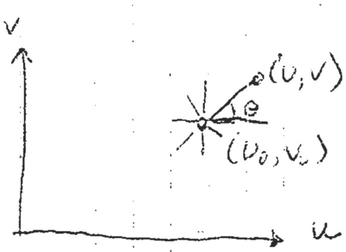
6.866
09/21/04

$$I = \iint (uE_x + vE_y + E_t)^2$$

$$I = \begin{vmatrix} (u-u_0) & (v-v_0) \end{vmatrix} \begin{vmatrix} \iint E_x^2 & \iint E_x E_y \\ \iint E_x E_y & \iint E_y^2 \end{vmatrix} \begin{vmatrix} u-u_0 \\ v-v_0 \end{vmatrix}$$

(u_0, v_0) is a solution of $\frac{dI}{du} = 0$ & $\frac{dI}{dv} = 0$



$$u - u_0 = \cos\theta$$
$$v - v_0 = \sin\theta$$

$$I = \begin{vmatrix} \cos\theta & \sin\theta \end{vmatrix} \begin{vmatrix} a & b \\ b & c \end{vmatrix} \begin{vmatrix} \cos\theta \\ \sin\theta \end{vmatrix}$$

$$I = a\cos^2\theta + 2b\sin\theta\cos\theta + c\sin^2\theta$$

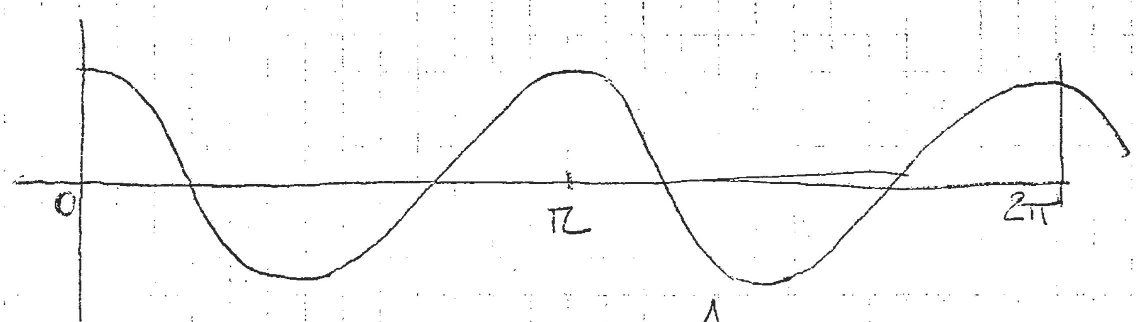
but:
$$\left.\begin{aligned} \cos^2\theta &= \frac{1}{2}(1+\cos 2\theta) \\ \sin^2\theta &= \frac{1}{2}(1-\cos 2\theta) \\ 2\sin\theta\cos\theta &= \sin 2\theta \end{aligned}\right\} \Rightarrow I = \frac{1}{2}(a+c) + \frac{1}{2}(a-c)\cos 2\theta + b\sin 2\theta$$

periodic $p = \pi$

or: $$I = \frac{1}{2}(a+c) + \sqrt{\frac{1}{4}(a-c)^2+b^2}\left(\underbrace{\frac{\frac{1}{2}(a-c)}{\sqrt{\quad}}}_{\cos 2\phi}\cos 2\theta + \underbrace{\frac{b}{\sqrt{\quad}}}_{\sin 2\phi}\sin 2\theta\right)$$

$$\boxed{I = \frac{1}{2}(a+c) + \sqrt{\frac{1}{4}(a-c)^2+b^2}\,\cos 2(\theta-\phi)}$$

$$\tan 2\phi = \frac{2b}{a-c}$$



$$I_{min,\,max} = \frac{1}{2}(a+c) \pm \frac{1}{2}\overbrace{\sqrt{(a-c)^2+4b^2}}^{\Delta}$$

$a \geq 0 \quad c \geq 0$

$ac \geq b^2$

I_{min} small $\longrightarrow$ poor result

- special case: $E_x = k E_y$ then $ac = b^2$

  and $\Delta = \sqrt{(a+c)^2} = a+c$

$$I_{min,max} = \frac{1}{2}(a+c) \pm \frac{1}{2}(a+c) \qquad \boxed{I_{min} = 0}$$

Eigenvalues / eigenvectors Reminder.

$$M\underset{\sim}{v} = \lambda \underset{\sim}{v}$$

$$(M - \lambda I)\underset{\sim}{v} = 0$$

if $\det(M) \neq 0$, 1 solution: $\underset{\sim}{v} = 0$.

if $\det(M) = 0$, 1+ solutions.

$$\det\left(\begin{vmatrix} a & b \\ b & d \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right) = 0$$

$$\det \begin{vmatrix} a-\lambda & b \\ b & d-\lambda \end{vmatrix} = 0$$

$$(a-\lambda)(d-\lambda) - bb = 0$$

$$(a-\lambda)(c-\lambda) - b^2 = 0$$

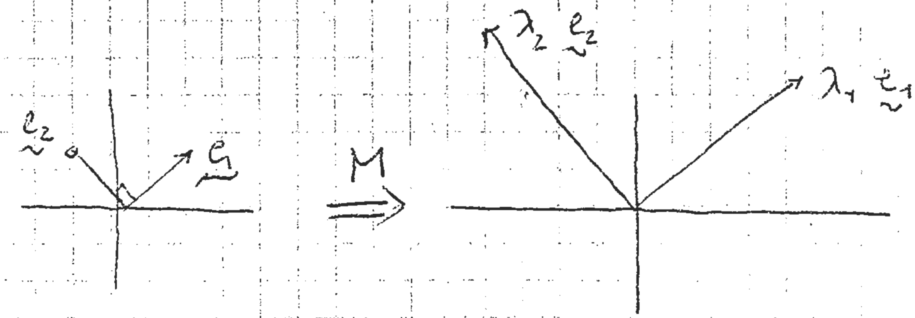
$$\lambda^2 - (a+c)\lambda + ac - b^2 = 0 \qquad \lambda_{+,-} = \frac{1}{2}(a+c) \pm \frac{1}{2}\sqrt{(a-c)^2 + 4b^2}$$

$$M \underset{\sim}{e_1} = \lambda_1 \underset{\sim}{e_1}$$

$$M \underset{\sim}{e_2} = \lambda_2 \underset{\sim}{e_2}$$

$$\underset{\sim}{e_1} \cdot \underset{\sim}{e_2} = 0$$

$$\det(M) = \lambda_1 \lambda_2$$



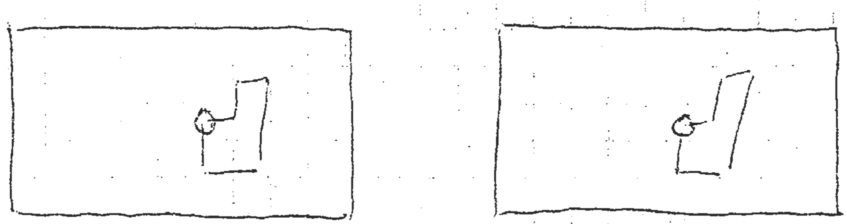
$$M^{-1} \underset{\sim}{e_1} = \frac{1}{\lambda_1} \underset{\sim}{e_1}$$

$$M^{-1} \underset{\sim}{e_2} = \frac{1}{\lambda_2} \underset{\sim}{e_2}$$

$\boxed{!}$ danger if $\lambda_1 \ll 1$ or $\lambda_2 \ll 1$

quality = magnitude of the smallest eigenvalue.

Binocular stereo

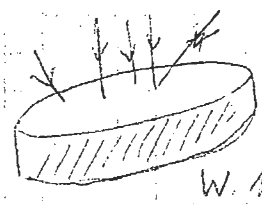


"interest operator"   compute a, b, c on a patch over the image

Read chapter 1 & 2. + 10?

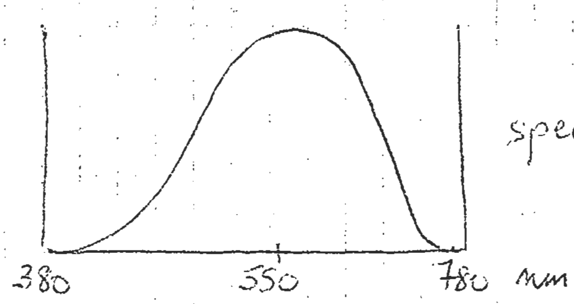
"Brightness"

① Radiance power per unit area



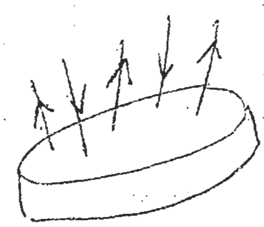
W. m^{-2}

$$E = \frac{\delta P}{\delta A}$$



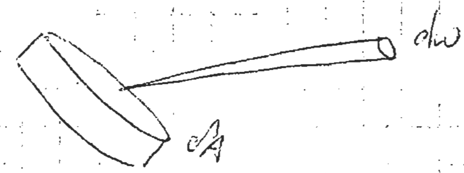
spectral weighting

"Emittance"



W. m^{-2}

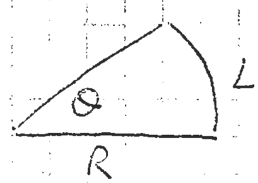
"Scene Radiance"



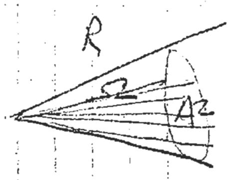
$$L = \frac{\delta^2 P}{\delta A \, \delta w}$$

W. m^{-2} sr^{-1}

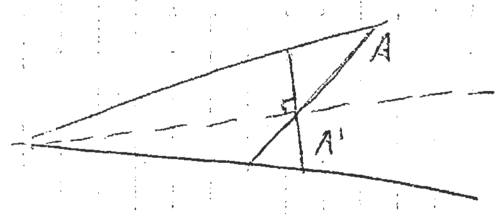
"Solid Angle"



$\theta = \frac{L}{R}$ max 2π



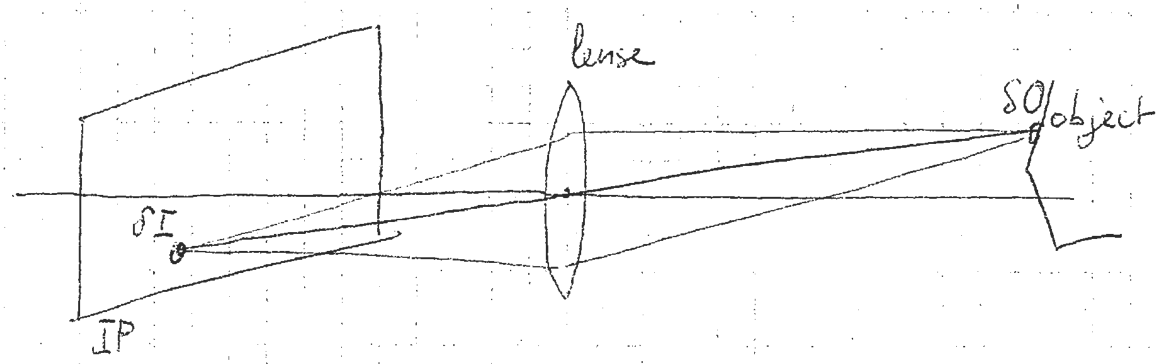
$\Omega = \frac{A}{R^2}$ (radians) max 4π

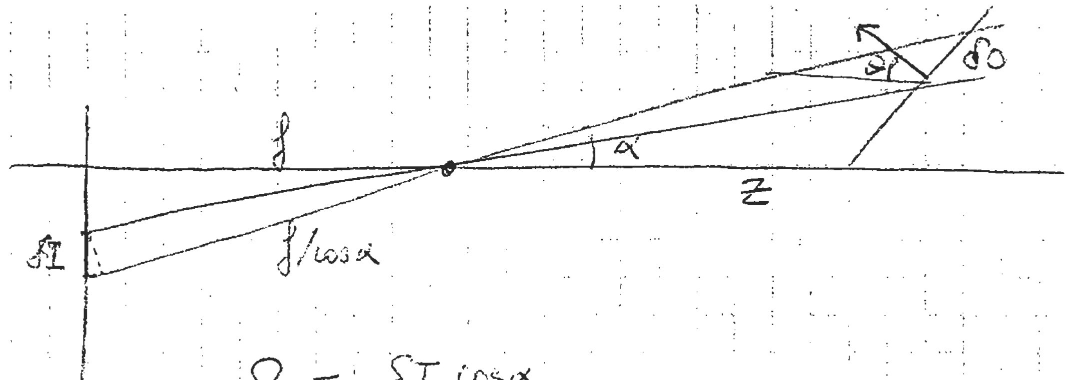


area A $\quad \Omega = \frac{A \cos\theta}{R^2}$

area $A' = A \cos\theta$

Image formation



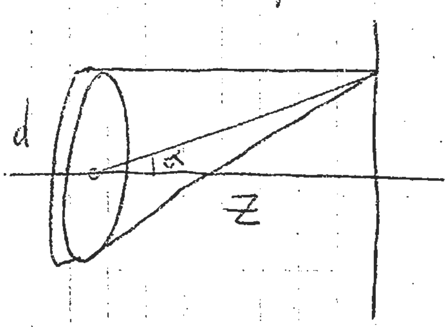


$$\Omega = \frac{\delta I \cos\alpha}{(f/\cos\alpha)^2}$$

and $$\Omega = \frac{\delta O \cos\theta}{(Z/\cos\alpha)^2}$$

thus: $$\boxed{\frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\theta}\left(\frac{Z}{f}\right)^2}$$

lense seen from the object,



$$\Omega_Z = \frac{\pi/4\, d^2 \cos\alpha}{(Z/\cos\alpha)^2} = \frac{\pi}{4}\left(\frac{d}{Z}\right)\cos^3\alpha$$

$$\delta P = L\, \delta O\, \Omega_Z \cos\theta$$

$$\delta P = L\, \delta O \cdot \frac{\pi}{4}\left(\frac{d}{Z}\right)^2 \cos^3\alpha \cos\theta$$

$$\boxed{E = \frac{\delta P}{\delta I} = L \frac{\pi}{4} \left(\frac{d}{f}\right)^2 \cos^4 \alpha}$$

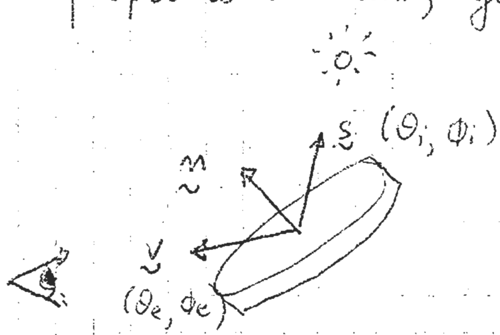
L scene radiance

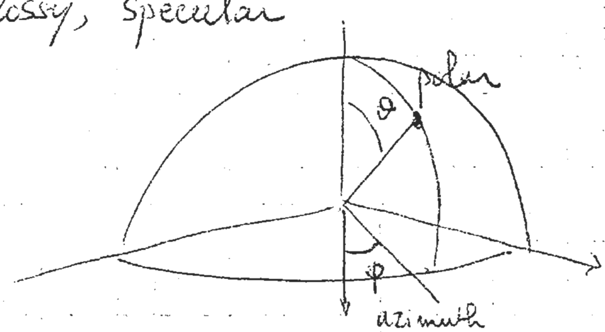
E image radiance

$\frac{f}{d} =$ f-stop (aperture)

Surface reflectance

properties : matt, glossy, specular





BRDF Bi Directional Reflectance Distribution Function

$$f(\theta_i, \varphi_i, \theta_e, \varphi_e) = \frac{\delta L(\theta_e, \varphi_e)}{\delta E(\theta_i, \varphi_i)}$$

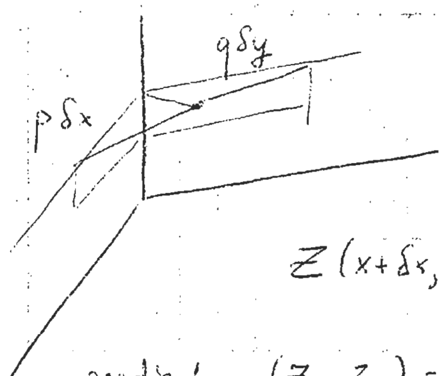
emitted (radiance)

incoming (irradiance)

for many surfaces, $(\varphi_i - \varphi_e)$ matters. $\longrightarrow$ 3 variables
(it does for velvet, hair, brushed aluminum)
thus; $$f(\theta_i, \varphi_i ; \theta_e, \varphi_e) = f(\theta_e, \varphi_e ; \theta_i, \varphi_i)$$
(Helmoltz reciprocity)

$\boxed{\text{Surface orientation}}$

unit normal $\hat{\underset{\sim}{n}}$ height $Z(x, y)$ of surface



$$Z(x+\delta x, y+\delta y) = Z(x, y) + \underbrace{\frac{\partial z}{\partial x}}_{p} \delta x + \underbrace{\frac{\partial z}{\partial y}}_{q} \delta y + \dots$$

gradient of surface: $(Z_x, Z_y) = (p, q)$

tangent: $(\delta x, 0, p\,\delta x)^T$
$(0, \delta y, q\,\delta y)^T$

normal $\underset{\sim}{N}$: $(-p, -q, 1)\,\delta x\,\delta y$

$$|N| = \frac{1}{\sqrt{1+p^2+q^2}} \qquad \hat{\underset{\sim}{n}} = \frac{1}{|N|} \cdot \underset{\sim}{N}$$