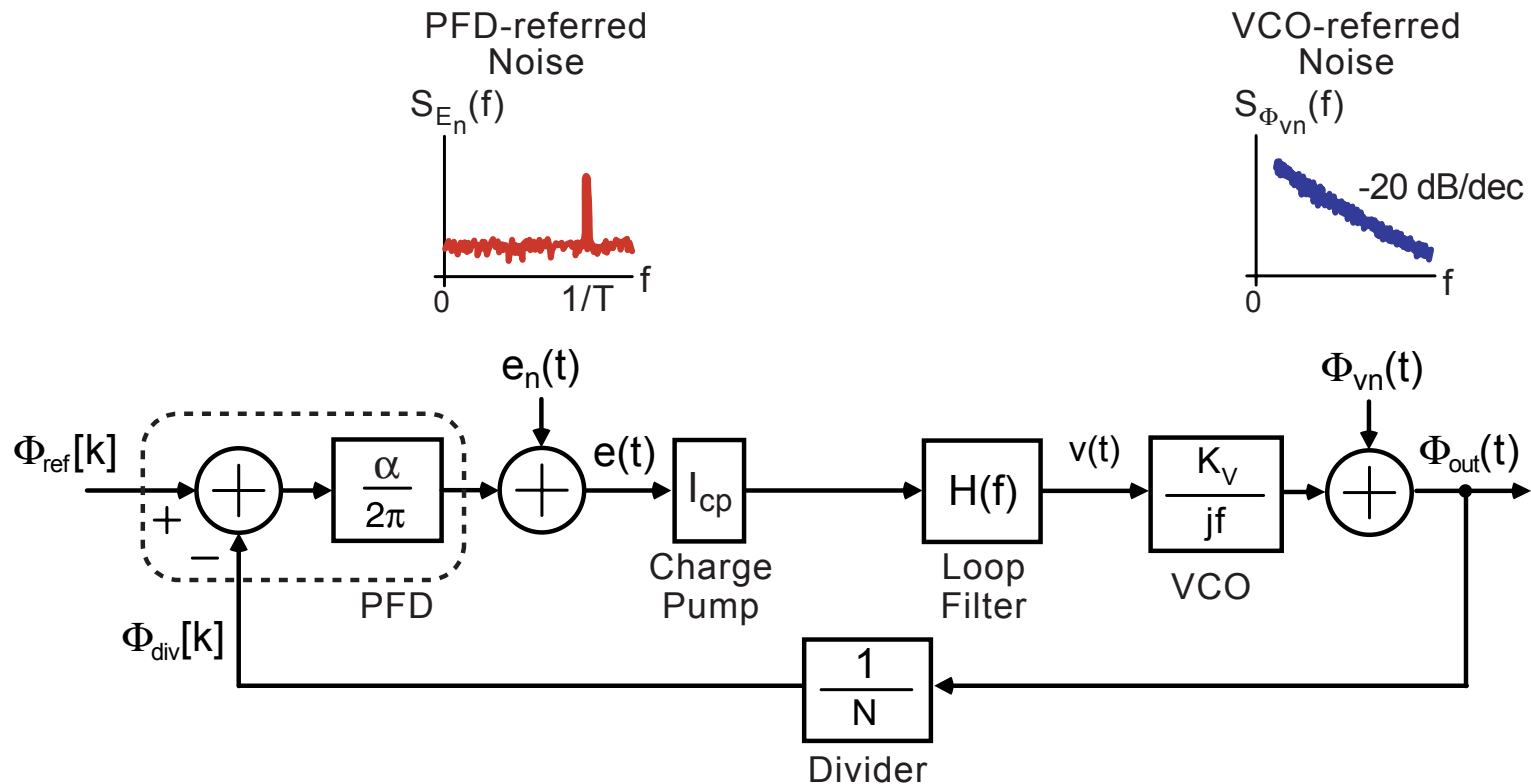
- Reference/divider jitter and reference feedthrough
- Charge pump noise

# Modeling the Impact of Noise on Output Phase of PLL



- **Determine impact on output phase by deriving transfer function from each noise source to PLL output phase**
  - **There are a lot of transfer functions to keep track of!**

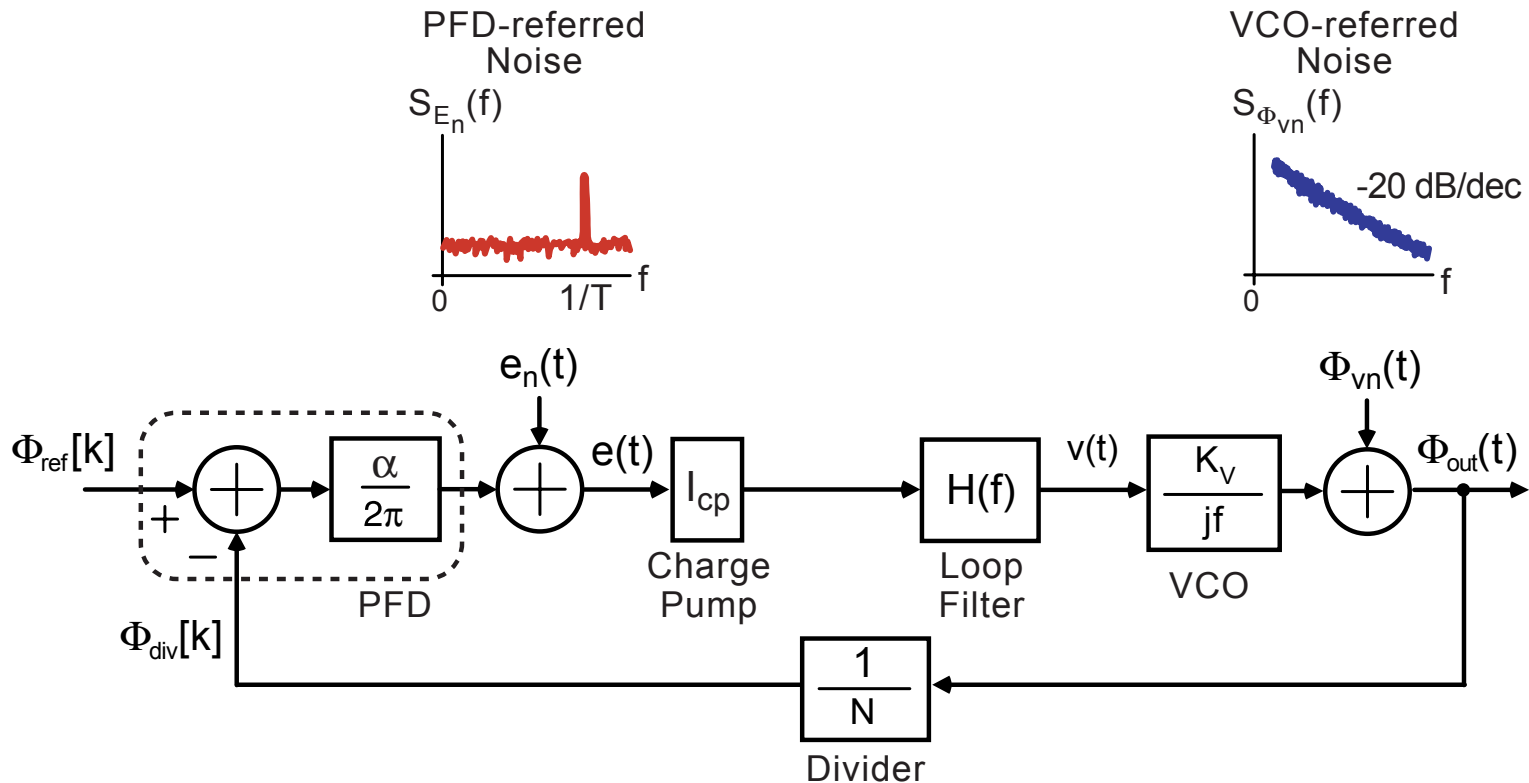
# Simplified Noise Model



- Refer all PLL noise sources (other than the VCO) to the PFD output
  - PFD-referred noise corresponds to the sum of these noise sources referred to the PFD output



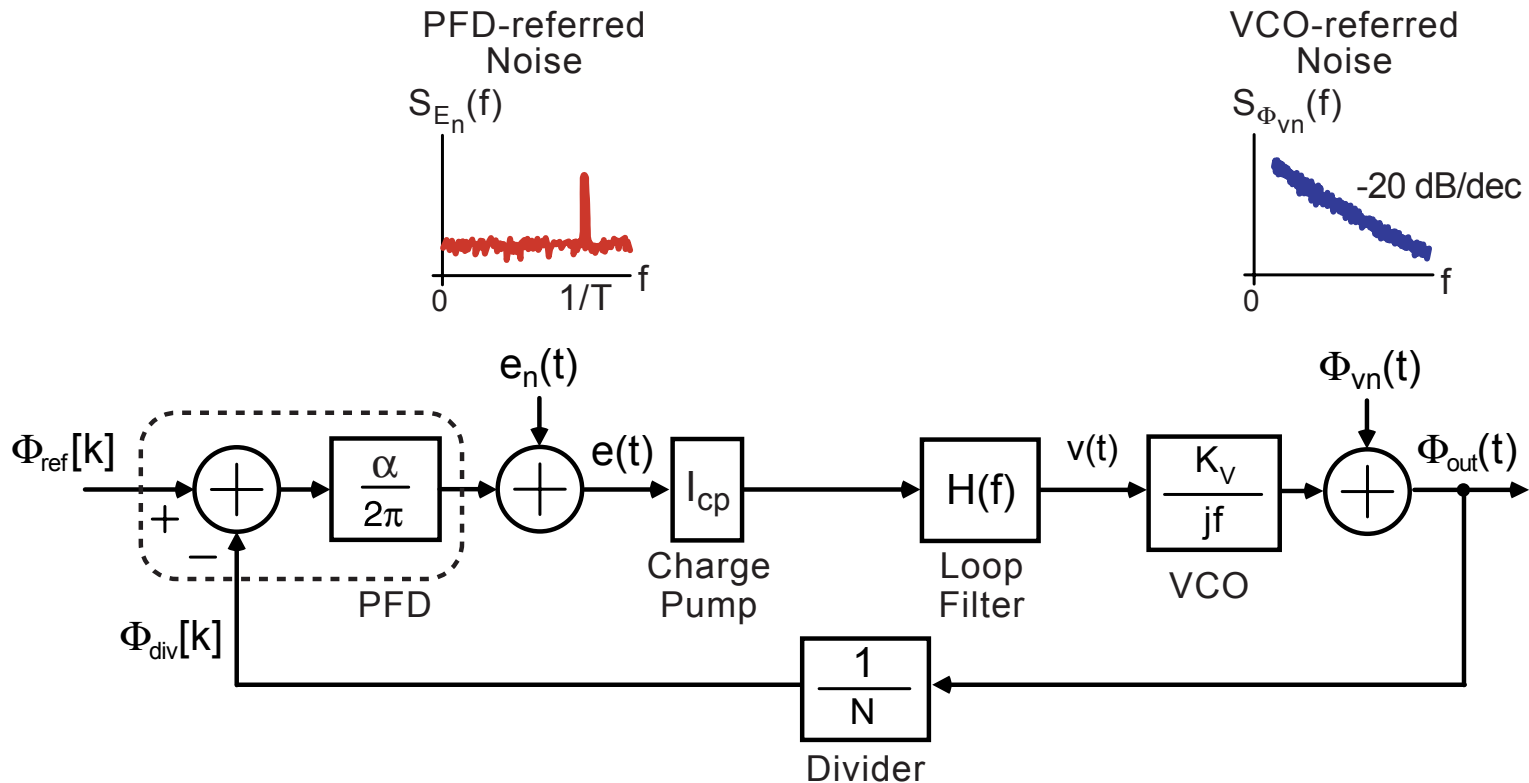
# Impact of PFD-referred Noise on Synthesizer Output



- Transfer function derived using Black's formula

$$\frac{\Phi_{out}}{e_n} = \frac{I_{cp}H(f)K_v/(jf)}{1 + \alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)}$$

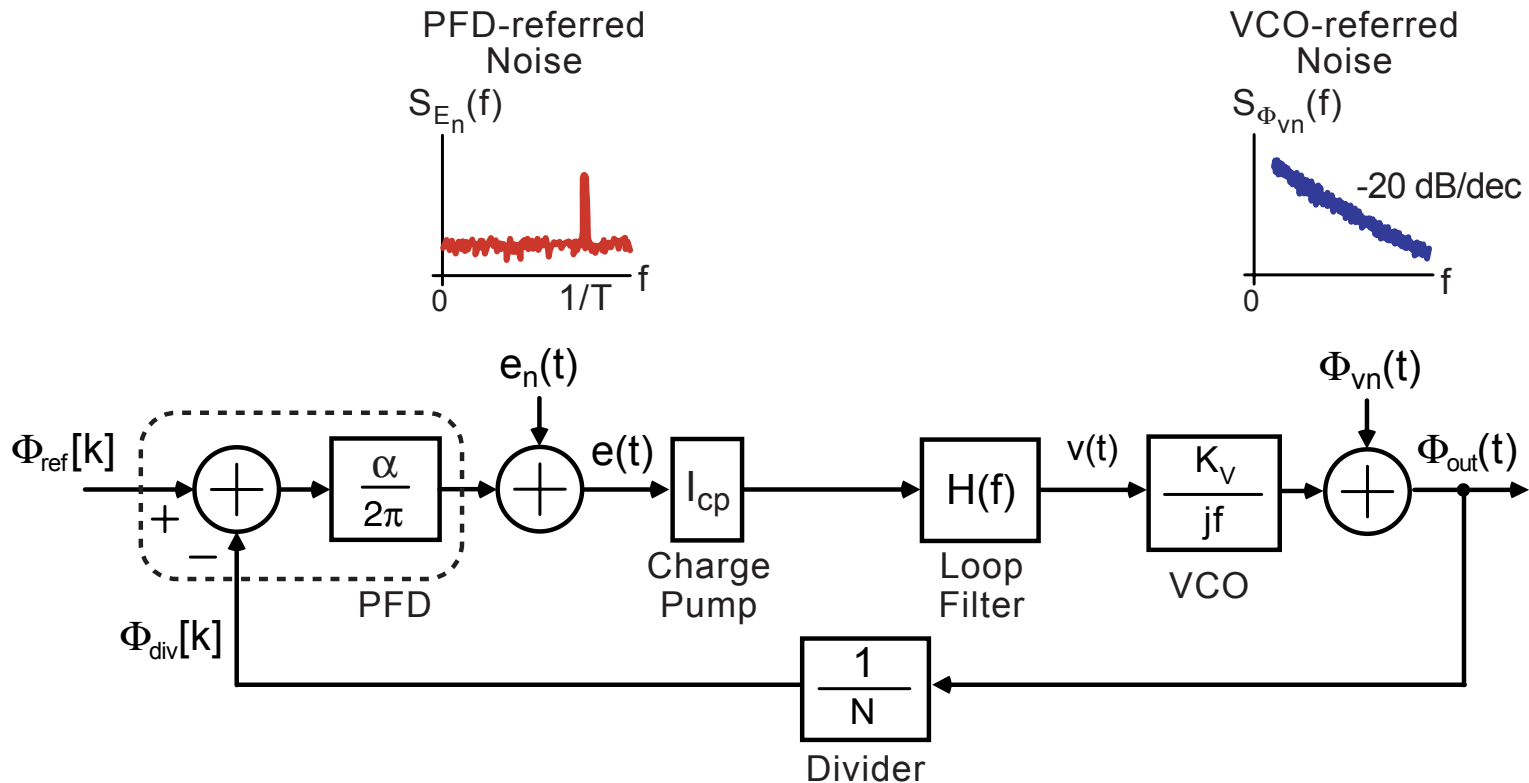
# Impact of VCO-referred Noise on Synthesizer Output



- Transfer function again derived from Black's formula

$$\frac{\Phi_{out}}{e_n} = \frac{1}{1 + \alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)}$$

# A Simpler Parameterization for PLL Transfer Functions



- Define  $G(f)$  as

$$G(f) = \frac{A(f)}{1 + A(f)}$$

Always has a gain of one at DC

- $A(f)$  is the open loop transfer function of the PLL

$$A(f) = \alpha / (2\pi) I_{cp} H(f) K_v / (j f) (1/N)$$

# Parameterize Noise Transfer Functions in Terms of $G(f)$

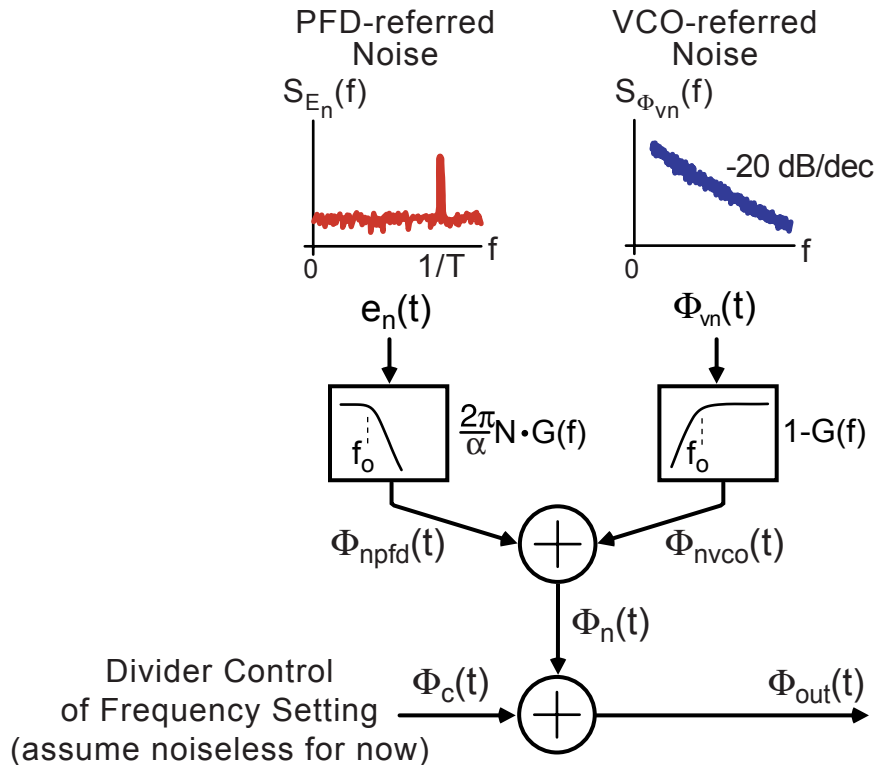
## ■ PFD-referred noise

$$\begin{aligned}\frac{\Phi_{out}}{e_n} &= \frac{I_{cp}H(f)K_v/(jf)}{1 + \alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)} \\ &= \frac{2\pi N}{\alpha} \frac{\alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)}{1 + \alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)} \\ &= \frac{2\pi N}{\alpha} \frac{A(f)}{1 + A(f)} = \boxed{\frac{2\pi N}{\alpha} G(f)}\end{aligned}$$

## ■ VCO-referred noise

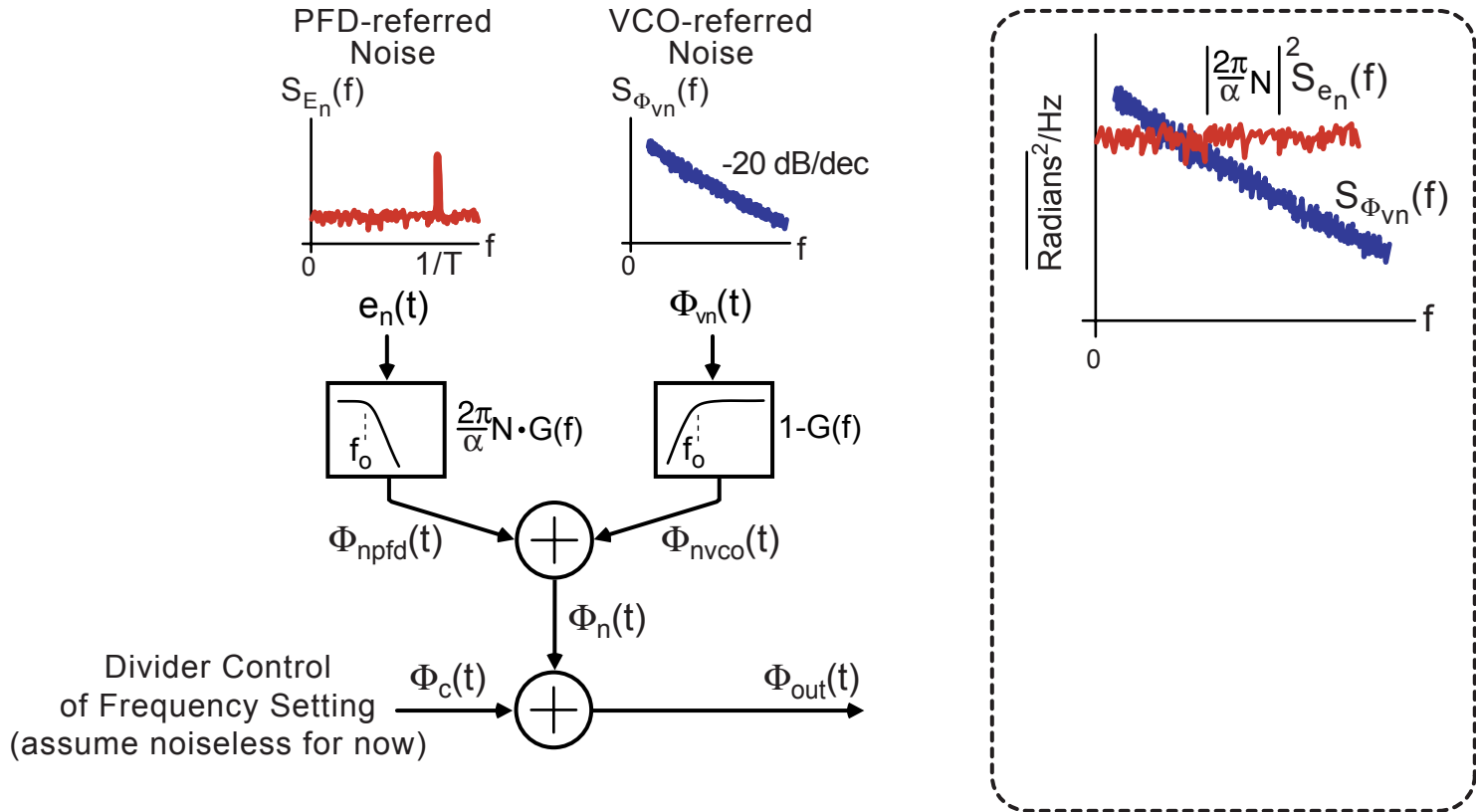
$$\begin{aligned}\frac{\Phi_{out}}{\Phi_{vn}} &= \frac{1}{1 + \alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)} \\ &= \frac{1}{1 + A(f)} = 1 - \frac{A(f)}{1 + A(f)} = \boxed{1 - G(f)}\end{aligned}$$

# Parameterized PLL Noise Model



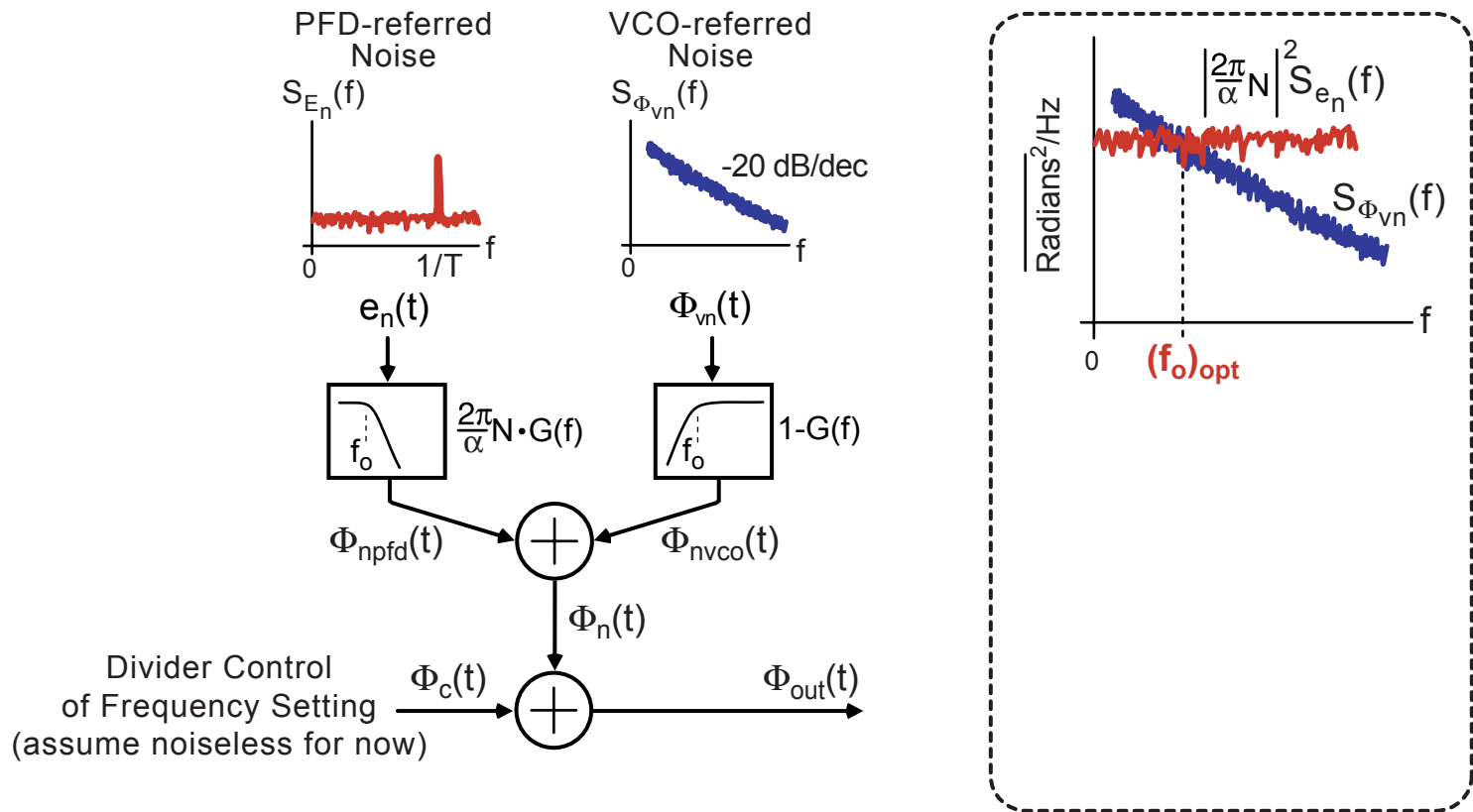
- PFD-referred noise is lowpass filtered
- VCO-referred noise is highpass filtered
- Both filters have the same transition frequency values
  - Defined as  $f_0$

# Impact of PLL Parameters on Noise Scaling



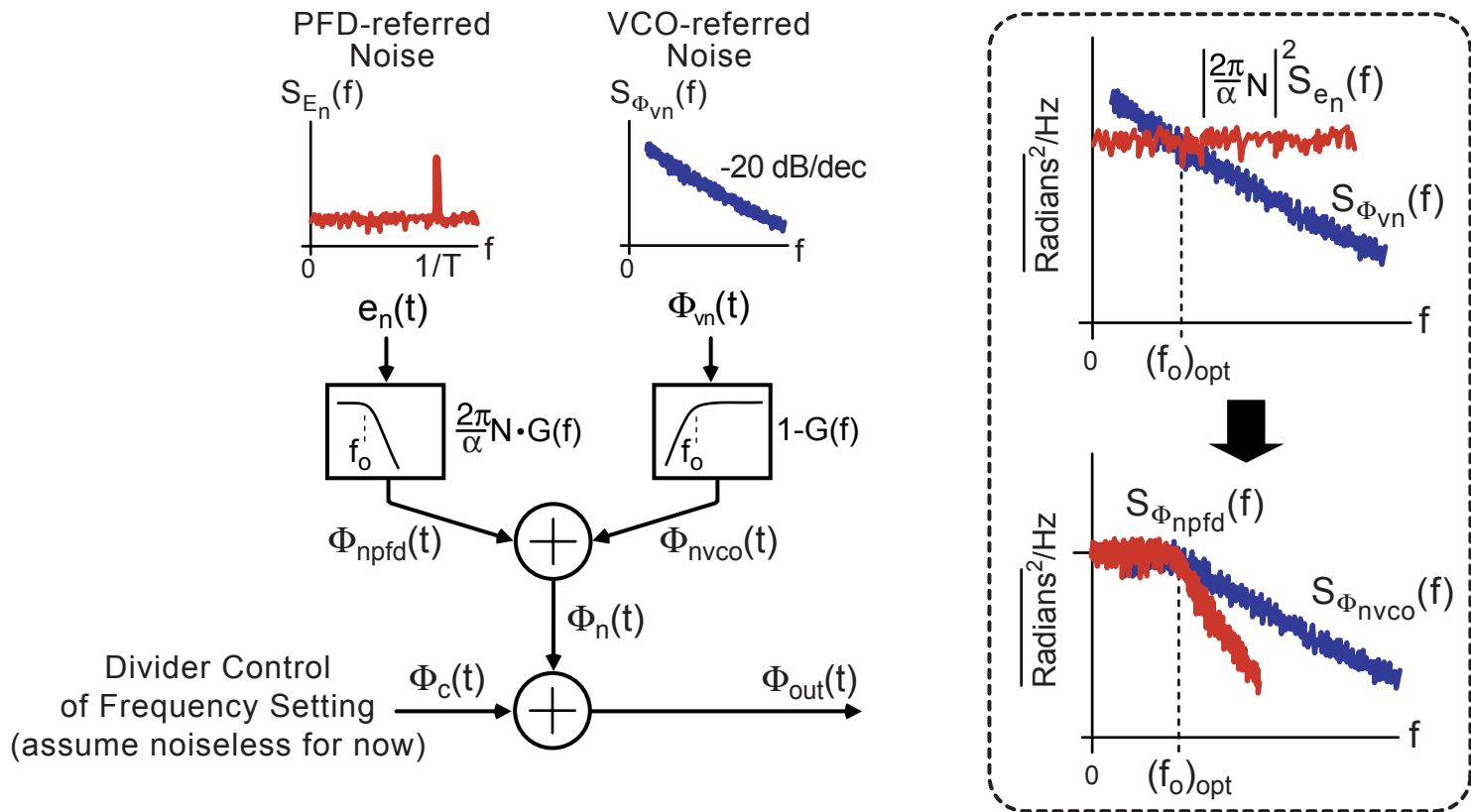
- **PFD-referred noise is scaled by square of divide value and inverse of PFD gain**
  - ▬ High divide values lead to large multiplication of this noise
- **VCO-referred noise is not scaled (only filtered)**

# Optimal Bandwidth Setting for Minimum Noise



- **Optimal bandwidth is where scaled noise sources meet**
  - Higher bandwidth will pass more PFD-referred noise
  - Lower bandwidth will pass more VCO-referred noise

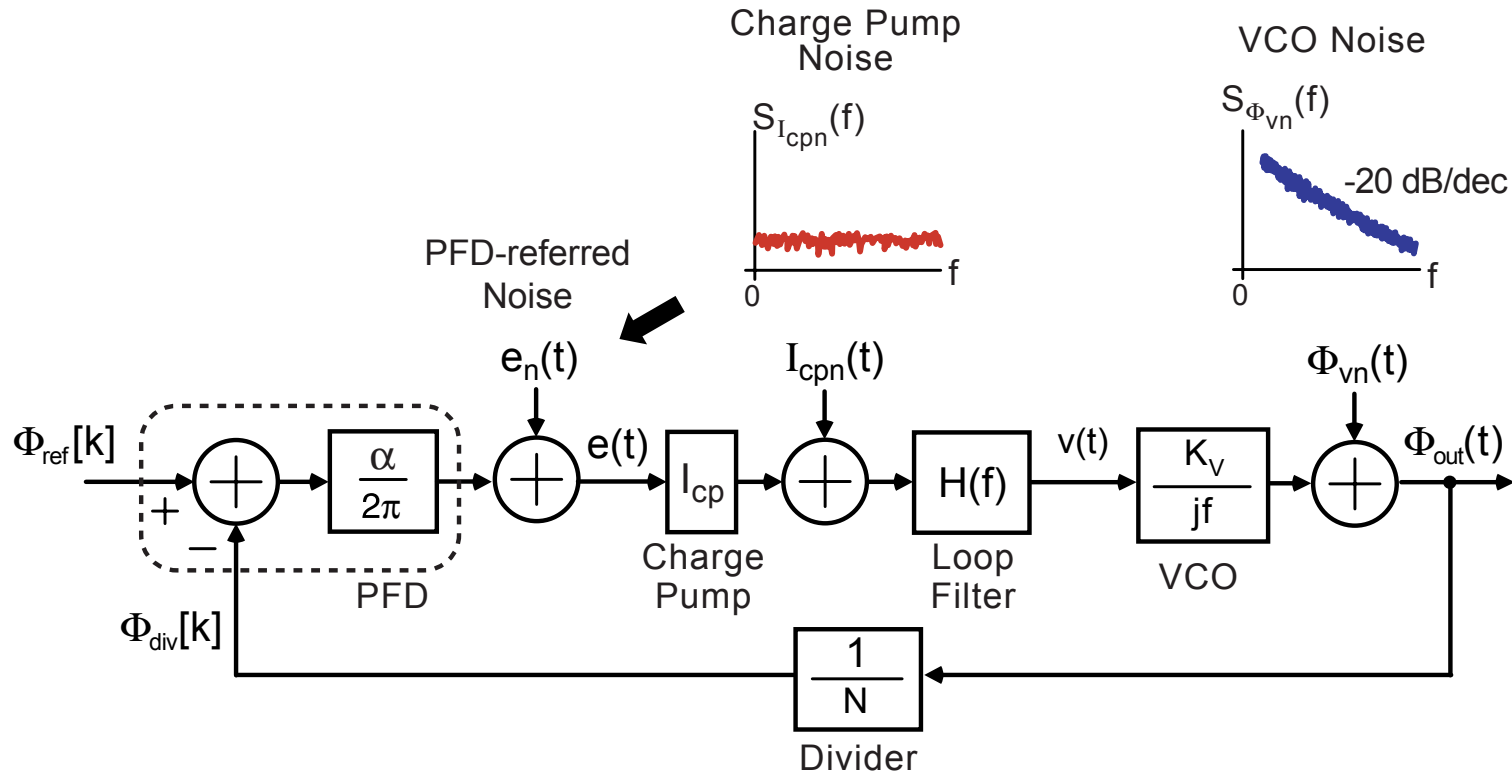
# Resulting Output Noise with Optimal Bandwidth



- **PFD-referred noise dominates at low frequencies**
  - Corresponds to close-in phase noise of synthesizer
- **VCO-referred noise dominates at high frequencies**
  - Corresponds to far-away phase noise of synthesizer



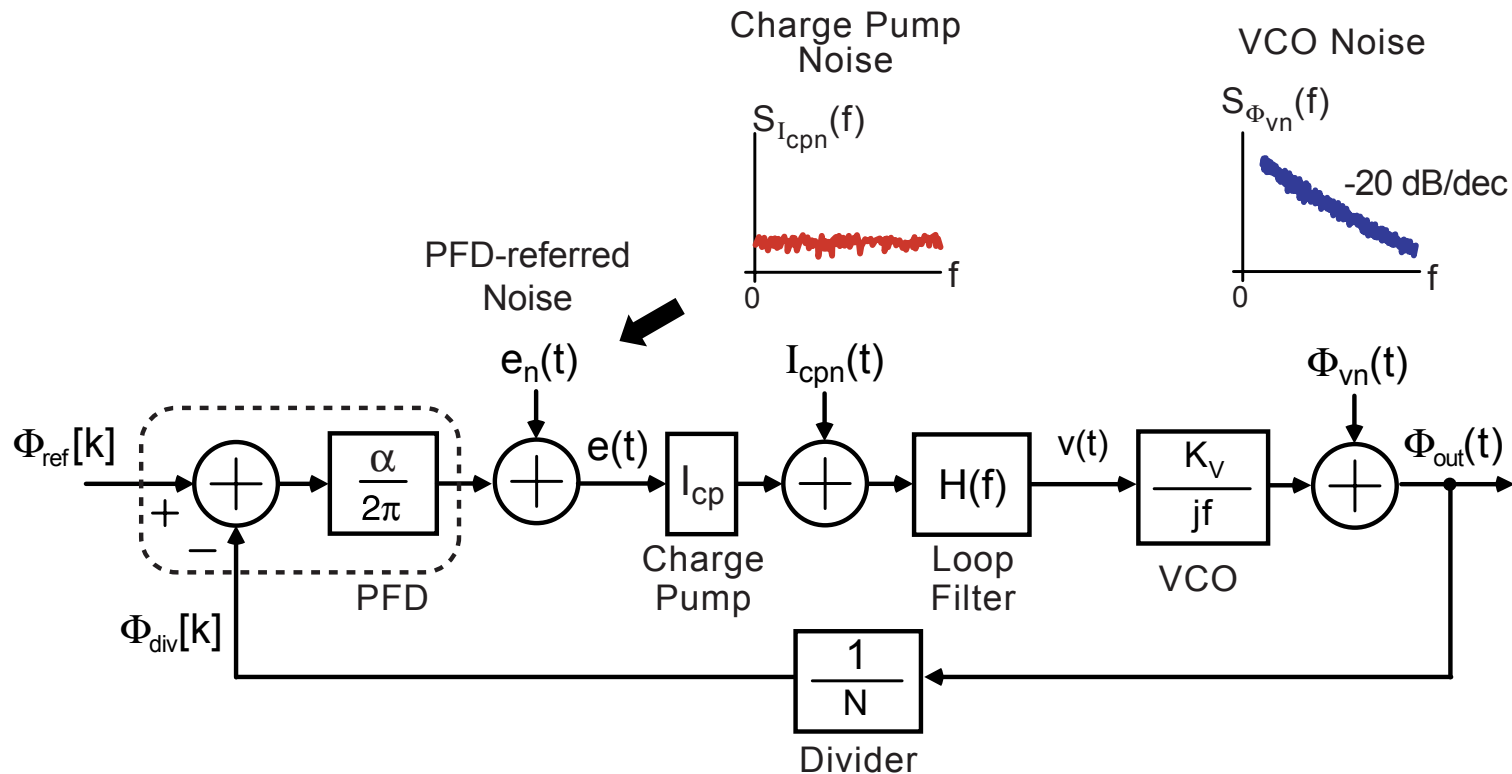
# Analysis of Charge Pump Noise Impact



- We can refer charge pump noise to PFD output by simply scaling it by  $1/I_{cp}$

$$\frac{\Phi_{out}}{I_{cpn}} = \left( \frac{1}{I_{cp}} \right) \frac{\Phi_{out}}{e_n} = \left( \frac{1}{I_{cp}} \right) \frac{2\pi}{\alpha} NG(f)$$

# Calculation of Charge Pump Noise Impact

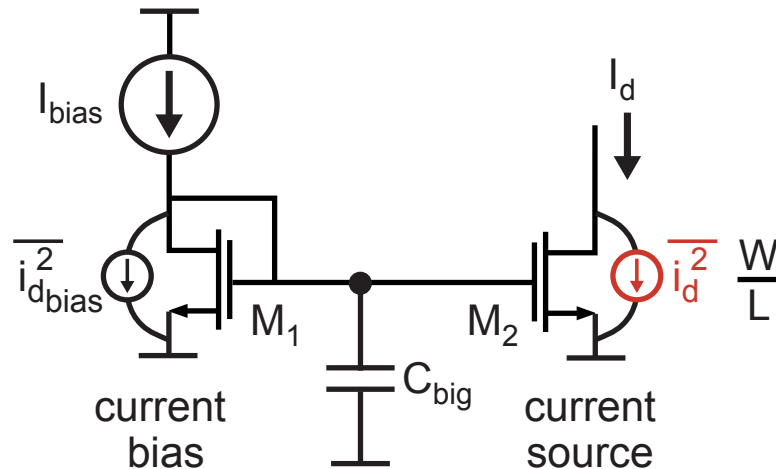


- **Contribution of charge pump noise to overall output noise**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{I_{cp}}\right)^2 \left(\frac{2\pi N}{\alpha}\right)^2 |G(f)|^2 S_{I_{cpn}}(f) + \text{other sources}$$

- **Need to determine impact of  $I_{cp}$  on  $S_{I_{cpn}}(f)$**

# Impact of Transistor Current Value on its Noise



- Charge pump noise will be related to the current it creates as

$$S_{I_{cpn}}(f) \propto \frac{\overline{I_d^2}}{\Delta f} = 4kT\gamma g_{do}$$

- Recall that  $g_{do}$  is the channel resistance at zero  $V_{ds}$ 
  - At a fixed current density, we have

$$g_{do} \propto W \propto I_d \Rightarrow \boxed{\overline{I_d^2} \propto I_d}$$

# Impact of Charge Pump Current Value on Output Noise

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- **Recall**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{I_{cp}}\right)^2 \left(\frac{2\pi N}{\alpha}\right)^2 |G(f)|^2 S_{I_{cpn}}(f) + \text{other sources}$$

- **Given previous slide, we can say**

$$S_{I_{cpn}}(f) \propto I_{cp}$$

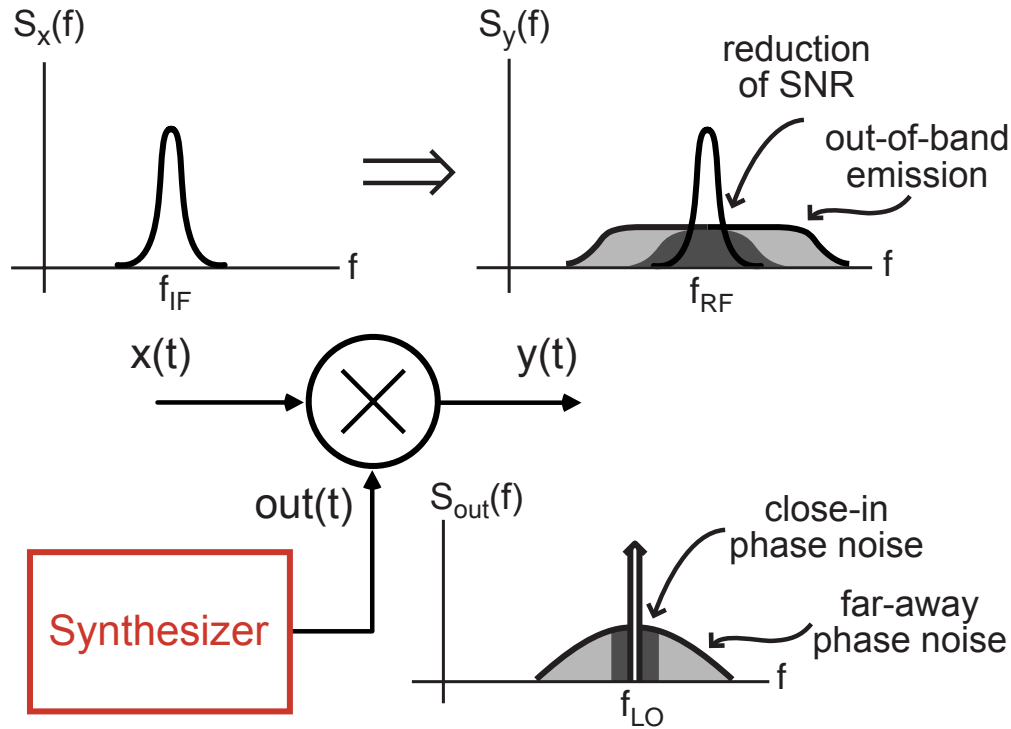
- Assumes a fixed current density for the key transistors in the charge pump as  $I_{cp}$  is varied

- **Therefore**

$$S_{\Phi_{out}}(f) \Big|_{\text{charge pump}} \propto \frac{1}{I_{cp}}$$

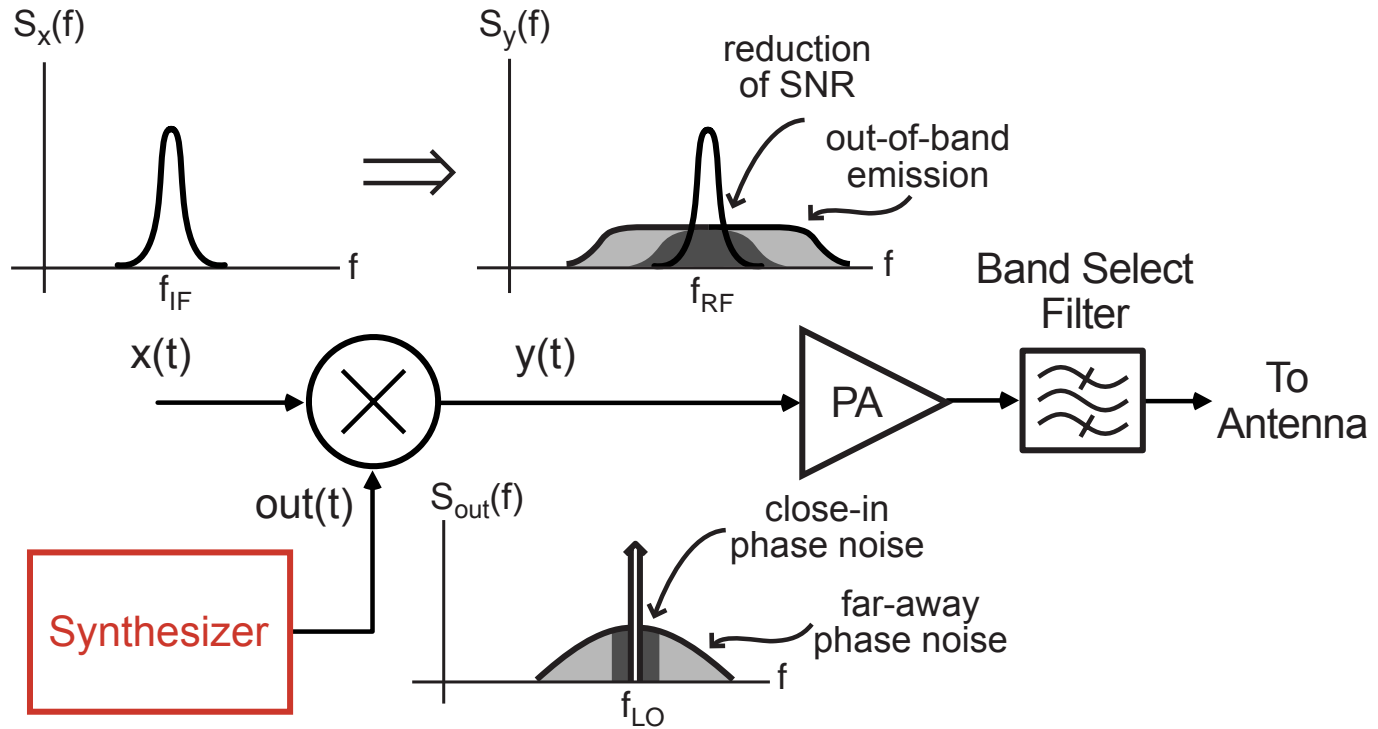
- Want high charge pump current to achieve low noise
- Limitation set by power and area considerations

# Impact of Synthesizer Noise on Transmitters



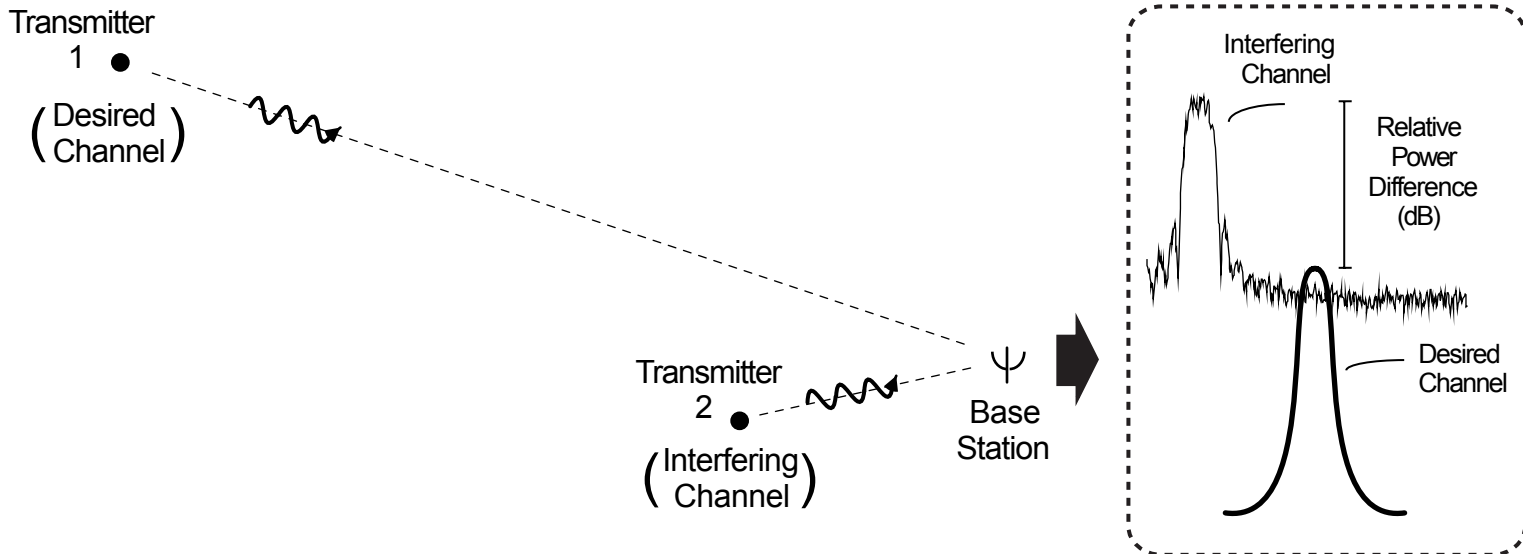
- **Synthesizer noise can be lumped into two categories**
  - **Close-in phase noise: reduces SNR of modulated signal**
  - **Far-away phase noise: creates spectral emissions outside the desired transmit channel**
    - This is the critical issue for transmitters

# Impact of Remaining Portion of Transmitter



- **Power amplifier**
  - Nonlinearity will increase out-of-band emission and create harmonic content
- **Band select filter**
  - Removes harmonic content, but not out-of-band emission

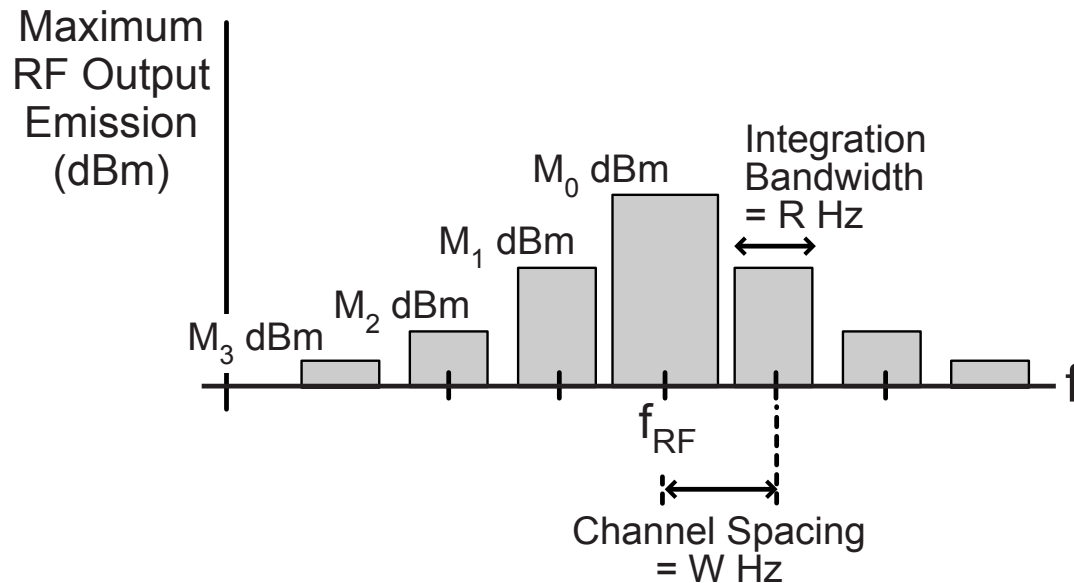
# Why is Out-of-Band Emission A Problem?



## ■ Near-far problem

- Interfering transmitter closer to receiver than desired transmitter
- Out-of-emission requirements must be stringent to prevent complete corruption of desired signal

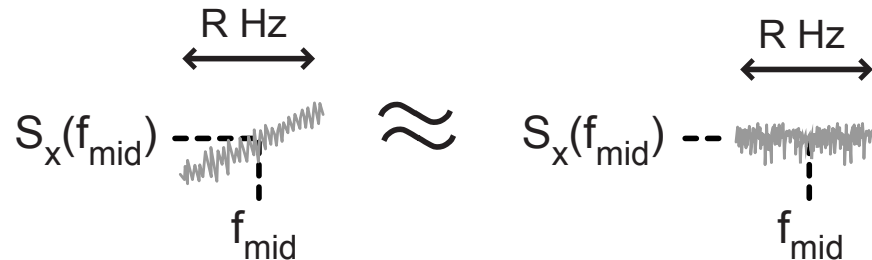
# Specification of Out-of-Band Emissions



- **Maximum radiated power is specified in desired and adjacent channels**
  - **Desired channel power:** maximum is  $M_0$  dBm
  - **Out-of-band emission:** maximum power defined as integration of transmitted spectral density over bandwidth  $R$  centered at midpoint of each channel offset



# Calculation of Transmitted Power in a Given Channel



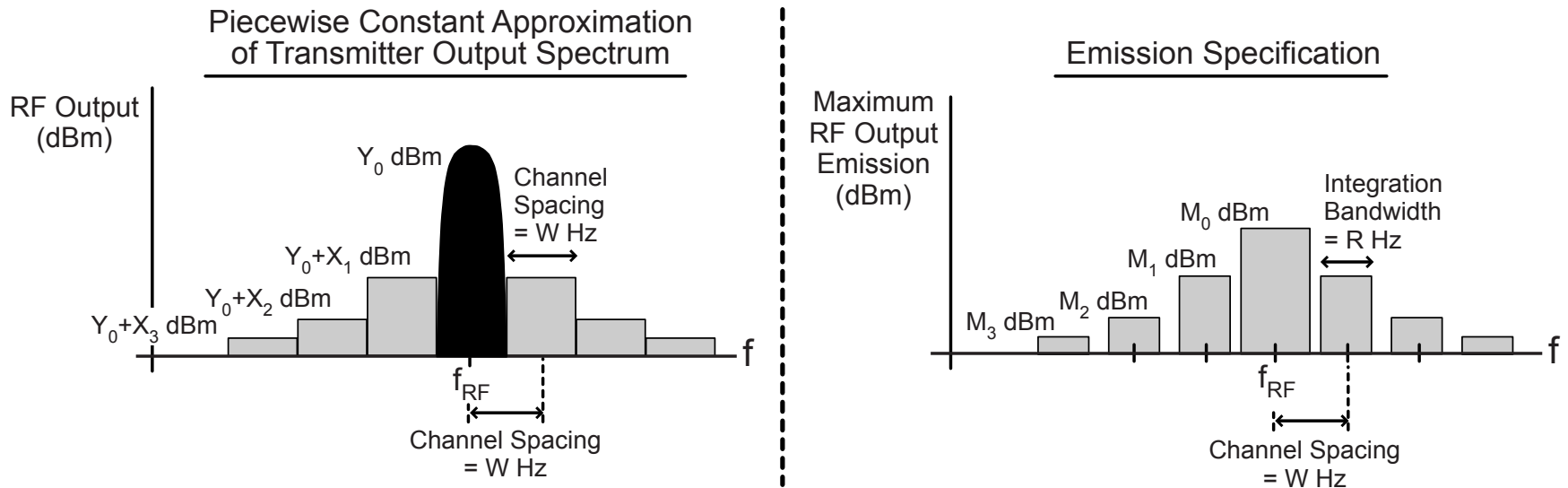
- For simplicity, assume that the spectral density is flat over the channel bandwidth
  - Actual spectral density of signal often varies with frequency over the bandwidth of a given channel
- Resulting power calculation (single-sided  $S_x(f)$ )

$$P_x = \int_{f_{mid}-R/2}^{f_{mid}+R/2} S_x(f) df \approx RS_x(f_{mid})$$

- Express in dB ( Note:  $\text{dB}(x) = 10\log(x)$  )

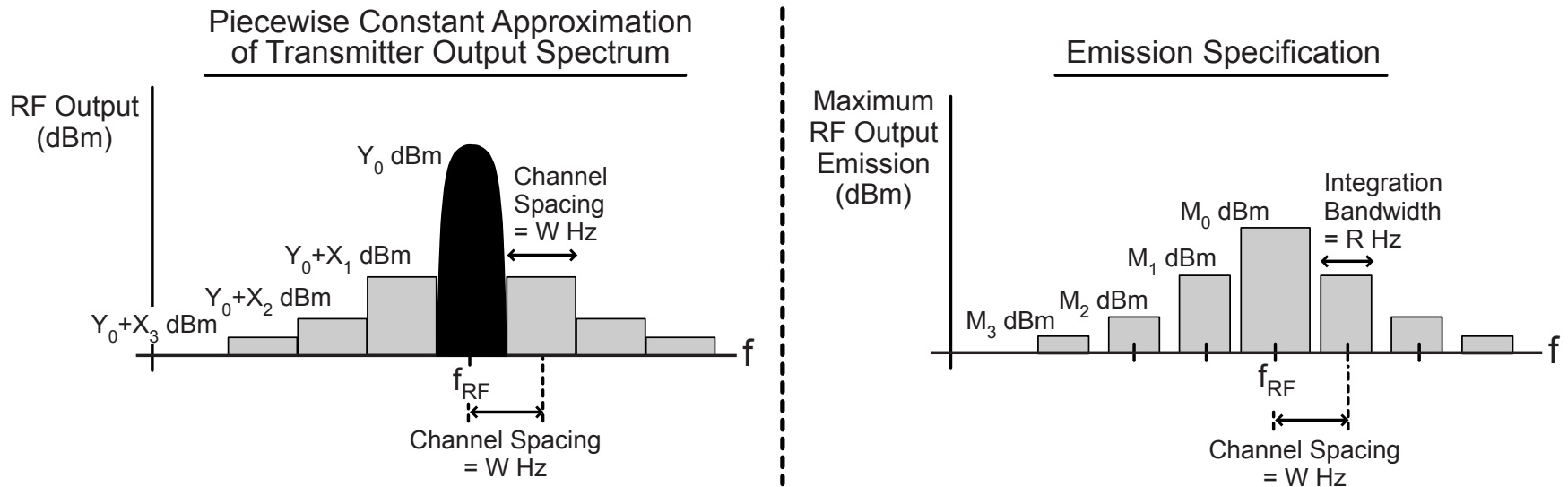
$$\text{dB}(P_x) \approx \text{dB}(RS_x(f_{mid})) = \text{dB}(S_x(f_{mid})) + \text{dB}(R)$$

# Transmitter Output Versus Emission Specification



- Assume a piecewise constant spectral density profile for transmitter
  - Simplifies calculations
- Issue: emission specification is measured over a narrower band than channel spacing
  - Need to account for bandwidth discrepancy when doing calculations

# Correction Factor for Bandwidth Mismatch



## ■ Calculation of maximum emission in offset channel 1

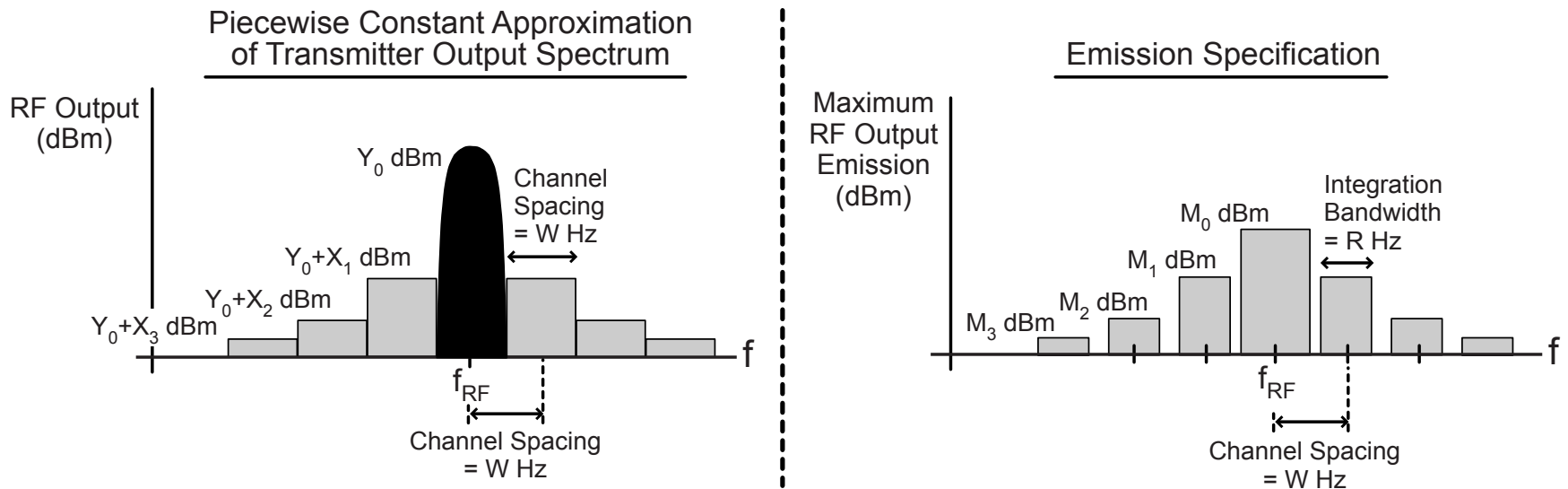
$$\text{dB}(S_{(Y_0+X_1)}R) \leq M_1$$

$$\Rightarrow \text{dB} \left( S_{(Y_0+X_1)} W \frac{R}{W} \right) \leq M_1$$

$$\Rightarrow \text{dB} \left( S_{(Y_0+X_1)} W \right) + \text{dB} \left( \frac{R}{W} \right) \leq M_1$$

$$\Rightarrow Y_0 + X_1 + \text{dB} \left( \frac{R}{W} \right) \leq M_1 \Rightarrow X_1 \leq M_1 - Y_0 + \text{dB} \left( \frac{W}{R} \right)$$

# Condition for Most Stringent Emission Requirement



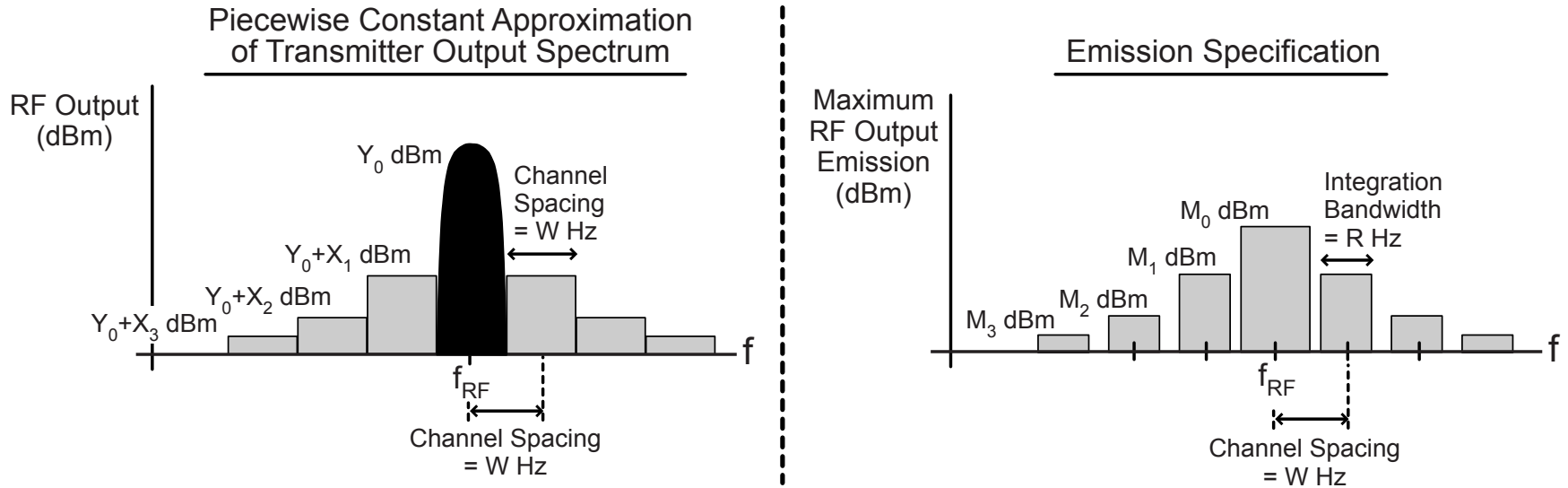
- **Out-of-band emission requirements are function of the power of the signal in the desired channel**
  - For offset channel 1 (as calculated on previous slide)

$$X_1 \leq M_1 - Y_0 + \text{dB} \left( \frac{W}{R} \right)$$

- **Most stringent case is when  $Y_0$  maximum**

$$\Rightarrow Y_0 = M_0$$

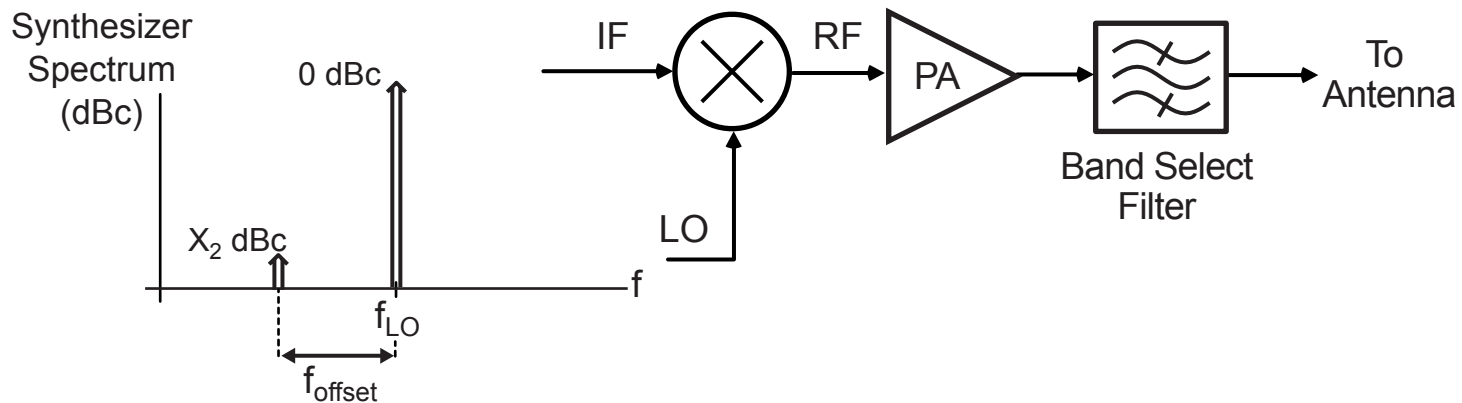
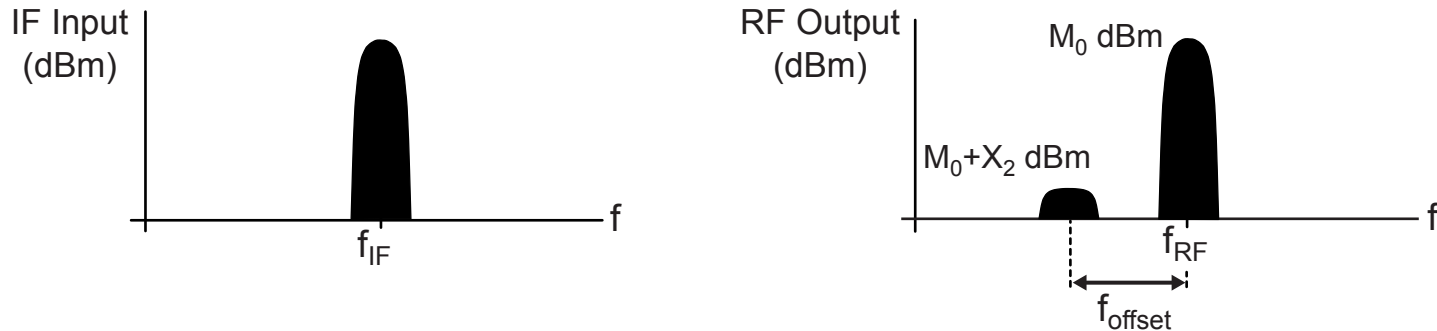
# Table of Most Stringent Emission Requirements



Channel Offset	Mask Power	Emission Requirements (Most Stringent)
0	$M_0$ dBm	$Y_0 = M_0$ (for most stringent case)
1	$M_1$ dBm	$X_1 = M_1 - M_0 + \text{dB}(W/R)$ dB
2	$M_2$ dBm	$X_2 = M_2 - M_0 + \text{dB}(W/R)$ dB
3	$M_3$ dBm	$X_3 = M_3 - M_0 + \text{dB}(W/R)$ dB

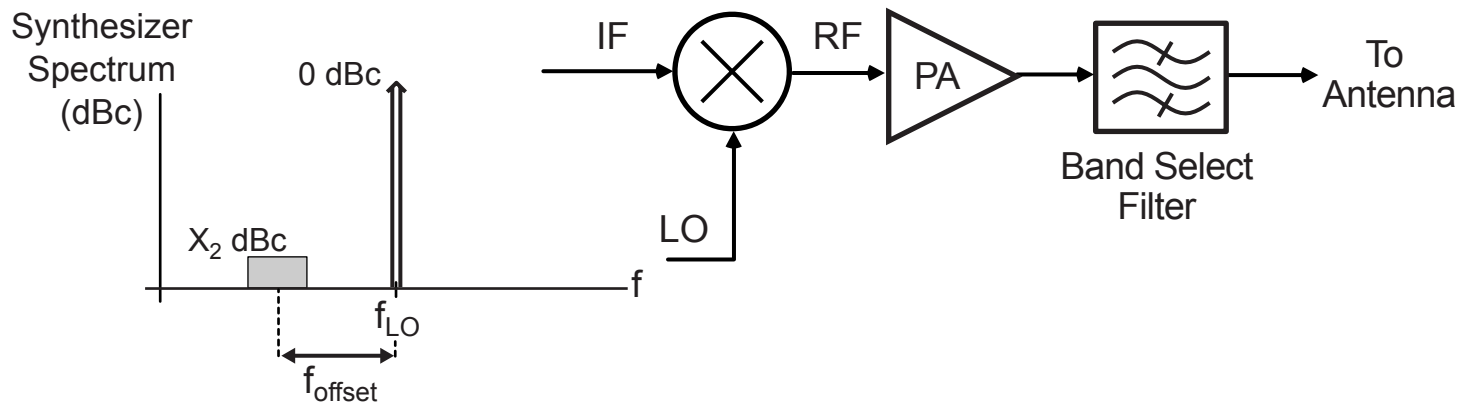
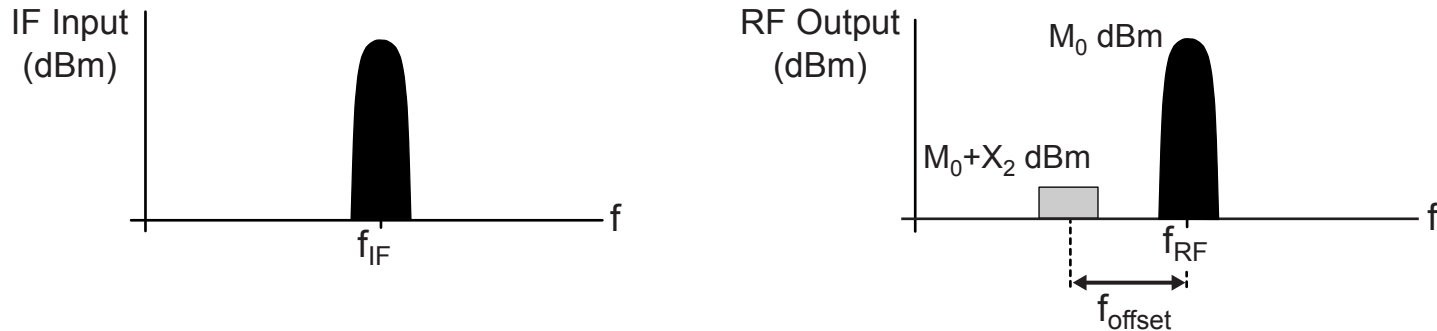
(Note :  $\text{dB}(W/R) = 10 \log(W/R)$ )

# Impact of Synthesizer Noise on Transmitter Output



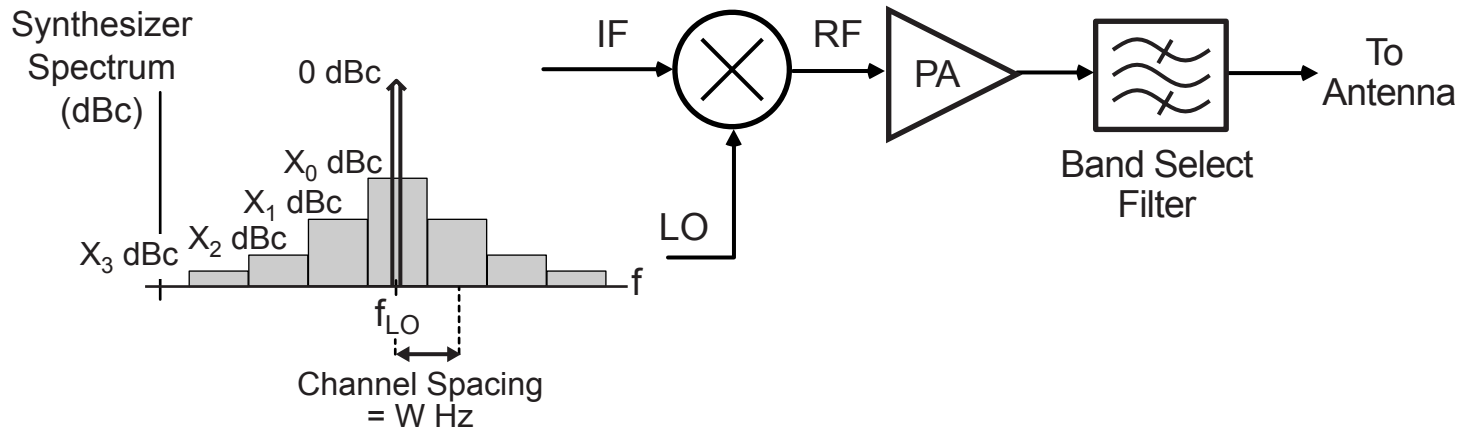
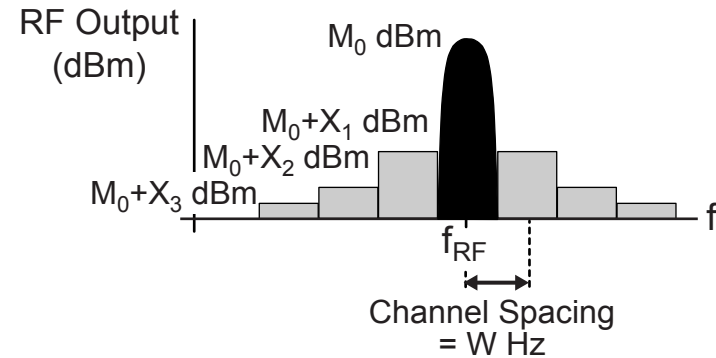
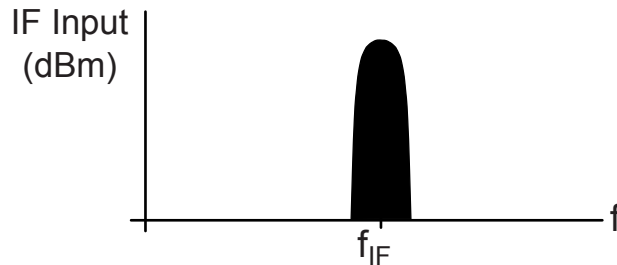
- Consider a spurious tone at a given offset frequency
  - Convolution with IF signal produces a replica of the desired signal at the given offset frequency

# Impact of Synthesizer Phase Noise (Isolated Channel)



- Consider phase noise at a given offset frequency
  - Convolution with IF signal produces a smeared version of the desired signal at the given offset frequency
    - For simplicity, approximate smeared signal as shown

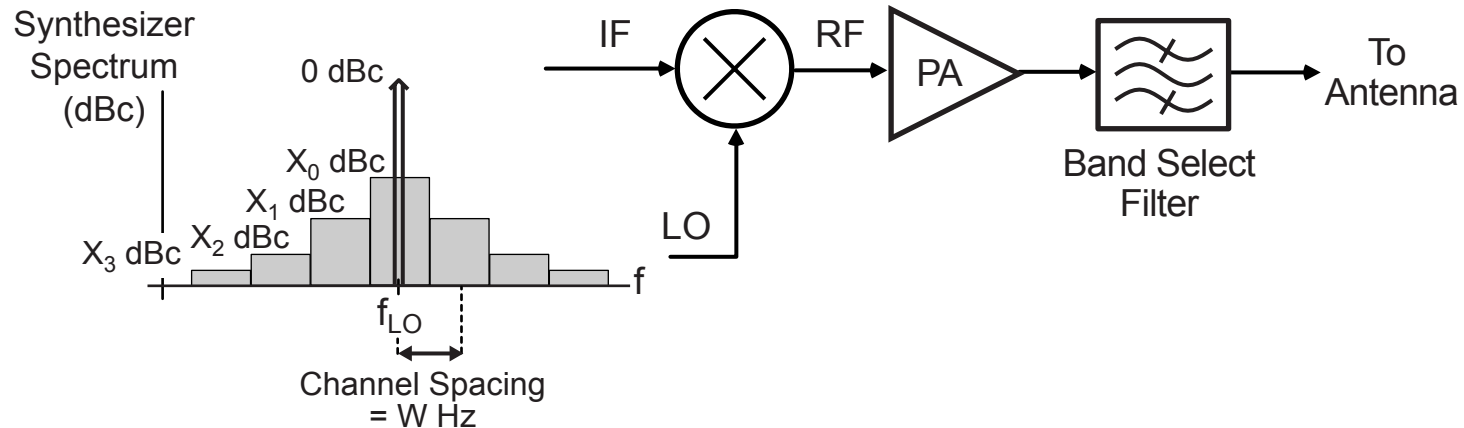
# Impact of Synthesizer Phase Noise (All Channels)



- Partition synthesizer phase noise into channels
  - Required phase noise power (dBc) in each channel is related directly to spectral mask requirements
    - Exception is  $X_0$  – set by transmit SNR requirements



# Synthesizer Phase Noise Requirements



- Impact of channel bandwidth (offset channel 1)

$$\text{dB}(S_{X_1}W) \leq X_1 \text{ dBc} \Rightarrow \text{dB}(S_{X_1}) \leq X_1 - \text{dB}(W) \text{ dBc/Hz}$$

- Overall requirements (most stringent, i.e.,  $Y_0 = M_0$ )

Channel Offset	Emission Requirements (Most Stringent)	Maximum Synth. Phase Noise (Most Stringent)
0	$Y_0 = M_0$	set by required transmit SNR
1	$X_1 = M_1 - M_0 + \text{dB}(W/R) \text{ dB}$	$X_1 - \text{dB}(W) \text{ dBc/Hz}$
2	$X_2 = M_2 - M_0 + \text{dB}(W/R) \text{ dB}$	$X_2 - \text{dB}(W) \text{ dBc/Hz}$
3	$X_3 = M_3 - M_0 + \text{dB}(W/R) \text{ dB}$	$X_3 - \text{dB}(W) \text{ dBc/Hz}$

## Example – DECT Cordless Telephone Standard

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- Standard for many cordless phones operating at 1.8 GHz
- Transmitter Specifications
  - Channel spacing:  $W = 1.728$  MHz
  - Maximum output power:  $M_0 = 250$  mW (24 dBm)
  - Integration bandwidth:  $R = 1$  MHz
  - Emission mask requirements

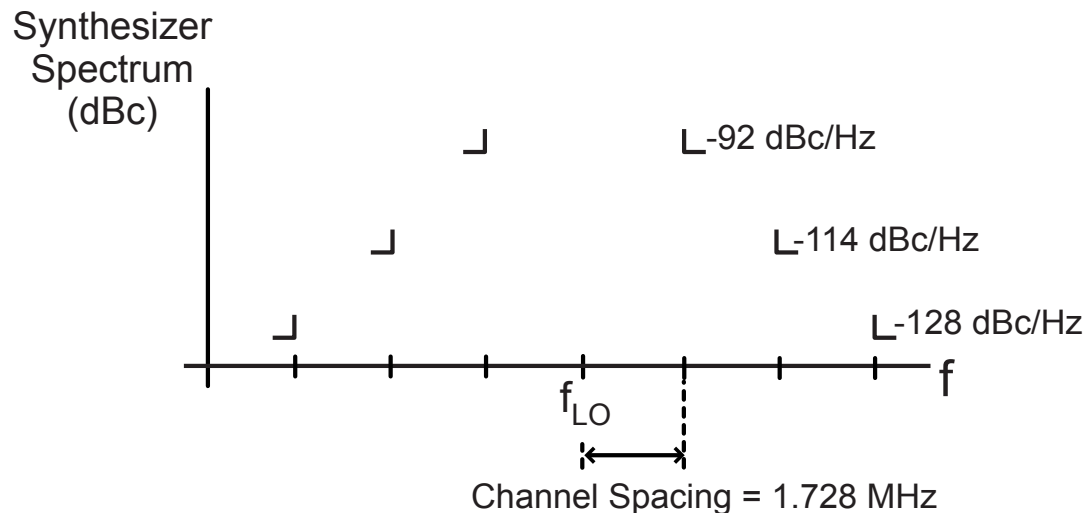
$f_{offset}$ (MHz)	Emission Mask (dBm)
0	$M_0 = 24$ dBm
1.728	$M_1 = -8$ dBm
3.456	$M_2 = -30$ dBm
5.184	$M_3 = -44$ dBm

# Synthesizer Phase Noise Requirements for DECT

## Using previous calculations with DECT values

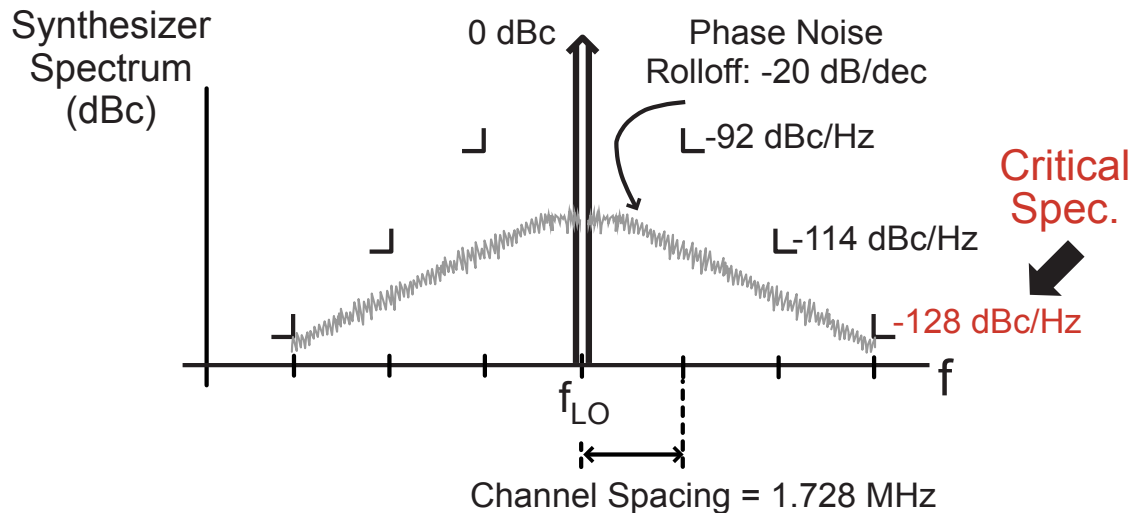
Channel Offset	Mask Power	Maximum Synth. Noise Power in Integration BW	Maximum Synth. Phase Noise at Channel Offset
0	24 dBm	set by required transmit SNR	
1.728 MHz	-8 dBm	$X_1 = -29.6$ dBc	-92 dBc/Hz
3.456 MHz	-30 dBm	$X_2 = -51.6$ dBc	-114 dBc/Hz
5.184 MHz	-44 dBm	$X_3 = -65.6$ dBc	-128 dBc/Hz

## Graphical display of phase noise mask

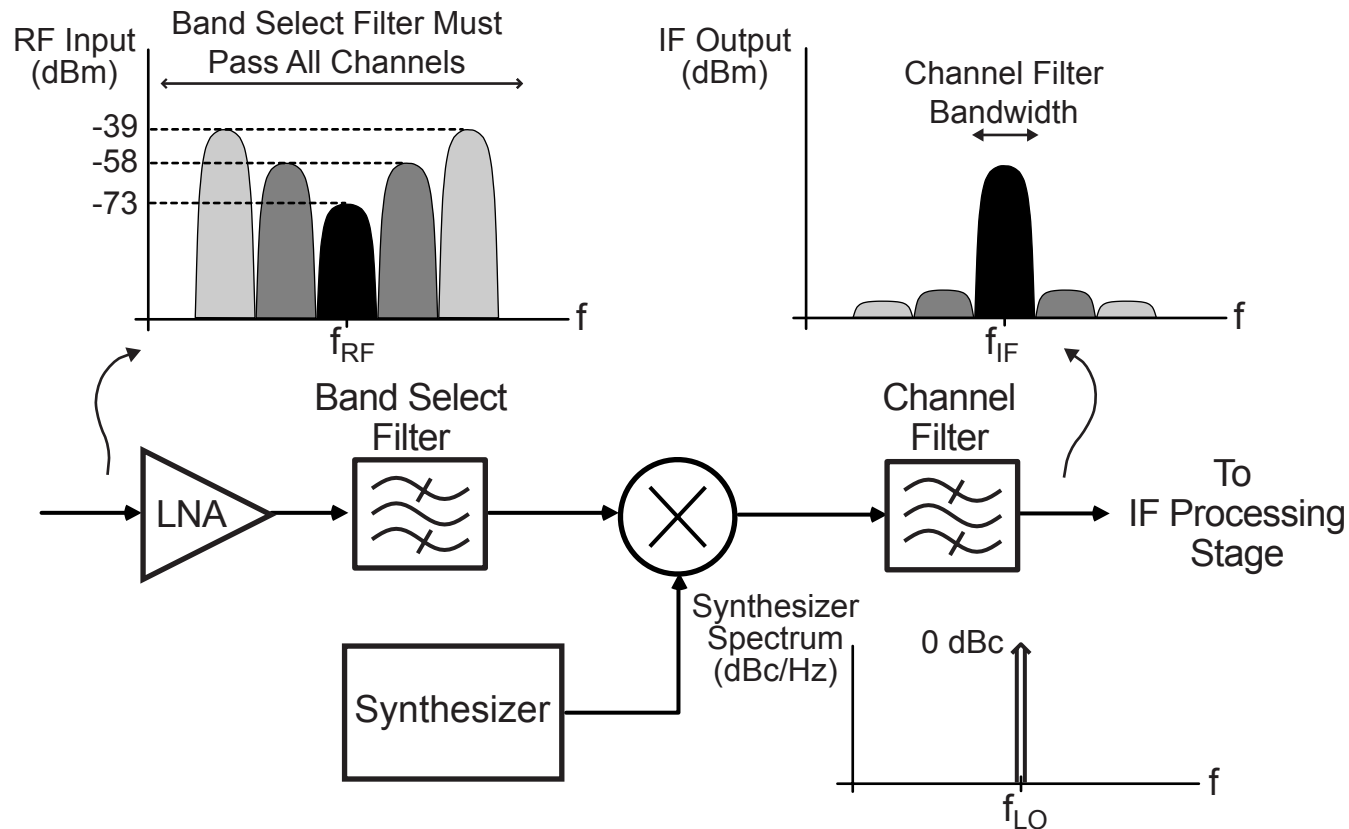


# Critical Specification for Phase Noise

- Critical specification is defined to be the one that is hardest to meet with an assumed phase noise rolloff
  - Assume synthesizer phase noise rolls off at -20 dB/decade
    - Corresponds to VCO phase noise characteristic
- For DECT transmitter synthesizer
  - Critical specification is -128 dBc/Hz at 5.184 MHz offset

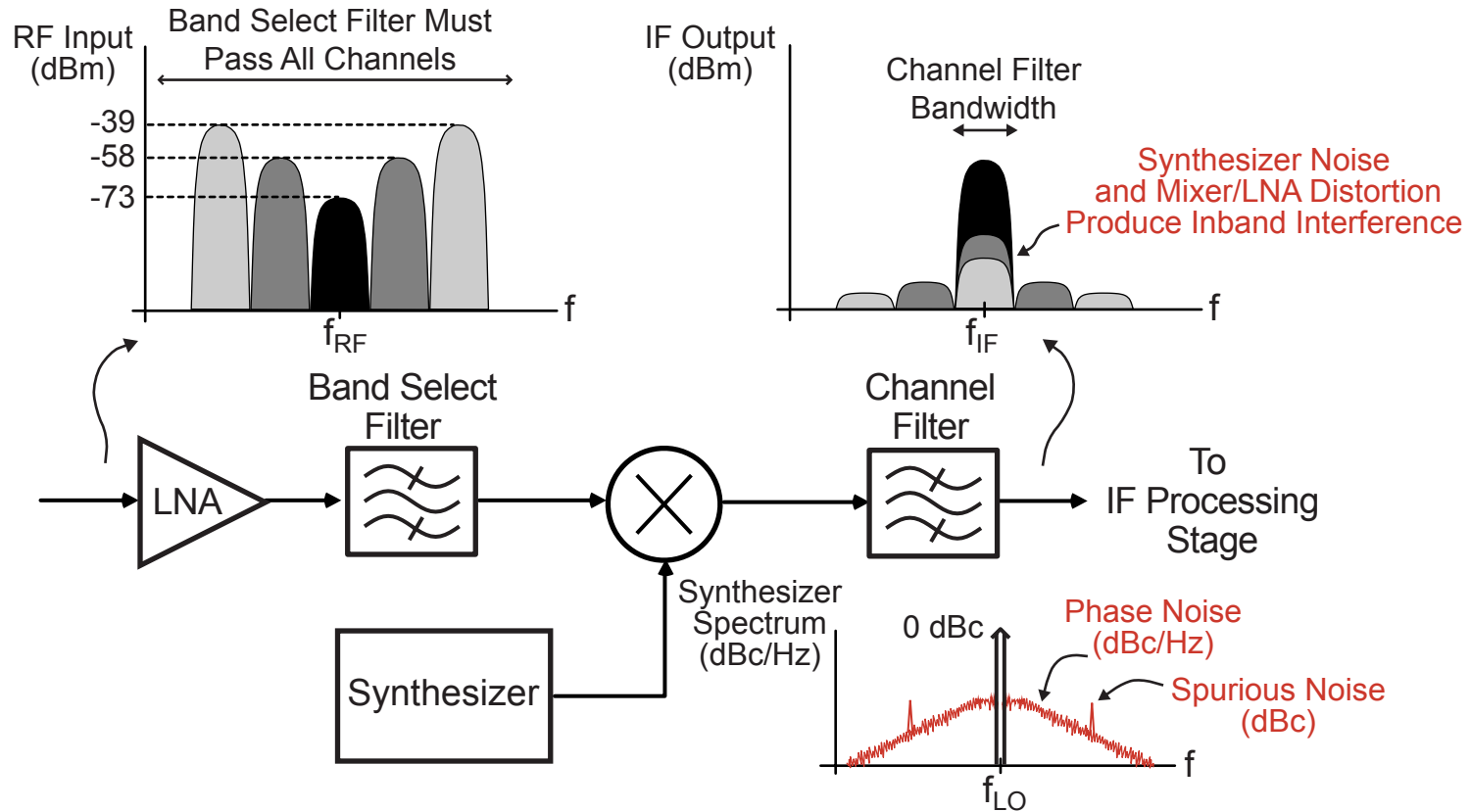


# Receiver Blocking Performance



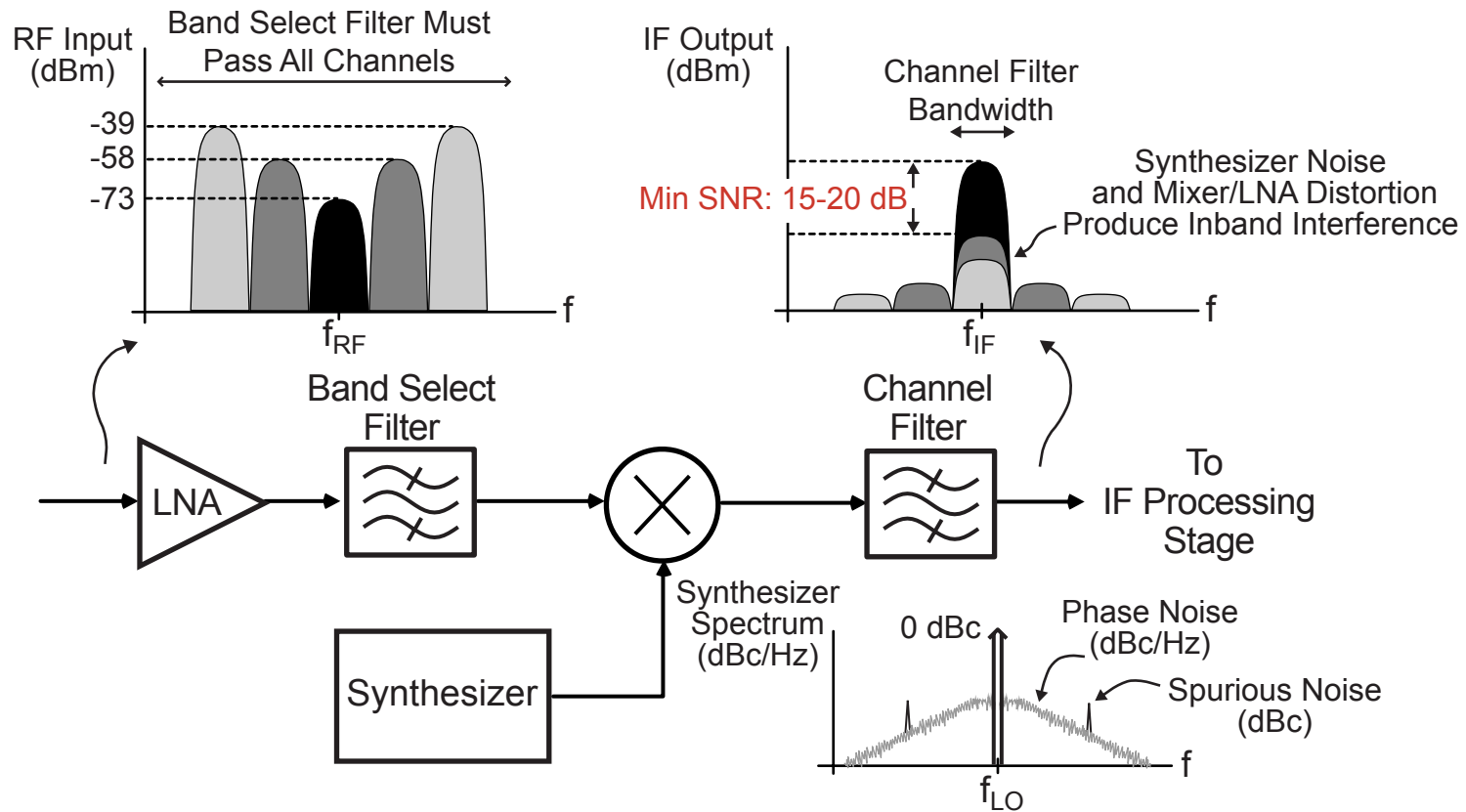
- Radio receivers must operate in the presence of large interferers (called blockers)
- Channel filter plays critical role in removing blockers
  - Passes desired signal channel, rejects interferers

# Impact of Nonidealities on Blocking Performance



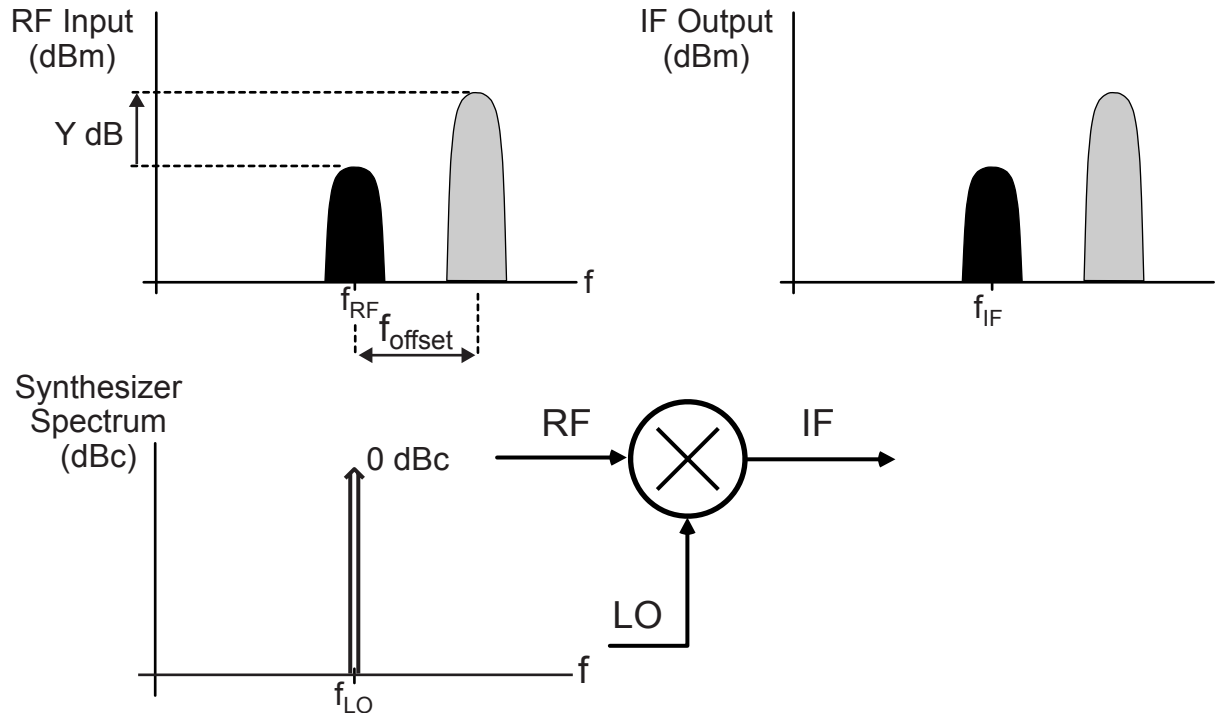
- **Blockers leak into desired band due to**
  - Nonlinearity of LNA and mixer (IIP3)
  - Synthesizer phase and spurious noise
- **In-band interference cannot be removed by channel filter!**

# Quantifying Tolerable In-Band Interference Levels



- Digital radios quantify performance with bit error rate (BER)
  - Minimum BER often set at  $1e-3$  for many radio systems
  - There is a corresponding minimum SNR that must be achieved
- Goal: design so that SNR with interferers is above  $SNR_{min}$

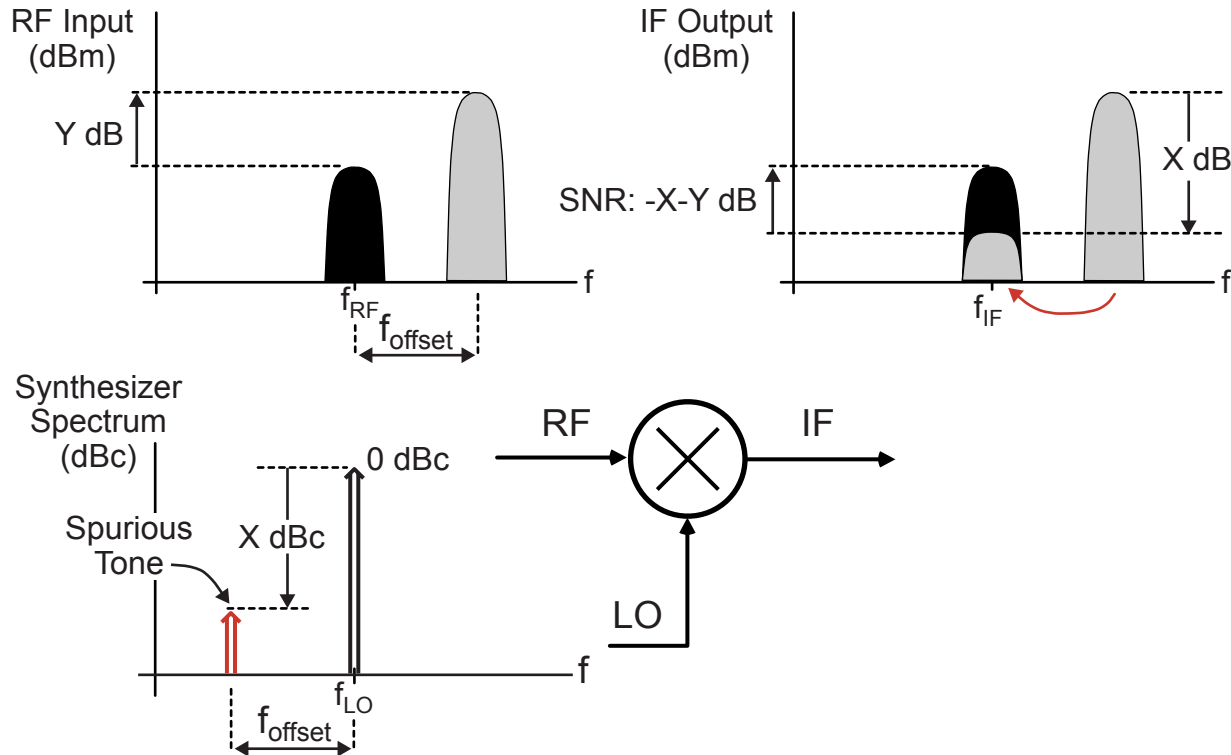
# Impact of Synthesizer on Blockers



- **Synthesizer passes desired signal and blocker**
  - Assume blocker is  $Y$  dB higher in signal power than desired signal

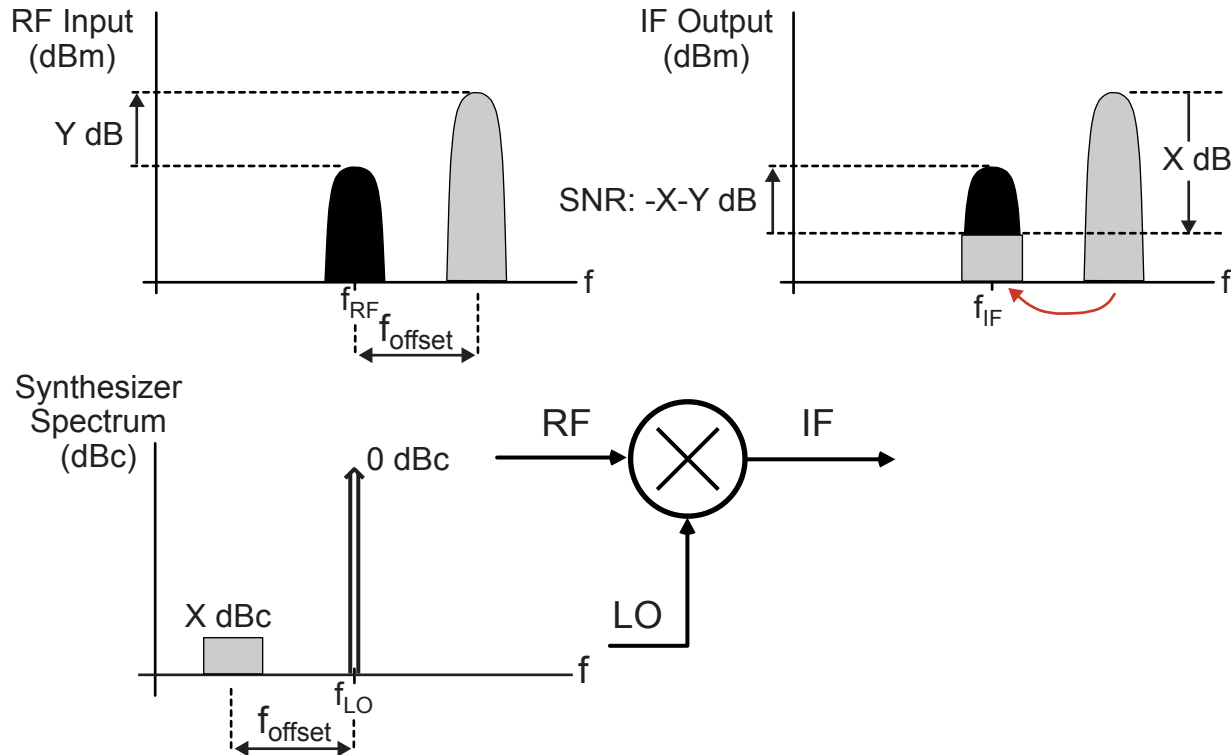


# Impact of Synthesizer Spurious Noise on Blockers



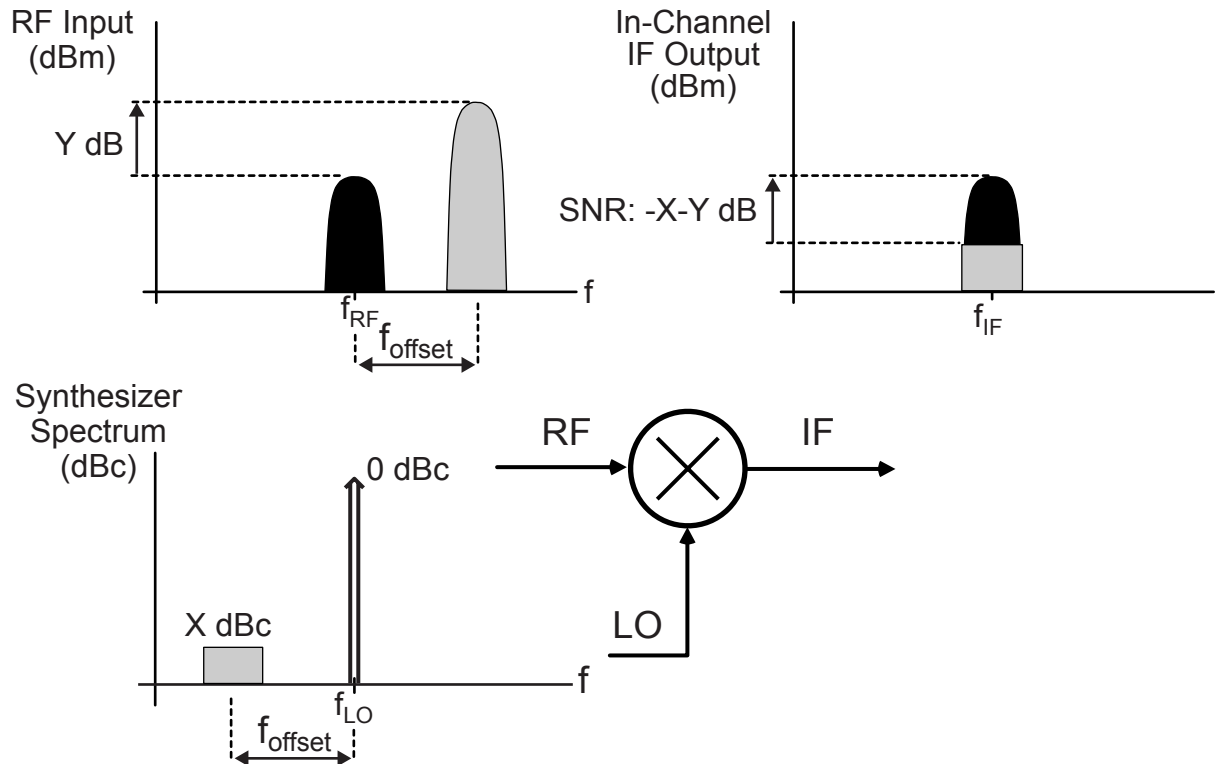
- **Spurious tones cause the blocker ( $Y$  dB) (and desired) signals to “leak” into other frequency bands**
  - In-band interference occurs when spurious tone offset frequency is same as blocker offset frequency
  - Resulting SNR =  $-X-Y$  dB with spurious tone ( $X$  dBc)

# Impact of Synthesizer Phase Noise on Blockers



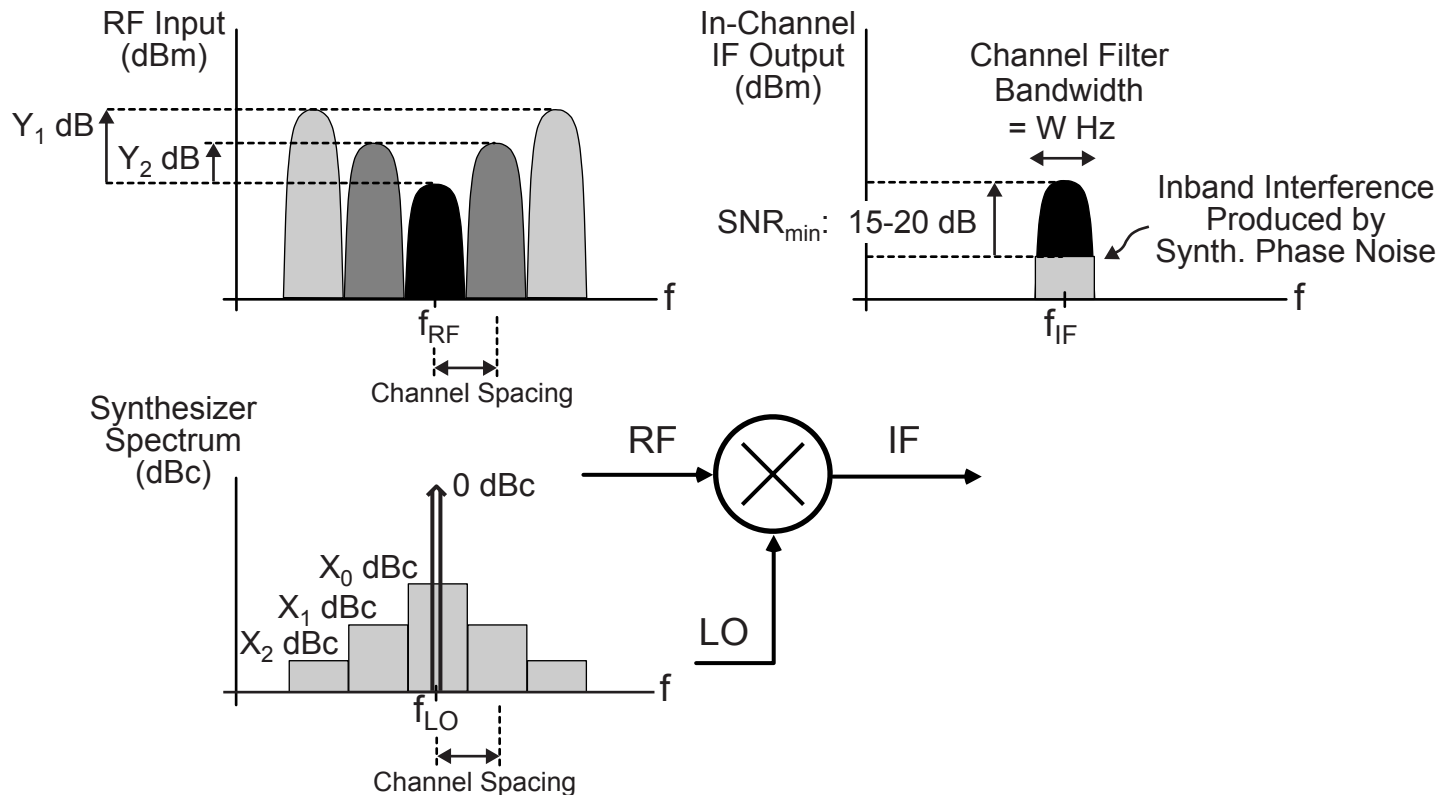
- Same impact as spurious tone, but blocker signal is “smeared” by convolution with phase noise
  - For simplicity, ignore “smearing” and approximate as shown above

# Blocking Performance Analysis (Part 1)



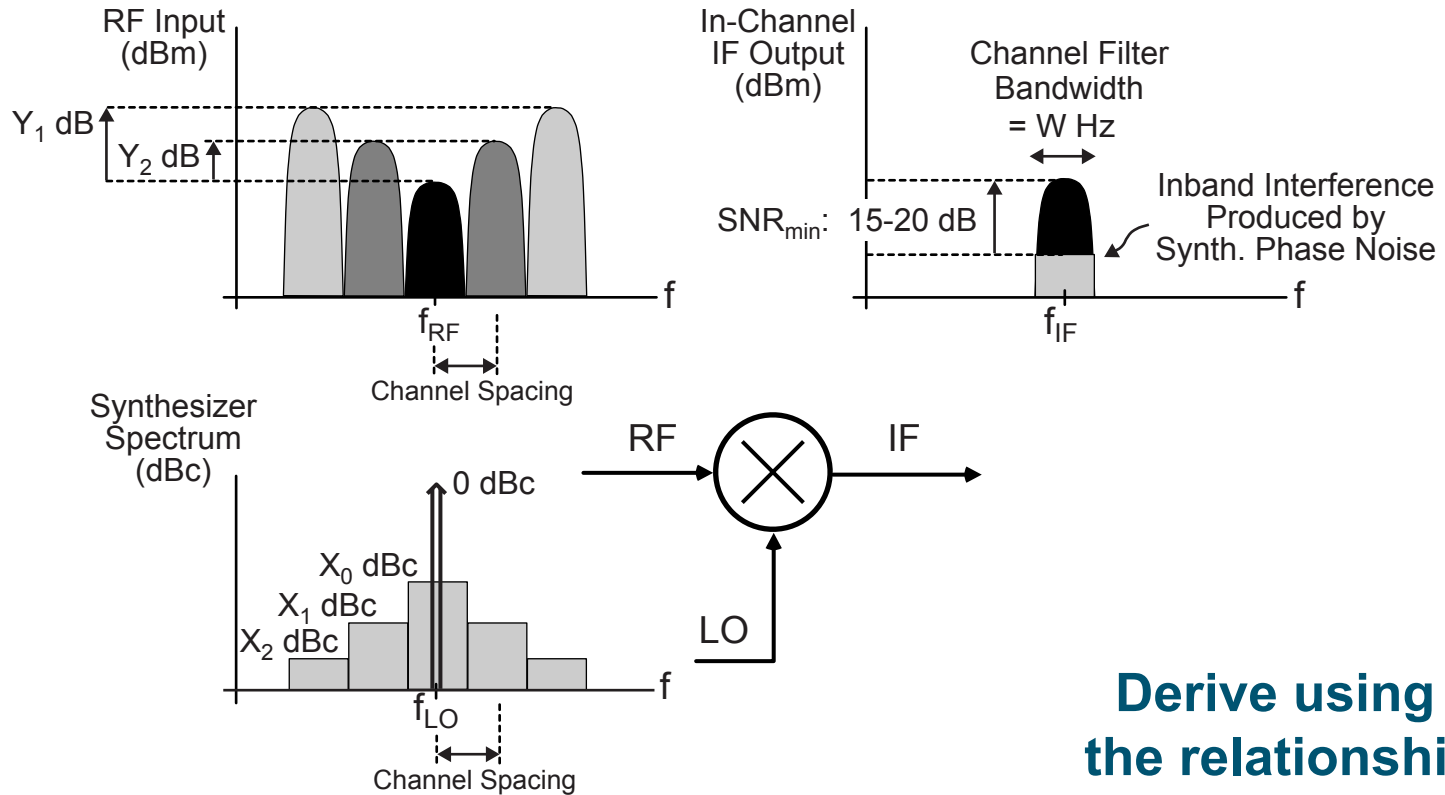
- **Ignore all out-of-band energy at the IF output**
  - Assume that channel filter removes it
  - Motivation: simplifies analysis

# Blocking Performance Analysis (Part 2)



- Consider the impact of blockers surrounding the desired signal with a given phase noise profile
  - $SNR_{min}$  must be maintained
  - Evaluate impact on SNR one blocker at a time

# Blocking Performance Analysis (Part 3)

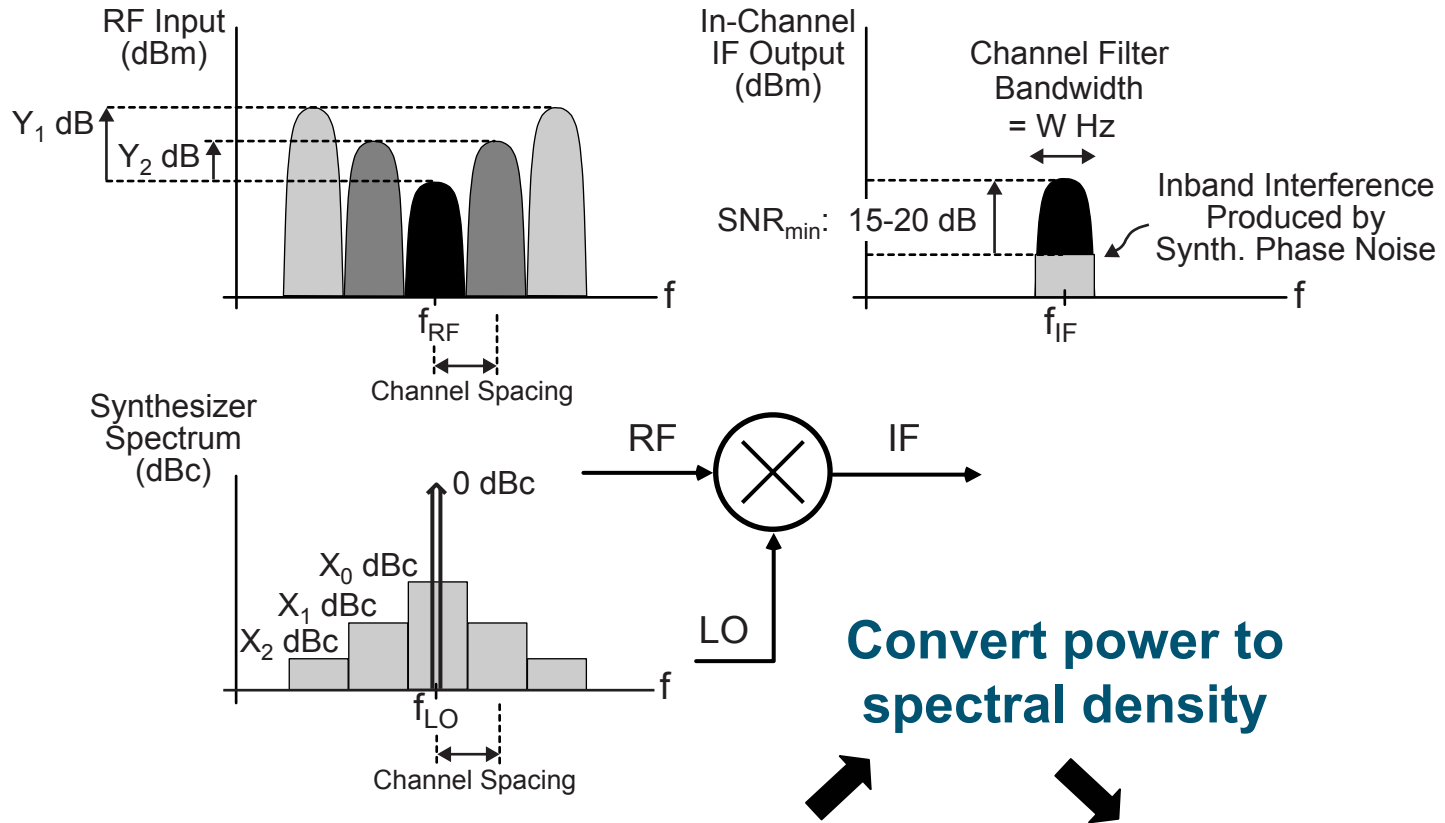


Derive using the relationship  
 $SNR = -X - Y \text{ dB} \geq SNR_{min}$



Channel Offset	Relative Blocking Power	Maximum Synth. Noise Power at Channel Offset
0	0 dB	$X_0 = -SNR_{min} \text{ dBc}$
1	$Y_1 \text{ dB}$	$X_1 = -SNR_{min} - Y_1 \text{ dBc}$
2	$Y_2 \text{ dB}$	$X_2 = -SNR_{min} - Y_2 \text{ dBc}$
3	$Y_3 \text{ dB}$	$X_3 = -SNR_{min} - Y_3 \text{ dBc}$

# Blocking Performance Analysis (Part 4)



Channel Offset	Relative Blocking Power	Maximum Synth. Noise Power at Channel Offset	Maximum Synth. Phase Noise at Channel Offset
0	0 dB	$X_0 = -SNR_{min}$ dBc	$X_0 - dB(W)$ dBc/Hz
1	$Y_1$ dB	$X_1 = -SNR_{min} - Y_1$ dBc	$X_1 - dB(W)$ dBc/Hz
2	$Y_2$ dB	$X_2 = -SNR_{min} - Y_2$ dBc	$X_2 - dB(W)$ dBc/Hz
3	$Y_3$ dB	$X_3 = -SNR_{min} - Y_3$ dBc	$X_3 - dB(W)$ dBc/Hz

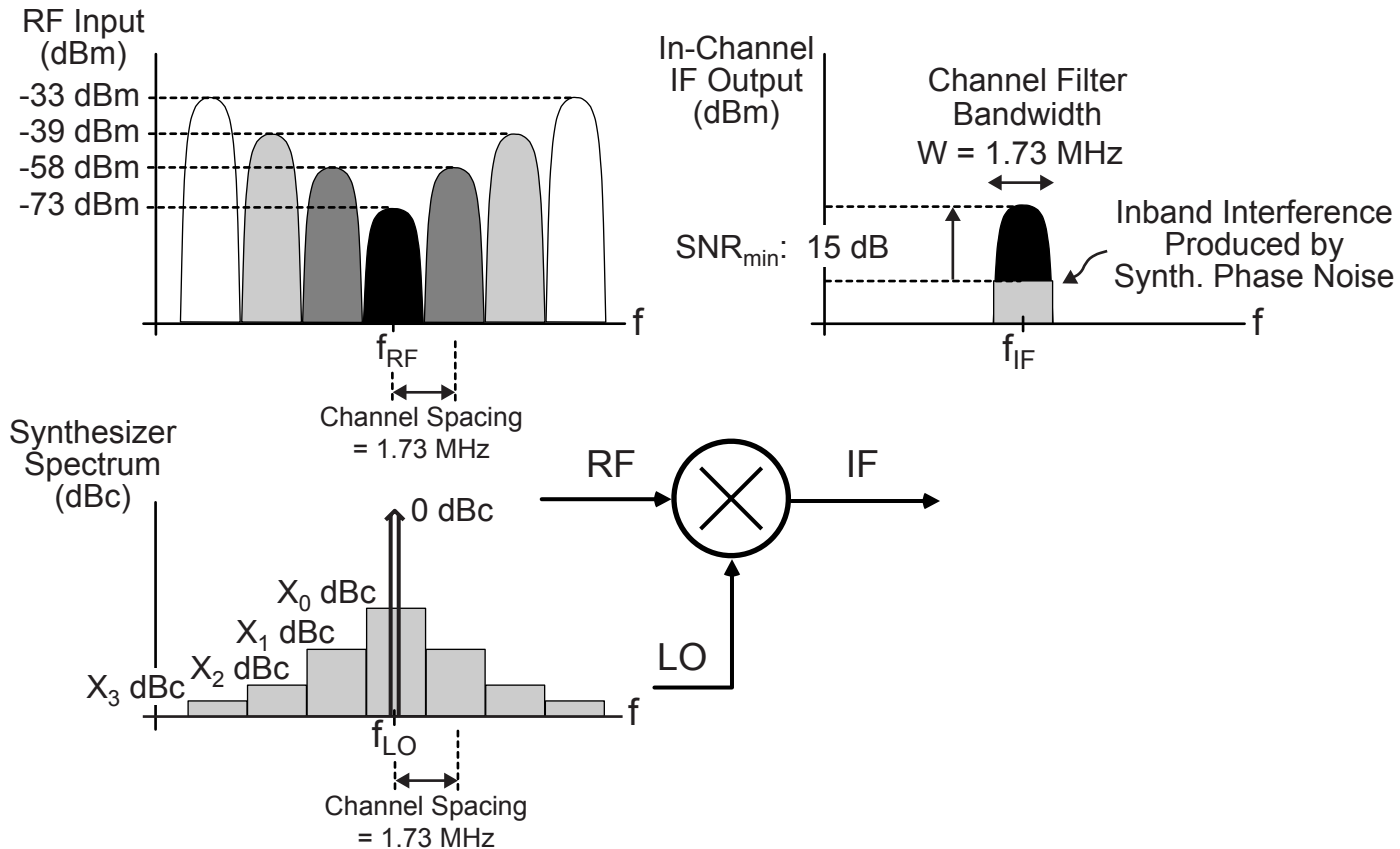
## Example – DECT Cordless Telephone Standard

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- Receiver blocking specifications
  - Channel spacing:  $W = 1.728$  MHz
  - Power of desired signal for blocking test: -73 dBm
  - Minimum bit error rate (BER) with blockers:  $1e-3$ 
    - Sets the value of  $SNR_{\min}$ 
      - Perform receiver simulations to determine  $SNR_{\min}$
      - Assume  $SNR_{\min} = 15$  dB for calculations to follow
  - Strength of interferers for blocking test

$f_{offset}$ (MHz)	Blocker Power (dBm)	Relative Strength
1.728	-58 dBm	$Y_1 = 15$ dB
3.456	-39 dBm	$Y_2 = 34$ dB
5.184	-33 dBm	$Y_3 = 40$ dB

# Synthesizer Phase Noise Requirements for DECT

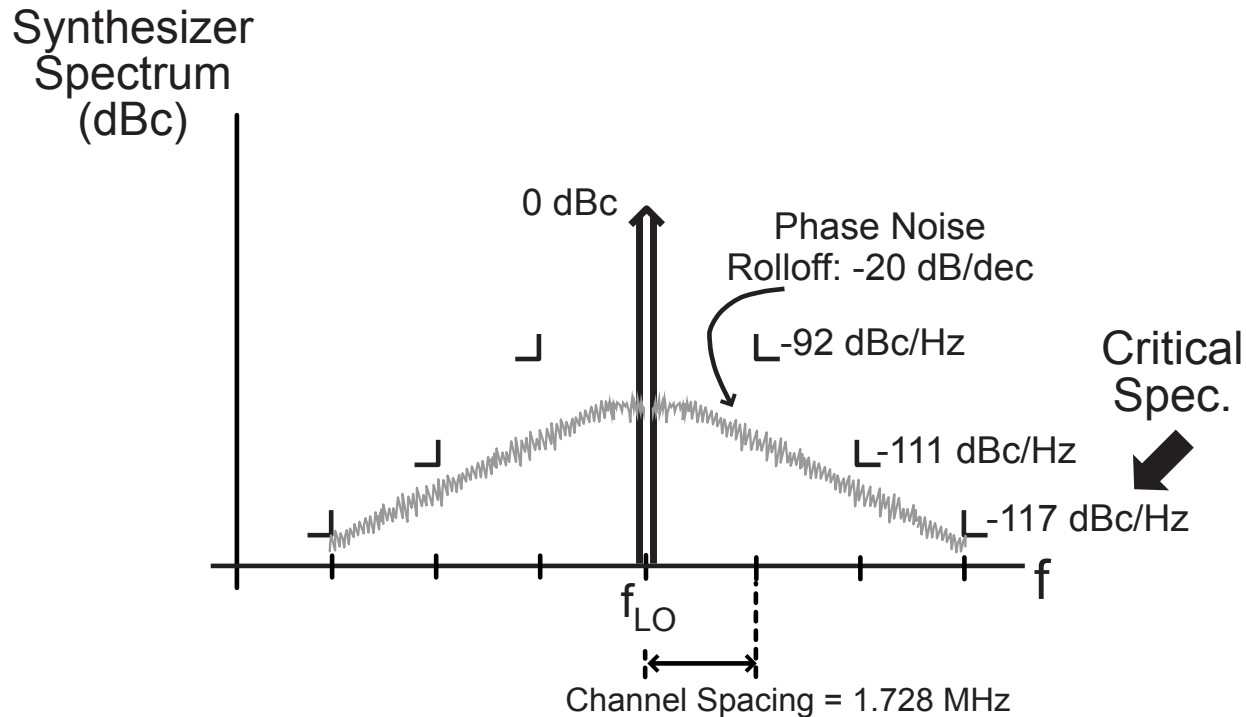


Channel Offset	Relative Blocking Power	Maximum Synth. Noise Power at Channel Offset	Maximum Synth. Phase Noise at Channel Offset
0	0 dB	$X_0 = -15$ dBc	-77 dBc/Hz
1.728 MHz	$Y_1 = 15$ dB	$X_1 = -30$ dBc	-92 dBc/Hz
3.456 MHz	$Y_2 = 34$ dB	$X_2 = -49$ dBc	-111 dBc/Hz
5.184 MHz	$Y_3 = 40$ dB	$X_3 = -55$ dBc	-117 dBc/Hz



# Graphical Display of Required Phase Noise Performance

- Mark phase noise requirements at each offset frequency



- Calculate critical specification for receive synthesizer
  - Critical specification is -117 dBc/Hz at 5.184 MHz offset
    - Lower performance demanded of receiver synthesizer than transmitter synthesizer in DECT applications!