6.856 — Randomized Algorithms

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Handout #17, October 30, 2002 — Homework 9, Due 11/6

1. Suppose you have an estimation algorithm that will find a $(1 \pm \epsilon)$ approximation to the correct value with probability 3/4. Show that you can reduce the failure probability exponentially fast from 1/4 to any desired δ by performing some number k of estimation experiments and taking the median value returned. Give the smallest upper bound you can on k as a function of δ.

2. Suppose you are able to sample from a probability distribution whose standard deviation is less than its mean. Give an (ϵ, δ)-approximation scheme for estimating the mean of this distribution with a number of samples polynomial in $1/\epsilon$ and $\log 1/\delta$. **Hint:** Consider the sum of n independent samples from the distribution and determine its mean and variance. Bound the probability that this sum deviates greatly from its mean. Now use the previous problem.

3. Suppose you are given a directed graph with n vertices and m unit-length edges. Consider the problem of estimating the number of vertices within distance d of each vertex. Give a fully polynomial (ϵ, δ) approximation scheme that solves this problem simultaneously for all vertices for any fixed d. Your running time should be $\tilde{O}((m + n)/\epsilon^2 \log 1/\delta)$.

4. Write a half-page or greater description of the project you plan to do. Include citation of relevant work. Typeset the writeup, and submit it on a separate sheet with your name. A group may submit a single writeup.