Geometry properties of binary images

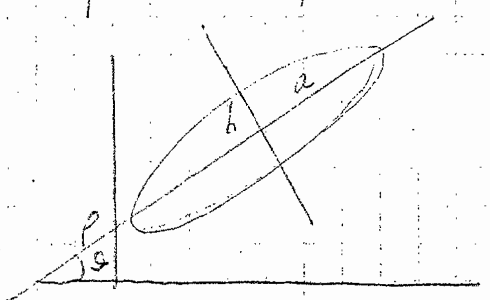
<u>Moments</u>: $M_{00} = Area = \iint b(x,y)\,dx\,dy$

$$M_{10} = x_0 A = \iint x\, b(x,y)\,dx\,dy \qquad (M_{ij} \quad x^i y^j)$$

$$M_{01} = y_0 A = \iint y\, b(x,y)\,dx\,dy$$

$$\tan 2\theta = \frac{2b}{a-c} = \frac{2M_{11}}{M_{20}-M_{02}}$$

<u>Équivalent ellipse</u>



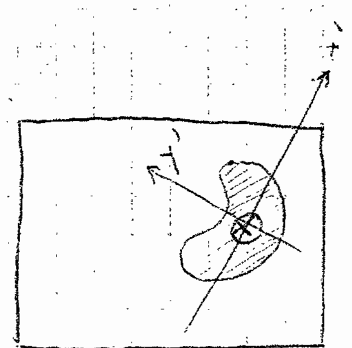
a & b are moment invariance

→ 2nd order, all info is in the ellipse

$A = \pi ab$

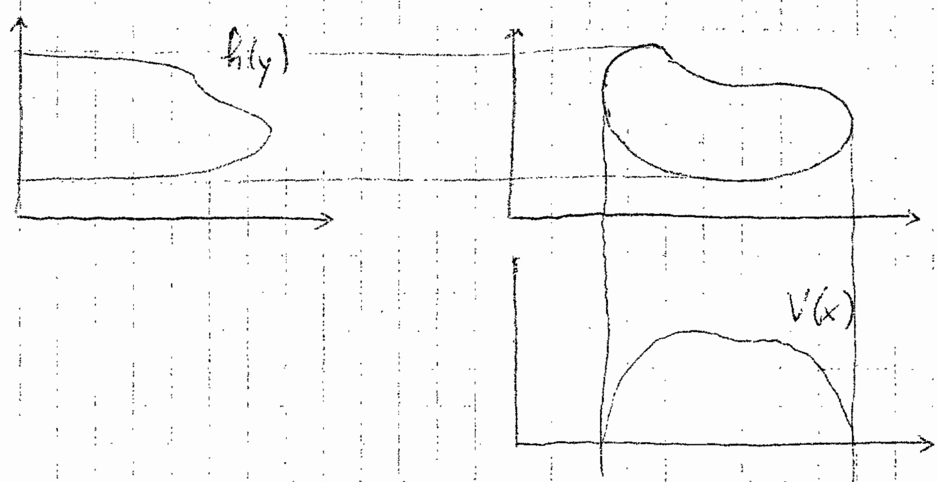
$e = \sqrt{1-(a/b)^2}$

Remove translational dependence: $x = x_0 + x'$
$y = y_0 + y'$



Remove the rotational dependence:

Projections



$V(x) = \int h(x,y)\,dy$

$h(y) = \int h(x,y)\,dx$

$A = \int V(x)\,dy = \int h(y)\,dx$

$x_0 A = \int x\,V(x)\,dx$ → faster

$y_0 A = \int y\,h(y)\,dy$

$M_{20} = \int x^2 V(x)\,dx$ $\qquad M_{02} = \int y^2 h(y)\,dy$

M_{11} ?

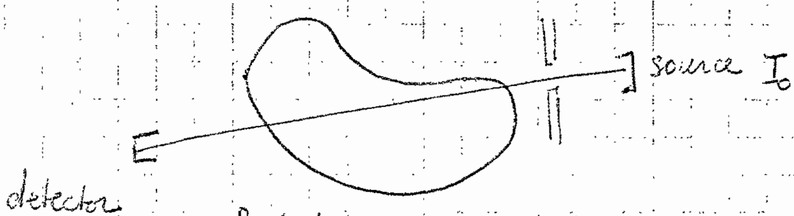
Diagonal projection: $\begin{cases} x' = \dfrac{x+y}{\sqrt{2}} \\ y' = \dfrac{x-y}{\sqrt{2}} \end{cases}$

We can compute $\iint x'^2\, b(x',y')$ and $\iint y'^2\, b(x',y')$ → M_{11}

Conclusion : 3 projections $\rightarrow$ all 2^{nd} order moments

$(n+1)$ projections $\rightarrow$ n^{th} order moments.

Computer Tomography (CT)



$$I = I_0 \, e^{-\int \mu(s)\,ds} \quad \rightarrow \quad \underbrace{\int \mu(s)\,ds}_{\text{given}} = \log I_0 - \log I$$

Many views $\rightarrow$ Radon Transform

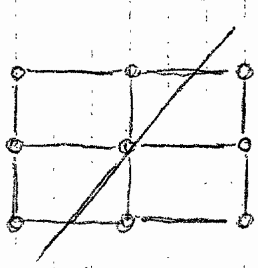
$P(\theta, \rho)$ — filtered back projection $\rightarrow$ $\mu(x,y)$

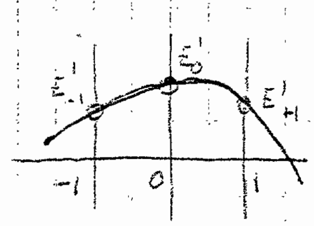
Edge detection

Compute slope on the image $\sqrt{E_x^2 + E_y^2}$ steepest ascent $\frac{1}{\sqrt{\dots}}(E_x, E_y)$

$\rightarrow$ find peaks in gradient magnitude (slope)

Discrete version :





$E' = a + bs + cs^2$

we find a, b, c using E'_{-1}, E'_0, E'_{+1}

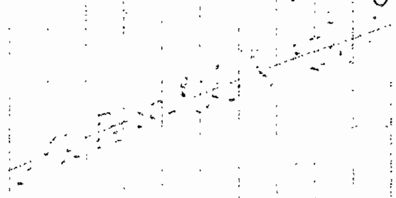
$$\frac{d}{ds}() = 0 \quad \rightarrow \quad s = -\frac{(E'_1 - E'_{-1})/2}{E'_1 - 2E'_0 + E'_{-1}}$$

$|s| \leq \frac{1}{2}$ → declare edge element at $x = x_1 + s\frac{E_x}{\sqrt{\dots}}$

$$y = y_1 + s\frac{E_y}{\sqrt{\dots}}$$

orientation $\left(\frac{-E_y}{\sqrt{\dots}}, \frac{E_x}{\sqrt{\dots}}\right)$.

Then: chain up edge fragments.



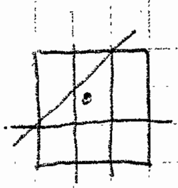
① nearest pixel
② interpolation
③ fit to 9 pixels

$$E' = a + bx + cy + dx^2 + exy + fy^2$$

$$\min_{a, \dots, f} \sum \left(E'_i - (a + bx_i + \dots)\right)^2$$

$\left\{ \frac{d}{da} = 0, \dots, \frac{d}{df} = 0 \right\}$ LSQ 6 equations, 6 unknowns

$$\left.\begin{aligned} \frac{d}{dx}(a + bx + \dots) &= 0 \\ \frac{d}{dy}(a + bx + \dots) &= 0 \end{aligned}\right\} \rightarrow x, y \text{ peak in the gradient}$$



peak along line $x = x_0 + s\frac{E_x}{\sqrt{\ldots}}$ $y = y_0 + s\frac{E_y}{\sqrt{\ldots}}$

plugin into $a + bx + \ldots \rightarrow f(s)$ $\frac{d}{ds}() = 0 \rightarrow s$