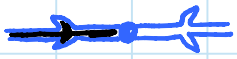
6.890	Lecture 19	Nov. 6, 2014

Recall: (L18)

Bounded 2-player Constraint Logic (2CL)

- each edge is either white or black
- each edge can be reversed only once
- goal: each player has target edge & wins if they reverse it

- PSPACE-complete for planar constraint graphs with white AND, SPLIT, OR, CHOICE & VARIABLE vertex 
  - reduction from impartial game positive CNF SAT
  - players take turns setting variables
  - positive ⇒ white wants true, black wants false
  - black can't win (edge irreversible)
  - white wins ⇔ formula satisfied
  - crossover gadget (only use of CHOICE)
- Can make OR protected using free edge
  no constraint at degree-1 end

Amazons: [Walter Zamkauskas 1988]

- queens on chessboard
- move = queen move + queen shot ↖
  destroy board position at queen-reachable location
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial # moves: shot consumes board
  - reduction from Bounded 2CL

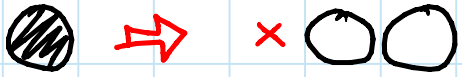
Konane [Hawaii – ancient Hawaiian Polynesians] (documented by Captain James Cook in 1778)

- move = jump your piece over 1 or more opponent pieces in a straight line:

  → remove captured opponent pieces
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial # moves: move consumes ≥ 1 piece
  - reduction from Bounded 2CL
  - conditional gadget for AND, SPLIT, shift:
    - can traverse input 2 → output 2 only after input 1 → output 1 (else captured)
    - ignore output 1 ⇒ AND
    - prime input 2 ⇒ SPLIT
    - both ⇒ parity shift

Cross Purposes: [Michael Albert 2004]

- black stones = 1×1×2 towers
- white stones = fallen towers
- move =  (right)
- Vertical player can only move up/down
- Horizontal player can only move left/right
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial # moves: move consumes black stone
  - reduction from Bounded 2CL
  - H forced to help V after variable settings
  - protected OR (& free edge) to avoid second activation terminating leaving H w/o move

Stochastic games: [Papadimitriou – JCSS 1985]

- one player (of 2) plays randomly "nature"
- PSPACE-complete to win with probability $> 1/2$ (via amplification)
- SSAT: $\exists x_1 : \forall x_2 : \exists x_3 : \forall x_4 : \cdots : \Pr\{F\} > 1/2$
- OPEN: real games?

Unbounded formula games: EXPTIME-complete

[Stockmeyer & Chandra - SICOMP 1979]

- start with arbitrary variable assignment
- can set variables to 0 or 1 many times (unlimited)
- all partizan: black & white variables, plus possibly "turn variable" $t = \begin{cases} 0 & \text{if player 2} \\ 1 & \text{if player 1} \end{cases}$

G_1: move = set all variables of your color & set (common) turn variable $t = \begin{cases} 0 & \text{if player 2} \\ 1 & \text{if player 1} \end{cases}$
lose if you satisfy (common) 4DNF formula
i.e. move must satisfy common 4CNF formula

G_2: move = set one variable of your color (can pass by not changing it)
win if you satisfy your 12DNF formula (2 of them)

G_3: move = flip one variable of your color (no pass)
lose if you satisfy your 12DNF formula (2 of them)

G_4: move = set one variable of your color (can pass)
win if you satisfy (common) 13DNF formula

($G_5 = G_6$ but without CNF constraint)

G_6: move = set one variable of your color (can pass)
player 1 wins if anyone satisfies (single) CNF formula

Peek: stack of plates with holes: 1 fixed plate;
(G_4) black & white plates have 2 states, in & out
- move = manipulate one plate (can pass)
- win if hole all the way through

Membership in EXPTIME = APSPACE [Chandra & Stockmeyer, Kozen - FOCS 1976]

alternating ∀,∃ guesses

- build set of "mate in k" states for $k = 0, 1, \ldots, c^n$

#moves ≤ #states

Unbounded graph games: EXPTIME-complete

[Stockmeyer & Chandra - SICOMP 1979]

HAM:

- given simple undirected graph
- each edge black or white & in or out
- move = toggle in/out of an edge of your color
- player 1 wins if in edges form a Hamiltonian cycle (after any move) (no passing)
- reduction from G_6

BLOCK:

- given 3 graphs on the same vertex set
- each player has tokens of their color on some of the vertices (≤1 token per vertex)
- move = move 1 token of your color along a path in one of the 3 graphs such that target & intermediate vertices have no tokens
- player i wins if they get a token to a vertex $\in W_i$
- reduction from G_3
  - variable & clause gadget

Real games that are EXPTIME-complete: $\Rightarrow \notin P$!

- Checkers [Robson – SICOMP 1981]
  - reduction from G_3 where about to lose after every turn
  - initially players adjust kings ☐/▣ between T/F
  - then player mounts an attack: move A or B forcing opponent to follow path, fork as desired
  - if all attack vars. set & no defense vars. set i.e. DNF clause satisfied then get x free moves
  - with x free moves can trigger outer spiral → huge material advantage
  - then can form picket lines > size(interior)

  $\Rightarrow$ win [Fraenkel, Garey, Johnson, Schaefer, Yesha – FOCS 1978]

- Chess [Fraenkel & Lichtenstein – JCTA 1981]
  - reduction from G_3

- Go with Japanese ko rule [Robson – IFIP 1983]

(Unbounded) 2CL:

- each edge is either white or black
- goal: each player has target edge & wins if they reverse it
- EXPTIME-complete even for planar graphs
  - reduction from G6
  - players flip variables
  - if formula satisfied: white (Player 1) will lock all variables & run formula
  - lock = reverse true or false edge
    - black must respond A (then B, C, D) to prevent white from fast win via F
    ⇒ black immobilized during locks
  - black's slow win is 1 move longer than formula satisfaction ⇒ white can't flip its variables after any locking (no time)
  - white slower win prevents black from flipping A early, e.g. instead of flipping a variable
  - formula uses path equalizer so all satisfying assignments take same time
  - NCL crossover

No-repeat rule: [Robson – MFCS 1984]

lose if ever repeat a past game configuration

⇒ G_1, G_2, G_3 become EXPSPACE-complete as do Chess & Checkers

– OPEN: is Go with superko (no-repeat) EXPSPACE-complete? (as in USA & China)

Conditional no-repeat rule: [Robson – MFCS 1984]

– two special variables x & y

– lose if ever repeat a past game configuration & at most 1 of x & y have changed since

⇒ G_1 becomes 2EXPTIME-complete

Private-information games: [Reif – JCSS 1984]

you can see some but not all of opponent's state

⇒ G_1 5DNF, G_2 DNF become 2EXPTIME-complete

↳ version of Peek with half of winning holes visible to each player

Blind games: [Reif – JCSS 1984]

player 1's entire state is hidden from player 2

⇒ G_2 DNF becomes EXPSPACE-complete

↳ version of Peek above

OPEN: Constraint Logic in all these settings

6.890 Algorithmic Lower Bounds: Fun with Hardness Proofs

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