

# 6.897: Selected Topics in Cryptography

## Lectures 15 and 16

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# Highlights of last week's lectures

- **Universal composition with joint state:** Allows analyzing a multi-instance system as separate instances, even when the instances use a joint “subroutine”. Examples:
  - Protocols in the  $F_{\text{crs}}$ -hybrid model
  - Protocols that use signature schemes
- **UC formulation of signature schemes:**
  - Motivation for providing a UC formulation of signatures
  - The signature functionality,  $F_{\text{sig}}$ .
  - Equivalence with CMA-security
- **Achieving authenticated communication:**
  - Defined  $F_{\text{cert}}$  (I.e,  $F_{\text{sig}}$  with binding to a party rather than a verif. key)
  - Realized  $F_{\text{cert}}$  given  $F_{\text{sig}}$  and public registries ( $F_{\text{sig}}$ ).
  - Realized  $F_{\text{auth}}$  given  $F_{\text{cert}}$ .

## This week:

- **Authenticated Key Exchange and Secure-Session protocols:**
  - Brief history of the problem and formalizations;
  - UC formalization of KE: The  $F_{ke}$  functionality.
  - Signature-based KE: Realizing  $F_{ke}$  in the  $F_{cert}$ -hybrid model
  - From KE to secure channels:
    - The secure session functionality  $F_{ss}$ .
    - Realizing  $F_{ss}$  in the  $F_{ke}$ -hybrid model.
  - Further relaxation of  $F_{ke}$ :
    - Non-information oracles
    - Equivalence with an earlier definition.
- **UC formulation of public-key encryption:**
  - Motivation for providing a UC formulation of encryption
  - The public-key encryption functionality,  $F_{pke}$ .
  - Equivalence with CCA2-security for non-adaptive adversaries
  - Problems and solutions for adaptive adversaries
  - Relaxing CCA security

# Background on key exchange protocols

- **Key Exchange:** A protocol for two parties to generate a common random key that is “secret” for external adversaries.  
Variants:
  - Assuming authenticated communication (the Diffie-Hellman model)
  - Unauthenticated communication (“Authenticated Key Exchange”)
- **Arguably the most widely used cryptographic protocol.**  
Typical use:
  - Run an AKE protocol to agree on a common secret
  - Derive keys for symmetric encryption and authentication functions
  - Use the derived keys to encrypt/authenticate the communication within the session.
- **Different ways to authenticate the exchange:**
  - Long-term public keys for signature or encryption, plus “public-key infrastructure”.
  - Long-term pre-shared keys
  - Trusted third parties (The Kerberos model)
  - Passwords

# Analyses of key exchange protocols

- AKE has been studied extensively but remained evasive:
  - Protocols were proposed, standardized, and later broken
  - Analytical approaches were proposed and later broken (e.g. the Needham-Schroeder<sup>78</sup> protocol was later proven secure in the Burrows-Abadi-Needham<sup>89</sup> logic, only to be broken in Lowe<sup>95</sup>)...
- First complexity-based notion of security by Bellare-Rogaway<sup>93</sup>:
  - Based on a “distinguishing game” for the adversary
  - Explicitly handles multiple concurrent sessions
- A bug and a fix by Rackoff (circa 95)
- Treatments that argue usability for secure sessions:
  - Bellare-C-Krawczyk<sup>98</sup>: simulation based (but has problems)
  - Shoup<sup>99</sup>: Points to problems in BCK<sup>98</sup> and some fixes.
  - C-Krawczyk<sup>01</sup>: based on BR<sup>93</sup> with a different system model, defines and obtains “secure sessions”.
  - CK<sup>02</sup>: A UC treatment of AKE

# The CK01 notion (based on BR93)

Consider an adversary  $A$  that interacts with a set of parties:

- Ideal initialization: The parties obtain secret keys and the public keys of everyone else for a generation function specified in the protocol.
- $A$  can activate a party  $P_i$  with input  $(\text{sid}, P_j)$  (i.e., to exchange a new key with  $P_j$  and session ID  $\text{sid}$ )
- $A$  obtains all messages sent by the parties and delivers arbitrary messages (models unauthenticated network).
- $A$  can corrupt parties and obtain their local data (either the session data or the long-term key or both).
- $A$  can ask to “reveal” a session  $(\text{sid}, P_i)$ ; in response it gets the local output of  $(\text{sid}, P_i)$ , which is of the form  $(\text{sid}, P_j, a)$  for  $a$  in  $\{0, 1\}^k$ .
- At some point,  $A$  chooses a “test session”  $(\text{sid}^*, P_j^*)$  within  $P_i^*$  and receives a “test value”  $a^*$ , where  $a^*$  is either taken from  $P_i^*$ 's output or is randomly chosen.
- Later,  $A$  outputs a guess whether  $a^*$  was taken from  $P_i^*$ 's output. It “wins” if neither the session  $(\text{sid}^*, P_j^*)$  within  $P_i^*$  nor the session  $(\text{sid}^*, P_i^*)$  within  $P_j^*$  are corrupted or revealed, and the guess is correct.

A protocol is “SK-secure” if for any  $A$ :

**Agreement:** Whenever  $P_i, P_j$  output  $(\text{sid}, P_j, a)$  and  $(\text{sid}, P_i, a')$  we have  $a = a'$ .

**Secrecy:**  $A$  wins the game with probability only negligibly more than  $\frac{1}{2}$ .

## More on the CK01 notion

In addition, they:

- Define a notion of secure (secret and authenticated) sessions, along the same lines.
- Prove that the composition of
  - A secure KE protocol
  - Key derivation using pseudorandom functions
  - Standard symmetric encryption and MAC using the derived keysIs a good secure-session protocol.

Caveats:

- No concrete assurance in the adequacy of the notion of secure-sessions.
- Multiple sessions have to be explicitly treated in the definition.
- There is no security guarantee w.r.t. other protocols
- Only idealized set-up.

## A UC treatment of key exchange

- Define and analyze key exchange protocols for a single session.
  - Use the JUC theorem to deduce security in the multi-session case.
  - Use the UC theorem to deduce security with respect to other protocols.
- Define a UC notion of a secure session (again, for a single session) and show how to realize it given UCKE.
- **Question:** How does the new definition relate to the old one?



# The key-exchange functionality $F_{KE}$ (I)

Wait to receive:

- $(sid, P_i, P_j)$  from party  $(sid, P_i)$
- $(sid, P_j, P_i)$  from party  $(sid, P_j)$

Then:

- Choose  $a \leftarrow_{\mathcal{R}} \{0, 1\}^k$
- Output  $(sid, P_i, P_j, a)$  to  $(sid, P_i)$  and  $(sid, P_j)$
- Send  $(sid, P_i, P_j)$  to the adv.
- Halt.

Too strong: Forces  $a$  to be random even if one of the parties is corrupted.

# The key-exchange functionality $F_{KE}$ (II)

Wait to receive:

- $(sid, P_i, P_j)$  from party  $(sid, P_i)$
- $(sid, P_j, P_i)$  from party  $(sid, P_j)$

Then:

- If one of the parties is corrupted then obtain a value  $a$  from the adv. Else, choose  $a \leftarrow_{\mathcal{R}} \{0,1\}^k$
- Output  $(sid, P_i, P_j, a)$  to  $(sid, P_i)$  and  $(sid, P_j)$
- Send  $(sid, P_i, P_j)$  to the adv.
- Halt.

Too strong: Requires “mutual authentication”

# The key-exchange functionality $F_{KE}$ (III)

When receiving  $(sid, P_i, P_j)$  from the first party,  $(sid, P_i)$ , do:

- Send  $(sid, P_i, P_j)$  to the adv, obtain (“ok”,  $P_i$ ) from the adv.
- If one of the parties is corrupted then obtain a value  $a$  from the adv. Else, choose  $a \leftarrow_R \{0, 1\}^k$
- Output  $(sid, P_i, P_j, a)$  to  $(sid, P_i)$

When receiving  $(sid, P_j, P_i)$  from the second party,  $(sid, P_j)$ , do:

- Send  $(sid, P_i, P_j)$  to the adv, obtain (“ok”,  $P_j$ ) from the adv.
- Output  $(sid, P_i, P_j, a)$  to  $(sid, P_i)$ .

## Signature-based key exchange

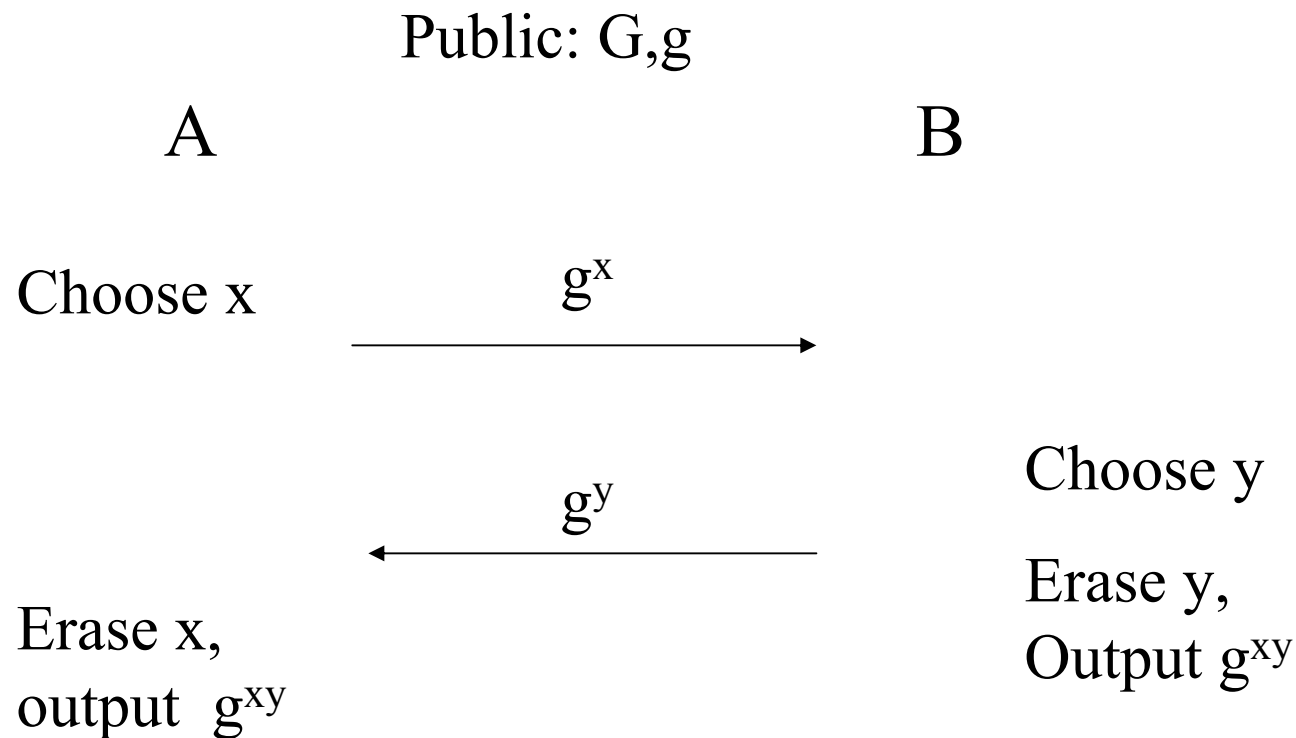
- Realize  $F_{ke}$  in the  $F_{cert}$ -hybrid model (here each session uses its own copy of  $F_{cert}$ ).
- Recall the protocol for realizing  $FF_{cert}$  using a single copy of  $F_{cert}$  (essentially, sign the ssid together with the message).
- Use the JUC theorem to obtain a multi-instance key-exchange protocol where each party uses a single signing/verification key pair.

## Reminder: The certification functionality, $F_{\text{cert}}$

1. On input  $(\text{sid}, \text{"sign"}, m)$  from  $(\text{sid}, S)$ , where  $\text{sid}=(S, \text{sid}')$ , forward  $(\text{sid}, m)$  to  $A$ , obtain a "signature"  $s$  from  $A$ , output  $s$  to  $(\text{sid}, S)$ , and record  $(m, s, 1)$ . Verify that no prior record  $(m, s, 0)$  exists.
2. On input  $(\text{sid}, \text{"verify"}, m, s)$  from any party, return  $(\text{sid}, f)$  where:
  - If  $(m, s, b)$  is recorded then  $f=b$ .
  - If  $S$  is uncorrupted and  $(m, s^*, 1)$  is not recorded for any  $s^*$ , then  $f=0$ .
  - Else, forward  $(m, s)$  to  $A$ , obtain  $f$  from  $A$ , and record  $(m, s, f)$ .

$F_{\text{cert}}$  is similar to  $F_{\text{sig}}$  except that the KeyGen interface is deleted. Instead, verification is done directly with respect to the signer's identity (which appears in the sid).

# Basic (unauthenticated) Diffie-Hellman



- This protocol guarantees secrecy of the key against eavesdroppers, under the Decisional Diffie-Hellman Assumption.
- More abstractly, can be based on any semantically secure encryption scheme.

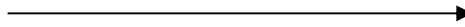
# Authenticated Diffie-Hellman: Attempt I

Sign the exchange:

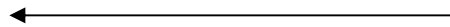
A

B

$g^x$



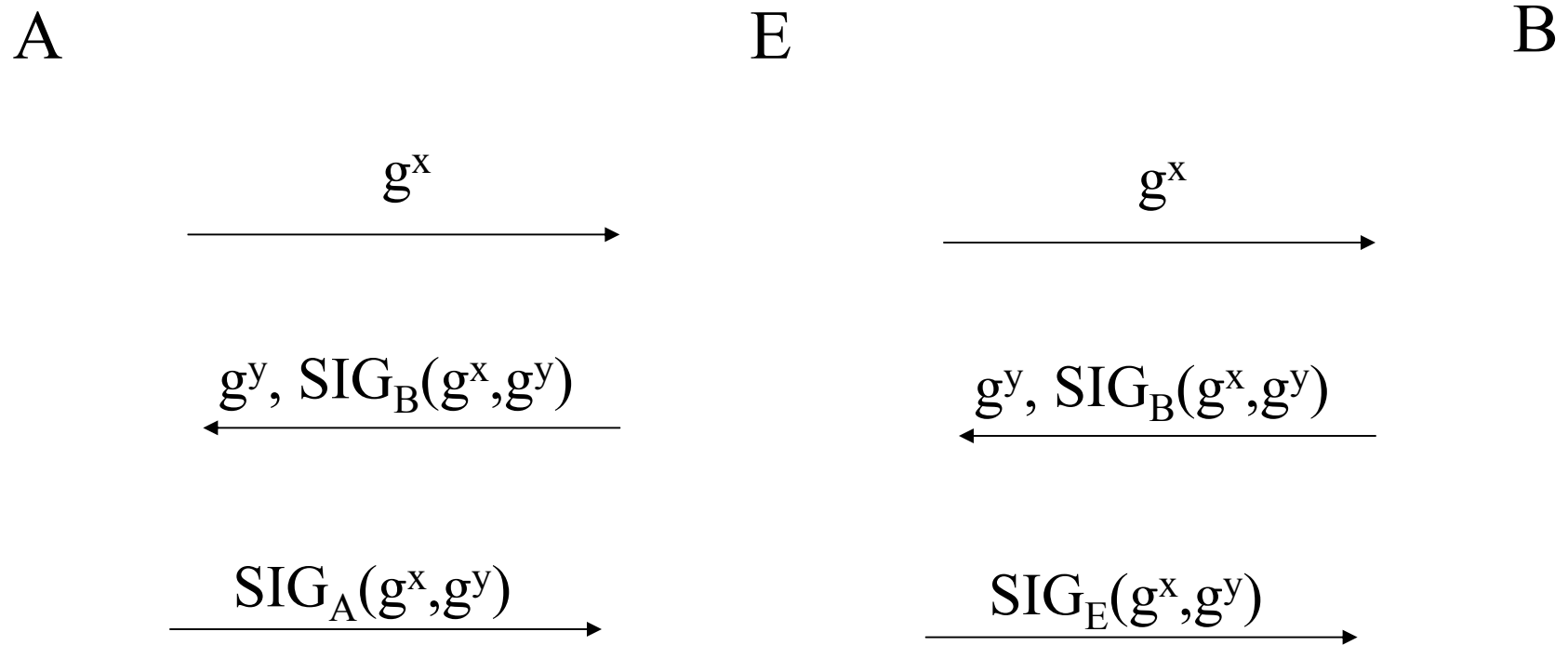
$g^y, \text{SIG}_B(g^x, g^y)$



$\text{SIG}_A(g^x, g^y)$

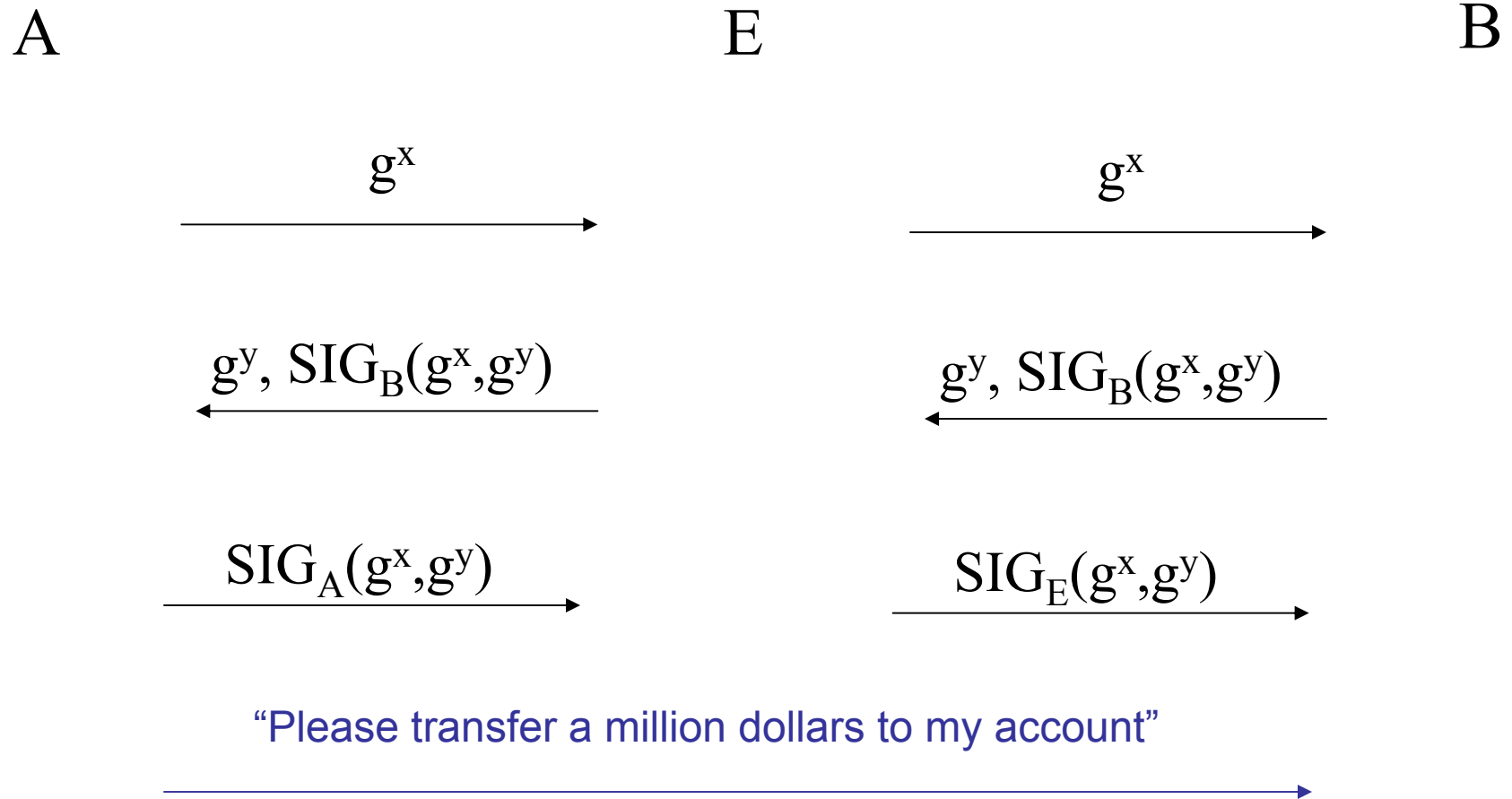


# Attack on Attempt I:



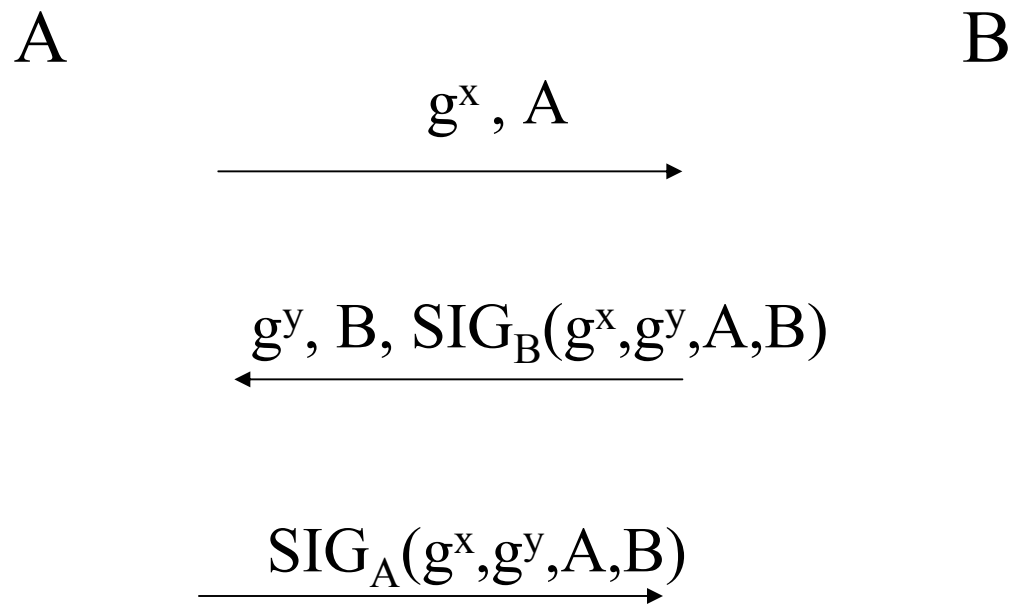


# Attack on Attempt I:



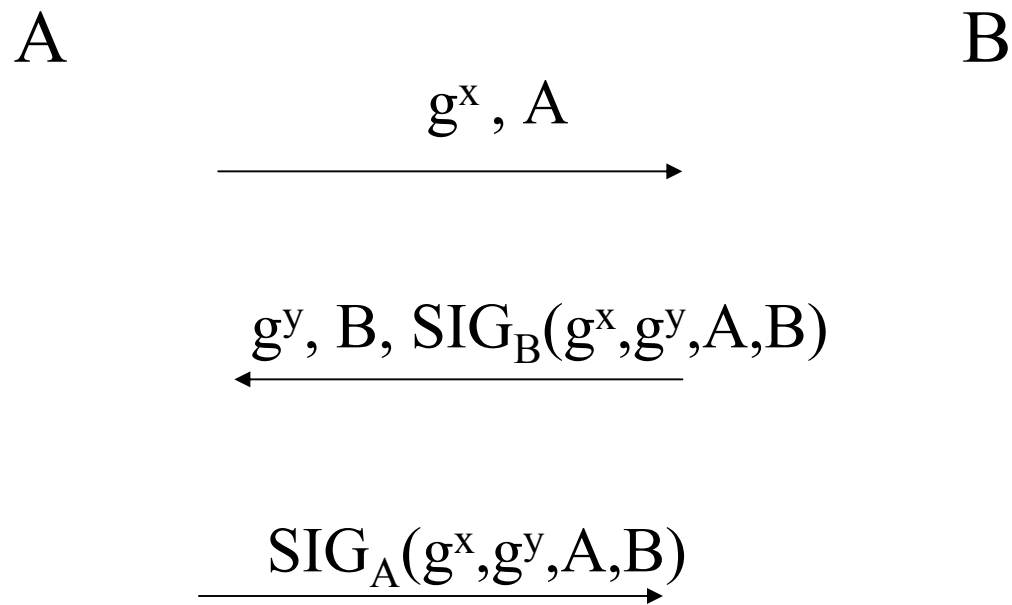
# Authenticated Diffie-Hellman: Attempt II

Idea: Include the identities in the signed text.



# Authenticated Diffie-Hellman: Attempt II

Idea: Include the identities in the signed text.



This is essentially the ISO 9798-3 protocol.

**Theorem:** The above protocol securely realizes  $F_{ke}$  in the  $F_{cert}$ -hybrid model.

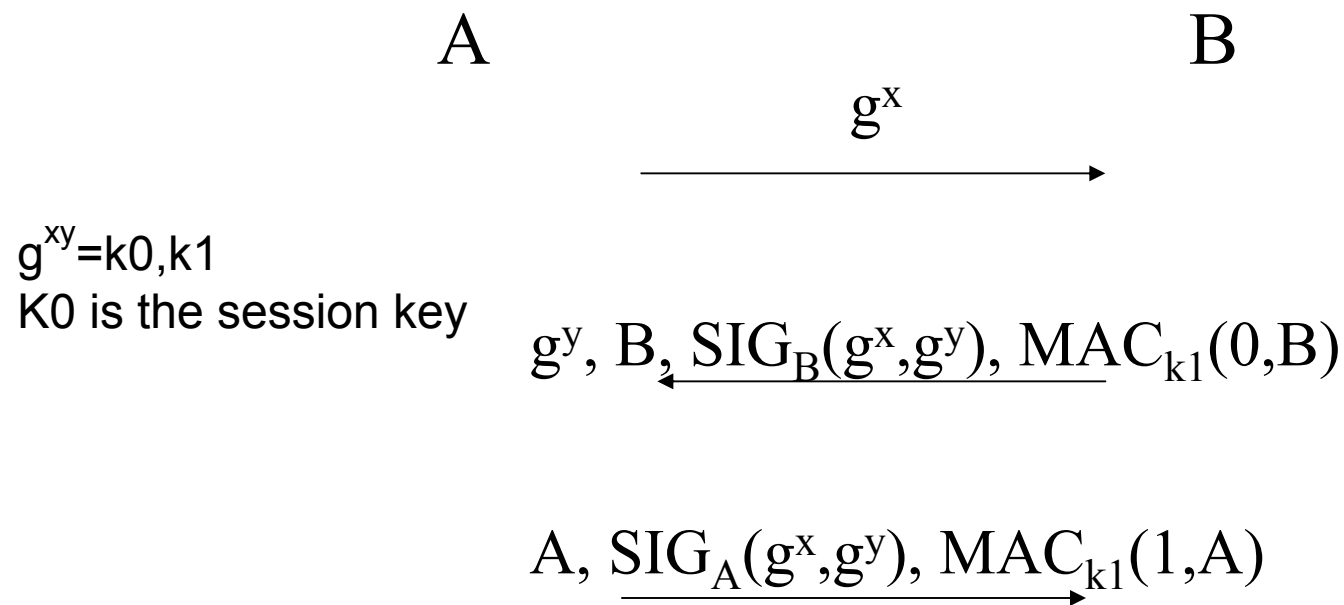
### Drawbacks of the protocol:

- Identities of A,B are transmitted in the clear (no identity protection).
- Leaves a non-repudiable proof of the exchange, including identities.
- Including the peer identity in the signature is harder to implement and introduces latency.

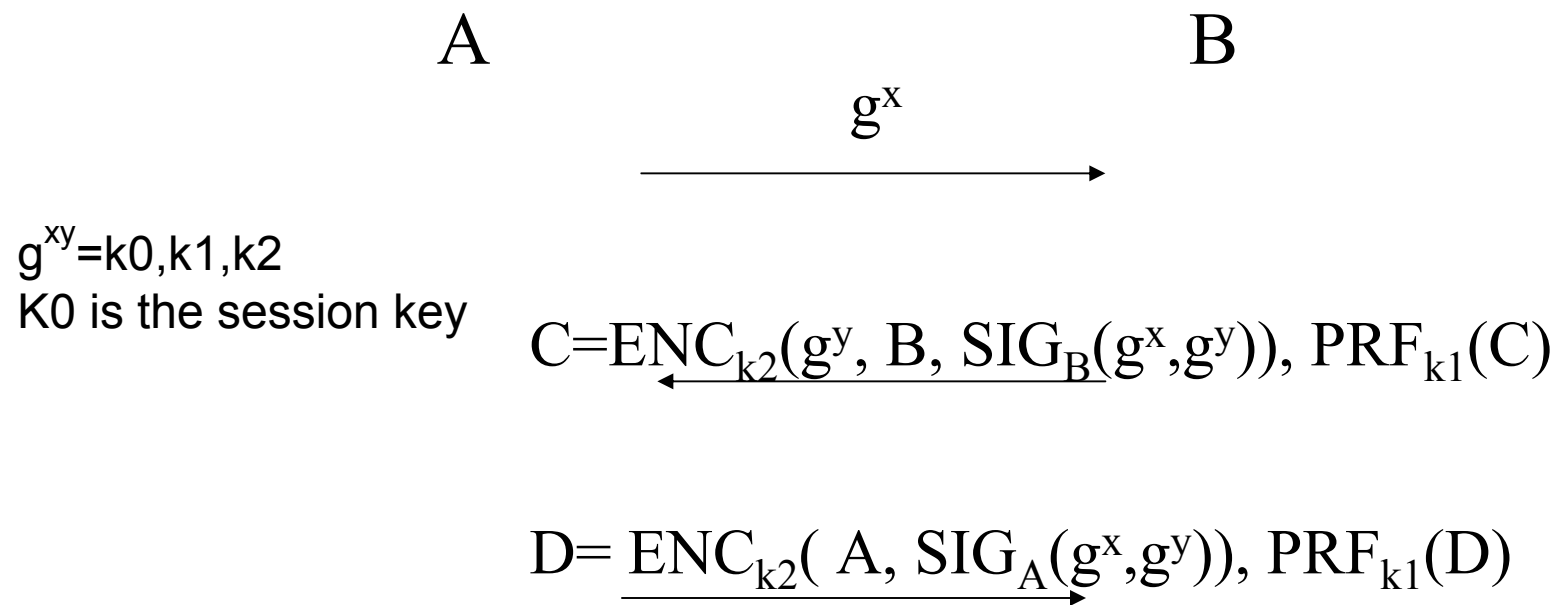
Can we do better?

Another approach: The SIGMA protocol [Krawczyk 95]  
 (based on ideas from the STS protocol of [Diffie-vanOorschot-Wiener92])

Basic version:



# The SIGMA protocol: Encrypted version

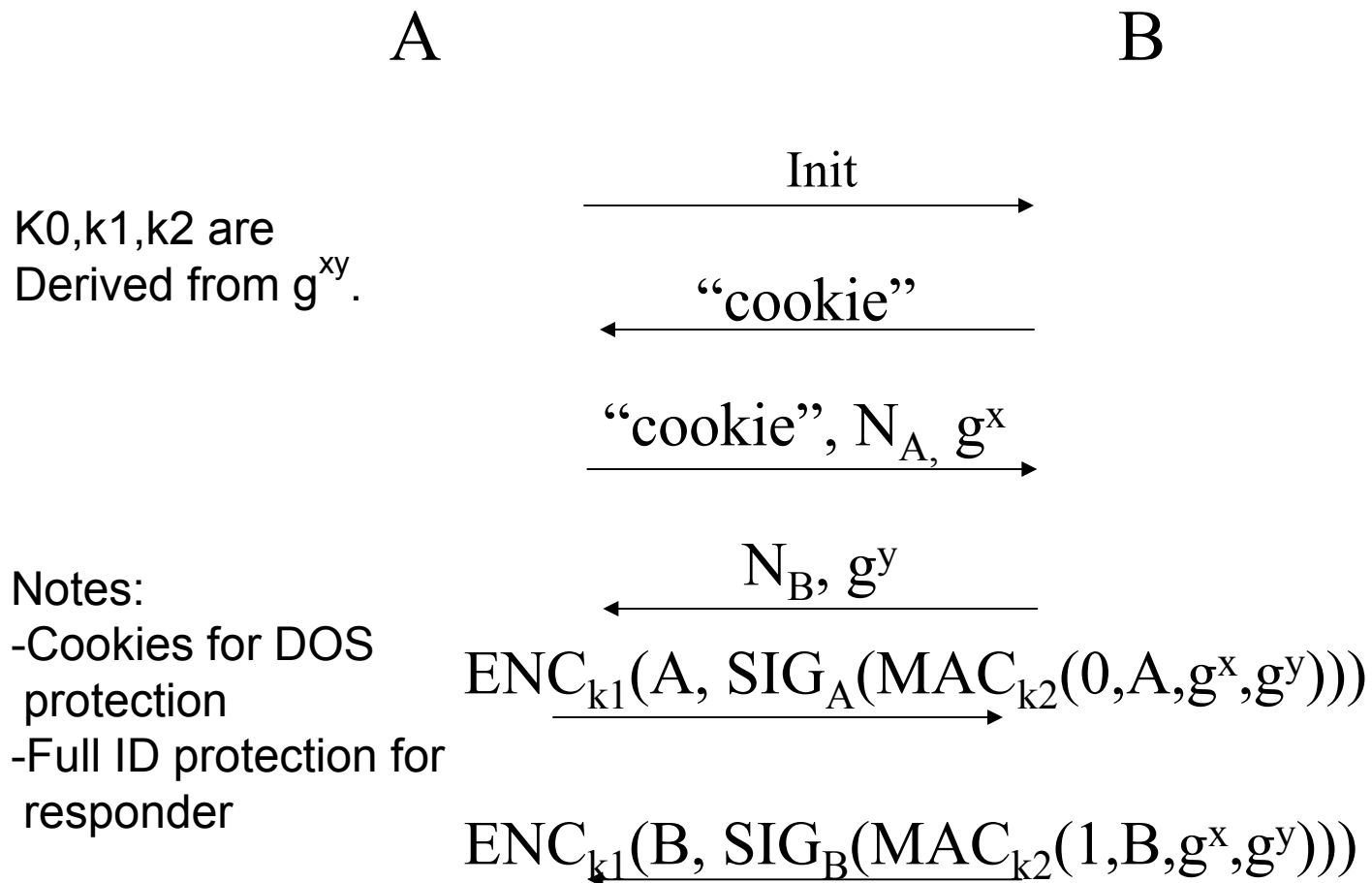


- Provides identity protection from eavesdroppers for both parties
- “Full identity protection” for the initiator.

**Theorem:** The SIGMA protocol securely realizes  $F_{ke}$  in the  $F_{cert}$ -hybrid model.

Note: While the “ISO protocol” provides mutual authentication, SIGMA does not.

# IPSEC's IKE protocol ("crypto core"):





## The secure session functionality, $F_{ss}$

1. When receiving input  $(sid, "init", P_i, P_j)$  from  $(sid, P_i)$ , record  $P_i$  and  $P_j$  as the peers of this session and send  $(sid, P_i, P_j)$  to the adv.
2. When receiving input  $(sid, m)$  from a peer  $(sid, P_i)$ , do:
  1. Output  $(sid, m)$  to the other peer
  2. Send  $(sid, P_i, |m|)$  to the adv.

# Realizing $F_{SS}$ in the $F_{ke}$ -hybrid model

## Primitives used:

- a pseudorandom function PRF
- a semantically secure symmetric encryption scheme (ENC,DEC)
- a symmetric message authentication function MAC

## Protocol:

- On input  $(sid, "init", P_i, P_j)$ ,  $(sid, P_i)$  does:
  - Calls  $F_{ke}$  with  $(sid.0, P_i, P_j)$ , obtains key  $k$ .
  - Lets  $k_0 = PRF_k(0), \dots, k_3 = PRF_k(3)$ .
  - Initializes counters IN and OUT to 0.
- On input  $(sid, m)$ , compute  $C = ENC_{k_0}(m)$ , and send  $(sid, C, MAC_{k_1}(C, OUT++))$ .
- When receiving  $(sid, C, D)$ , verify that  $D = MAC_{k_3}(C, IN++)$ , and if so then outputs  $(sid, DEC_{k_3}(C))$ .

## Remaining Questions:

- What's the relation between the UC notion of KE and the prior one (SK-security)?
- Why does adaptive security require special encryption schemes?

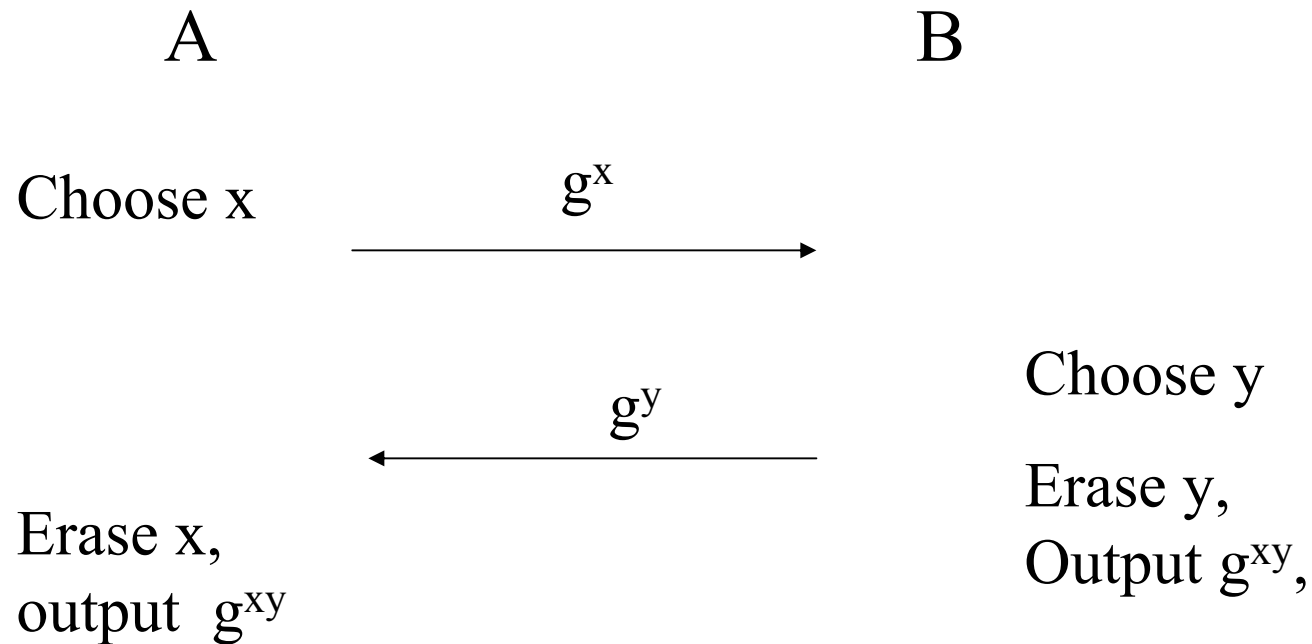
- Restrict the discussion to “session-wise protocols”, ie protocols where the only shared state across sessions is the long-term authentication module.

**Theorem:** If a protocol realizes  $F_{ke}$  then it is SK-secure.

The other direction doesn't hold...

# Example: DH exchange (assuming authenticated channels)

## Protocol 2DH



# Proof of security

(based on DDH assumption):

Assume an adversary  $A$  that breaks 2DH.

Construct a distinguisher of  $(g^x, g^y, g^{xy})$  from  $(g^x, g^y, g^z)$ .

Given  $(a, b, c)$  do:

- Choose  $i$  at random. (Hope that the  $i$ th session will be the test session. If not, then output a random bit.)
- Run  $A$ . In the  $i$ th session, give  $A$  the values  $a, b$  as the messages sent by the parties, and  $c$  as the test value.
- Output whatever  $A$  outputs.

*Note:  $A$  never expects to see the secret exponents, since it never corrupts the test session.*

## 2DH does not realize $F_{ke}$

Z that distinguishes between real and ideal:

- Prompt P1,P2 to exchange a key.
- Obtain P1's message,  $a$ , from A.
- Obtain P2's message,  $b$ , from A.
- Before P2's message is delivered to P1:
  - Obtain the session key  $k$  from P2's output.
  - Instruct A to corrupt P1, obtain the secret exponent  $x$ .
- Output “real” if  $g^x=a$  and  $b^x=k$ .

*Analysis:*

- *In a real execution, Z always outputs “real”.*
- *In the ideal process  $k$  is independent of  $a,b$ . Thus Z outputs “real” w.p.  $1/|group|$ .*

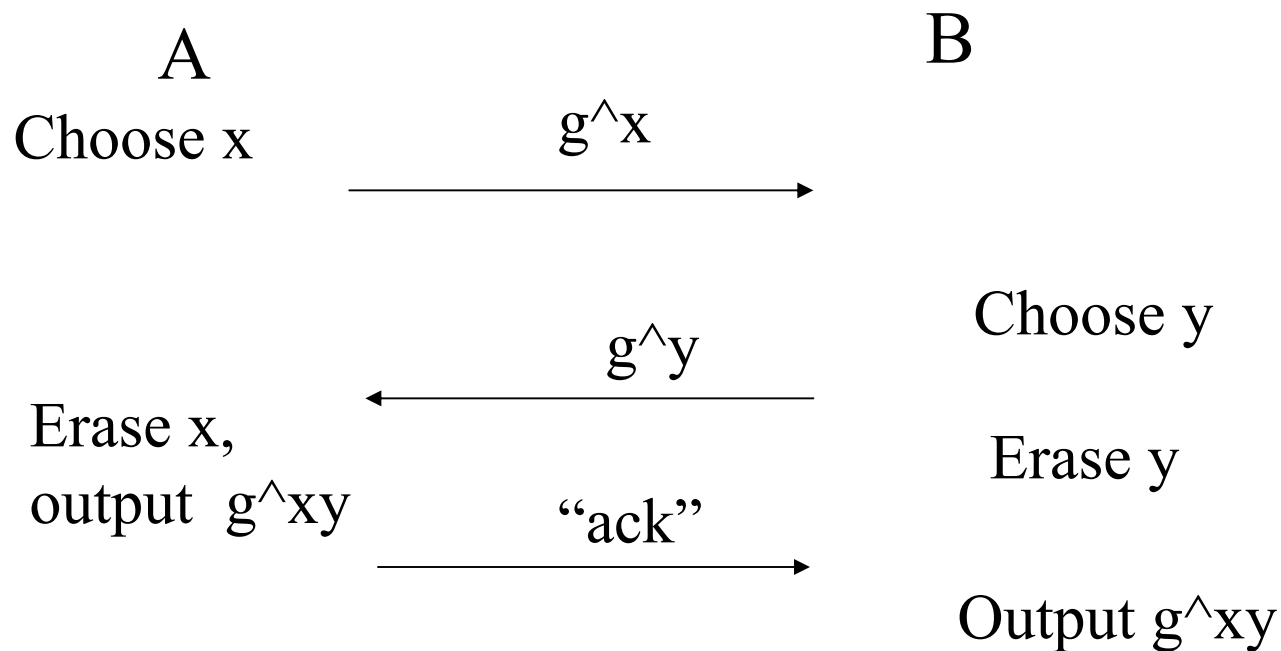
# Reflections

The essence of the problem: P2 outputs the key while P1 still holds “sensitive info”.

- Is this a “real” security problem with 2DH?  
Or is it only a “technicality” of the definition?
- Are there reasonable ways to strengthen 2DH, or alternatively to relax the definition?



# Solution 1: An addition to 2DH:



*-Can show that a similar addition turns **any** SK-secure protocol into a UC-secure one.*

## Solution 2: Relax $F_{KE}$

The idea: Allow  $F_{KE}$  to “release information” on the secret key, as long as this information is indistinguishable from random. That is:

- An ITM  $M$  is a ***non-information oracle*** if no adversary, after interacting with  $M$ , can distinguish the local output of  $M$  from a random value.
- Modify  $F_{KE}$  as follows:
  - $F_{KE}$  will allow the adversary to interact with a **NIO**  $M$ .
  - The session key will be the local output of  $M$ .
  - When either party is corrupted, the adversary gets the internal randomness of  $M$ .

# The Weak key-exchange functionality $F_{\text{wkE}}$ (with non-information oracle $M$ )

When receiving  $(\text{sid}, P_i, P_j)$  from the first party,  $(\text{sid}, P_i)$ , do:

- Send  $(\text{sid}, P_i, P_j)$  to the adv.
- Let the adv. interact with a copy  $M_i$  of  $M$ .
- If  $P_i$  is corrupted then reveal the local state of  $M_i$  to the adv, and let the adv. set the output of  $M_i$ .

When receiving  $(\text{sid}, P_j, P_i)$  from the second party,  $(\text{sid}, P_j)$ , do:

- Send  $(\text{sid}, P_j, P_i)$  to the adv.
- Let the adv. interact with a copy  $M_j$  of  $M$ .
- If  $P_j$  is corrupted then reveal the local state of  $M_j$  to the adv, and let the adv. set the output of  $M_j$ .

When either  $M_i$  or  $M_j$  generate output  $a$ , output  $(\text{sid}, P_i, P_j, a)$  to the corresponding party. When the other copy of  $M$  generates output, output  $(\text{sid}, P_i, P_j, a)$  to the other party.

**Theorem:** A KE protocol  $P$  is SK-secure iff there exists a non-information oracle such that  $P$  realizes  $F_{wke}^M$ .

**Theorem:** Any protocol that securely realizes  $F_{ss}$  in the  $F_{ke}$ -hybrid model, realizes  $F_{ss}$  also in the  $F_{wke}^M$ -hybrid model for any NIO  $M$ .

A similar trick works for the “adaptive encryption” problem in realizing  $F_{ss}$ :

- Relax  $F_{ss}$  by adding a “non-information oracle” for encryption.
- Can show that any semantically secure encryption can be used to realize  $F_{wss}$  for adaptive adversaries.

# Modeling public-key encryption as an ideal functionality

Same motivation as for signatures:

- Re-assert validity of known notions
- Facilitate modular analysis of protocols using PKE (e.g., via the JUC theorem)
- A step towards formal/automated analysis of protocols.

# Functionality $F_{\text{pke}}(I)$

(parameterized by domain  $M$ )

On input  $(\text{sid}, \text{KeyGen})$  from  $D$ , do:

- Verify that  $\text{sid}=(D,\text{sid}')$
- Hand  $(\text{KeyGen}, \text{sid})$  to adv, get value  $e$ .
- Return  $e$  to  $D$ .

On input  $(\text{sid}, \text{Encrypt}, e', m)$  from any  $P$ , do:

- If  $m$  not in  $M$  then return an error message
- Else, hand  $(\text{Encrypt}, \text{sid}, |m|)$  to adv.  
(If  $e' \neq e$  then give the full  $m$  to the adv.)
- Get  $c$  from adv, record  $(m, c)$  and return  $c$  to  $P$ .

On input  $(\text{sid}, \text{Decrypt}, c)$  from  $D$  (and  $D$  only), do:

- If a pair  $(m, c)$  is recorded, then return  $m$  to  $D_i$ .
- Else, hand  $c$  to adv, get value  $m$ , and return  $m$  to  $D$ .

# Equivalence with CCA security

Given an encryption scheme  $E=(\text{Gen}, \text{Enc}, \text{Dec})$ ,  
construct the protocol  $P_E$ :

- On input  $(\text{sid}, \text{KeyGen})$ , verify that  $\text{sid}=(D, \text{sid}')$  where  $D$  is the local pid; then run  $(e, d) \leftarrow \text{Gen}()$ , return  $e$  and record  $d$ .
- On input  $(\text{sid}, \text{Encrypt}, m, e)$ , return  $\text{Enc}_e(m)$ .
- On input  $(\text{sid}, \text{Decrypt}, c)$ ,  $D$  retrieves  $d$  and returns  $\text{Dec}_d(c)$ .

## Theorem:

An encryption scheme  $E$  is CCA2-secure iff protocol  $P_E$  securely realizes  $F_{\text{pke}}$  for non-adaptive adversaries.



## Reminder: CCA2 Security

An encryption scheme  $(\text{Gen}, \text{Enc}, \text{Dec})$  is CCA2 secure for domain  $D$  if:

- **Validity:** For all  $m$  in  $D$ :  $\text{Dec}_d(\text{Enc}_e(m))=m$
- **CCA2 security:**
  - Define game for adversary  $A$ :
    - $(e,d) \leftarrow G(k)$ ;  $A \leftarrow e$
    - $c \leftarrow A$ ;  $A \leftarrow D_d(c)$
    - $(m_0, m_1) \leftarrow A$ ;  $A \leftarrow c^* = E_e(m_b)$ ,  $b \leftarrow_R \{0, 1\}$
    - $c \leftarrow A$ ;  $A \leftarrow D_d(c)$ , unless  $c=c^*$
    - $b' \leftarrow A$
  - Scheme secure if any  $A$  outputs  $b$  with prob.  $< \frac{1}{2} + \text{negl.}$

## Proof of equivalence:

$P_E$  realizes  $F_{pke} \rightarrow E$  is CCA2-secure:

**Validity:** Assume  $E$  is not valid, then construct an environment  $Z$  and adversary  $A$  that distinguish a run of  $P_E$  from the ideal process for  $F_{pke}$ :  $Z$  invokes a simple  $\text{KeyGen} \rightarrow \text{Encrypt} \rightarrow \text{Decrypt}$  sequence for an uncorrupted decryptor.

**CCA2 security:** Assume there exists a breaker  $B$  for  $E$ .  $Z$  runs  $B$ :

- $Z$  Invokes an uncorrupted  $D$  with  $\text{KeyGen}$ , obtains  $e$ , gives to  $B$ .
- When  $B$  asks to encrypt  $(m_0, m_1)$ ,  $Z$  chooses  $b \leftarrow \{0, 1\}$ , asks  $D$  to encrypt  $m_b$ , obtains  $c^*$ , gives  $c^*$  to  $B$  as test ciphertext.
- When  $B$  asks to decrypt  $c$ ,  $Z$  asks  $D$  to decrypt  $c$ .
- When  $B$  outputs a guess  $b'$ ,  $Z$  outputs  $b + b'$ .

**Analysis:** In a run of  $P^H$ ,  $Z$  outputs 1 with non-neglig. probability.  
In the ideal process,  $Z$  never outputs 1.

## Analysis of Z:

- If Z interacts with  $P_E$  then the view of the simulated B sees is exactly that of an interaction with E. Thus, if B has advantage  $f$  then Z outputs 1 w.p.  $\frac{1}{2}+f$ .
- If Z interacts with some S in the ideal process for  $F_{pke}$  then the view of B is independent from  $b$ . (This is so since B sees only  $e$  and the ciphertexts and decryptions generated by S, and the view of S is also independent of  $b$ .) Thus Z outputs 1 w.p.  $\frac{1}{2}$ .

$E$  is CCA2-secure  $\rightarrow P_E$  realizes  $F_{pke}$ :

Let  $Z$  be an environment that distinguishes a run of  $P_E$  from ideal interaction with  $F_{pke}$  w.p.  $f$  for any ideal-process adversary  $S$ . In particular,  $Z$  works for the following “generic  $S$ ”:

- When asked by  $F_{pke}$  to generate a key,  $S$  runs  $(e,d) \leftarrow \text{Gen}()$  and returns  $e$ .
- When asked by  $F_{pke}$  to generate a ciphertext,  $S$  runs  $c \leftarrow \text{Enc}(e, 0^{|m|})$  and returns  $c$ .
- When asked by  $F_{pke}$  to decrypt( $c$ ),  $S$  returns  $m \leftarrow \text{Dec}(d,c)$ .

Let  $n$  be the number times  $Z$  asks to encrypt a message. Define  $n+1$  hybrid interactions  $H_0 \dots H_n$ :

In  $H_i$  the first  $i$  ciphertexts are computed as  $\text{Enc}(e, 0^{|m|})$ , and the rest are computed as  $\text{Enc}(e, m)$  (where  $m$  is the plaintext in the request).

Then there is an  $i < n$  s.t.  $Z$  distinguishes between  $H_i$  and  $H_{i+1}$  with probability  $f/n$ .

Given  $Z$ , construct a breaker  $B$  for  $E$ .  $B$  runs  $Z$ :

- When  $Z$  activates the signer with  $\text{KeyGen}$ ,  $B$  gives  $Z$  the  $e$  from  $B$ 's input.
- In the first  $i$  times that  $Z$  asks a party to encrypt a message  $m$ ,  $B$  returns  $\text{Enc}(e, 0^{|m|})$ .
- In the  $i+1$ st time that  $Z$  asks a party to encrypt a message  $m$ ,  $B$  gives  $(m, 0^{|m|})$  to its  $\text{Enc}(e, 0^{|m|})$ . encryption oracle, gets  $c^*$ , and gives  $c^*$  to  $Z$ .
- In the rest of the times that  $Z$  asks a party to encrypt a message  $m$ ,  $B$  returns  $\text{Enc}(e, m)$ .
- When  $Z$  asks  $D$  to decrypt a ciphertext  $c$  that was generated by  $B$ , then  $B$  returns the corresponding  $m$ . If  $c$  was not generated by  $B$ , then  $B$  consults its decryption oracle and returns the answer to  $Z$ . ( $B$  never asks to decrypt  $c^*$ ).

Claim: If  $c^*$  is an an encryption of  $m$  then  $Z$  sees  $H_i$ . If  $c^*$  is an encryption of  $0^{|m|}$  then  $Z$  sees  $H_{i+1}$ .



Note: If  $Z$  asks to corrupt  $D$  during the interaction then  $F$  is stuck...

## A definitional caveat:

$F_{pke}$  allows an “illegal ciphertext” to be decrypted to a value that became known only after the ciphertext was generated.

This seems to allow “functional malleability”...

**Q:** Is that a contradiction to the equivalence with CCA2-security?

**A:** No... since the equivalence was proven only to encryption schemes where the decryption algorithm does not get external information other than the ciphertext (e.g., network messages), thus the problem does not arise.

**Q:** How to guarantee this property without so restricting the decryption algorithm?

# Functionality $F_{\text{pke}}$ (II)

(parameterized by domain  $M$ )

On input  $(\text{sid}, \text{KeyGen})$  from  $D$ , do:

- Verify that  $\text{sid}=(D,\text{sid}')$
- Hand  $(\text{KeyGen}, \text{sid})$  to adv, get  $e$  and algorithms  $\text{Enc},\text{Dec}$ .
- Return  $e$  to  $D$ .

On input  $(\text{sid},\text{Encrypt},e',m)$  from any  $P$ , do:

- If  $m$  not in  $M$  then return an error message
- Else, hand  $(\text{Encrypt},\text{sid},|m|)$  to adv.  
(If  $e' \neq e$  then give the full  $m$  to the adv.)
- Compute  $c=E(m)$ , record  $(m,c)$  and return  $c$  to  $P$ .

On input  $(\text{sid},\text{Decrypt},c)$  from  $D$  (and  $D$  only), do:

- If a pair  $(m,c)$  is recorded, then return  $m$  to  $D_i$ .
- Else, compute  $m=D(c)$  and return  $m$  to  $D$ .

# Properties of the new formulation

- The decryption value of a ciphertext is not influenced by events occurring after the ciphertext was generated, regardless of how the decryption procedure works.
- Equivalence with CCA2-security still works.



# Summary of course up till now

- Saw two frameworks for analyzing security of cryptographic protocols:
  - “basic security”: Provides the basic ideas, but limited scope (synchronous, non-reactive), limited composability (non-concurrent).
  - “UC security”: More general scope, general composability, but more restrictive.
- Saw general construction and proof techniques (“how to realize any ideal functionality”) within the UC security framework.
- Saw an impossibility result (there are more...)
- Saw how to model common tasks within the UC framework (signatures, authentication, key-exchange, encryption...)

# Things to keep in mind

- Composability is an integral part of a security requirement.
- Notions of security are not set in stone.  
(Deciding what theorems to prove on a protocol is sometimes harder than actually proving...)
- Formulating ideal functionalities “right” is very tricky.
- There is usually no such thing as “THE right notion of security”. Different notions have different properties and are good for different purposes.