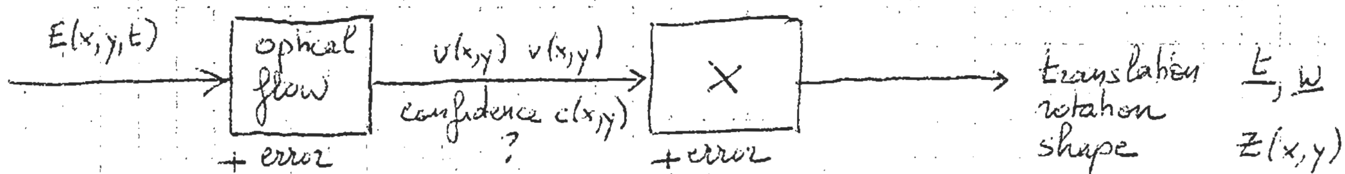
Nov. 2, 2004 (go voting!) 6.866 Proposal due Nov. 9

Motion Vision $E(x, y, t)$

2 stage method:



Is there any direct way?

Motion field $\quad \underline{r} = \begin{pmatrix} x \\ y \\ f \end{pmatrix} \qquad \frac{1}{f}\underline{r} = \frac{1}{\underline{R}\cdot\hat{\underline{z}}}\underline{R}$

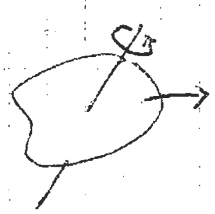
diff: $\quad \frac{1}{f}\dot{\underline{r}} = \frac{1}{\underline{R}\cdot\hat{\underline{z}}}\dot{\underline{R}} - \frac{\dot{\underline{R}}\cdot\underline{z}}{(\underline{R}\cdot\underline{z})^2}\underline{R} = \frac{(\underline{R}\cdot\hat{z})\dot{\underline{R}} - (\dot{\underline{R}}\cdot\hat{z})\underline{R}}{(\underline{R}\cdot\hat{\underline{z}})^2} = \frac{1}{(\underline{R}\cdot\underline{z})^2}\left(\hat{z}\times(\dot{\underline{R}}\times\underline{R})\right)$

thus, $\quad \dot{\underline{r}}\cdot\hat{\underline{z}} = 0 \quad \therefore \quad \dot{\underline{r}} = \begin{pmatrix} u \\ v \\ 0 \end{pmatrix} \quad (z = f \text{ plane})$

$\dot{\underline{r}} = 0 \quad \text{if} \quad \dot{\underline{R}}\times\underline{R} = 0$

Also: $\quad \dot{r} = \frac{1}{\underline{R}\cdot\hat{\underline{z}}}\left(\hat{\underline{z}}\times(\dot{R}\times\underline{r})\right)$

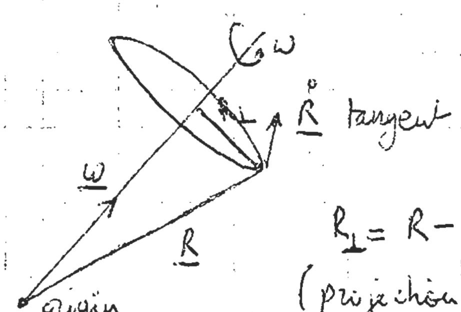
Rigid Body Motion (application)



6-DOF

Rotation: $\underline{\omega} = \omega\hat{\underline{\omega}}$

Translation $\underline{t}$



$R_\perp = R - (\hat{\underline{\omega}}\cdot\underline{R})\hat{\omega}$

(projection of $\underline{R}$ onto $\underline{\omega}$)

We have $\underline{R}_\perp \cdot \dot{\underline{R}} = 0$ and $\underline{R} \cdot \dot{\underline{R}} = 0$

Thus $\dot{R} \parallel R_\perp$ and $R = R_\perp \times R = \underline{\hat{\omega}} \times \underline{R}$

And $\|\dot{R}\| = \|\underline{\omega} \times \underline{R}\| = R_\perp \omega$

Finally $\boxed{\dot{\underline{R}} = \underline{\hat{\omega}} \times \underline{R}}$ (Rotation only)

__Parallel Axis Theorem__

If $\underline{R} \to \underline{R} + \underline{R_0}$, $\underline{\omega} \times (\underline{R} + \underline{R_0}) = \underline{\omega} \times \underline{R} + \underbrace{\underline{\omega} \times \underline{R_0}}_{\text{translation}}$

Any rotation = rotation with axis/origin + translation

__General case__ $\boxed{\dot{\underline{R}} = -\underline{t} - \underline{\omega} \times \underline{R}}$

Now let us combine:

$$\frac{1}{f}\dot{\underline{r}} = -\left(\underline{z} \times \left(\underline{r} \times \left(\underline{r} \times \underline{\omega} - \frac{1}{\underline{R} \cdot \hat{\underline{z}}}\underline{t}\right)\right)\right)$$

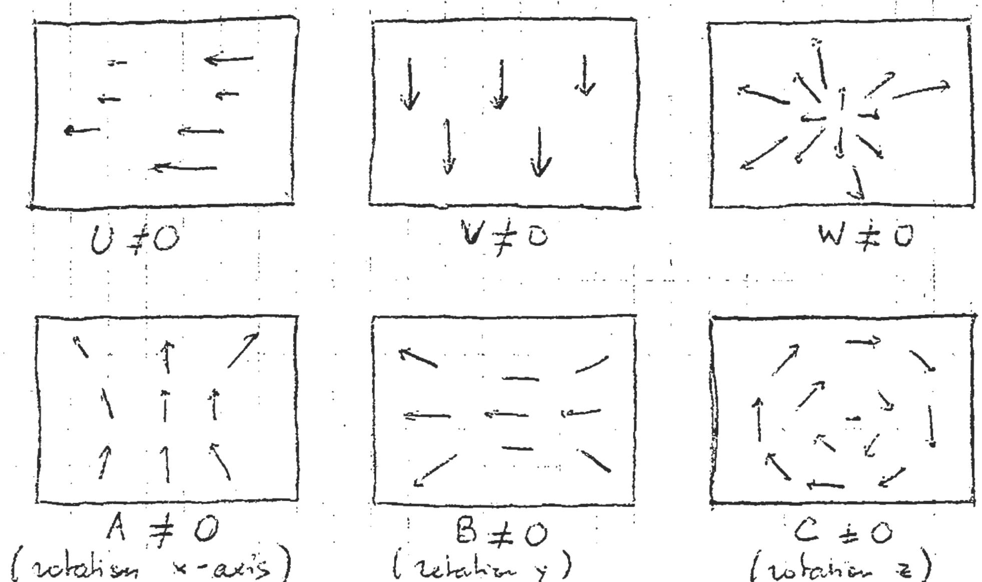
__Scale factor ambiguity:__

$$\left.\begin{array}{l} \underline{R} \to k\underline{R} \\ \underline{t} \to k\underline{t} \end{array}\right\} \text{We get the same image motion!}$$

Now: $$\frac{1}{f}\begin{pmatrix} u \\ v \\ 0 \end{pmatrix} = \begin{pmatrix} \dfrac{-U + \frac{x}{f}W}{Z} + A\dfrac{xy}{f^2} - B\left(1 + \dfrac{x^2}{f^2}\right) + C\,y/f \\ \dfrac{-V + \frac{y}{f}W}{Z} + A\left(1 + \dfrac{y^2}{f^2}\right) - B\dfrac{xy}{f^2} - C\,x/f \end{pmatrix}$$

$$\underbrace{}_{\text{translation, } \frac{1}{Z}} \qquad \underbrace{}_{\text{rotation}}$$

Problem is much easier if we don't have translation but we have much less information!



$$uE_x + vE_y + E_t = 0$$

$$\nabla E \cdot \dot{r} + E_t = 0$$

$$E_t - E_r \cdot \left(\underline{\hat{z}} \times \left(\underline{r} \times \left(\underline{r} \times \underline{\omega} - \frac{1}{\underline{R} \cdot \underline{\hat{z}}} \underline{t}\right)\right)\right) = 0$$

But:

$\underline{E_r} \cdot (\underline{\hat{z}} \times (\underline{r} \times \underline{t})) = (\underline{E_r} \times \underline{\hat{z}}) \cdot (\underline{r} \times \underline{t}) = ((\underline{E_r} \times \underline{\hat{z}}) \times \underline{r}) \cdot \underline{t}$ unknowns pulled out

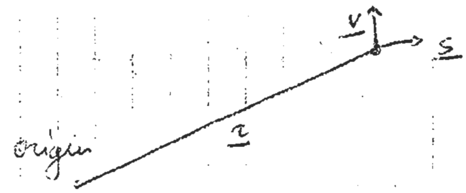
And do the same for the rot. component: ... $(\times\times\times) \cdot \underline{\omega}$

Thus: $\boxed{E_t + \underline{v} \cdot \underline{\omega} + \frac{1}{Z} \underline{s} \cdot \underline{t} = 0}$

$$\underline{s} = (E_r \times \underline{\hat{z}}) \times \underline{r}$$

$$\underline{v} = -\underline{s} \times \underline{r}$$

$\underline{s} \cdot \underline{r} = 0 \qquad \underline{v} \cdot \underline{r} = 0 \qquad \underline{s} \cdot \underline{v} = 0$



$$\underline{s} = \begin{pmatrix} -E_x \\ -E_y \\ \left(\frac{x}{f} E_x + \frac{y}{f} E_y\right) \end{pmatrix} \qquad \underline{v} = (\ldots)$$

Special cases:

- <u>pure rotation</u> can't recover depth! $\|\underline{t}\| = 0$

$E_t + \underline{v} \cdot \underline{w} = 0$ 3 pixels $\rightarrow$ 3 eq $\rightarrow$ ok but noise! $\Rightarrow$ LSQ

$$\min_{\underline{\omega}} \iint (E_t + \underline{v} \cdot \underline{\omega})^2 \, dx\,dy \qquad \frac{d}{d\underline{\omega}} () = 0$$

$$2 \iint (E_t + \underline{v} \cdot \underline{\omega}) \, \underline{v} \, dx\,dy = 0$$

$$\iint (v \cdot \omega) \, \underline{v} = -\int E_t \underline{v}$$

$$\underbrace{\left(\iint v \cdot v^T\right)}_{3\times 3} \underline{\omega} = \underbrace{\iint E_t \underline{v}}_{1\times 3}$$

<u>Depth known</u> $z(x,y) \longrightarrow \underline{\omega}$ & $\underline{t}$?

$$\min_{\underline{\omega}, \underline{t}} \iint \left(E_t + \underline{v} \cdot \underline{\omega} + \frac{1}{z} (\underline{s} \cdot \underline{t})^2\right) dx\,dy$$

$\frac{d}{d\omega} () = 0 \qquad \iint \ldots$ boring calculus $dx\,dy$

$\frac{d}{d\underline{t}} () = 0 \qquad \hookrightarrow$ six linear eq in $\underline{w}$ and $\underline{t}$

- Pure translation

$\|\omega\| = 0 \qquad E_t + \frac{1}{z}\underline{s}.\underline{t} = 0 \qquad \min_{\underline{t}, z} \iint \left(E_t + \frac{1}{z}\underline{s}.\underline{t}\right)^2 dx\,dy$

there is a trivial solution : $z = \frac{-\underline{s}.\underline{t}}{E_t}$ given any $\underline{t}$ ... it's ill-posed!

Problem is that z may be negative ...

If $E_t \ll 1$, dynamic range issue. We can try to find $\underline{t}$ such as

$$\min_{\underline{t}} \iint z^2\,dx\,dy \quad \text{or} \quad \min_{\underline{t}} \iint \left(\frac{\underline{s}.\underline{t}}{E_t}\right)^2 dx\,dy$$

$\Rightarrow \min \; \underline{t}^T S \underline{t}$ good, but it has a trivial solution $\underline{t} = 0$

1. force $\underline{t}$ to be unit vector
2. Raleigh quotient $\frac{\underline{t}^T S \underline{t}}{\underline{t}^T \underline{t}}$