6.728 Applied Quantum and Statistical Physics:

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**PROBLEM SET 1**

Problem Set Out: 9/6/06 Problem Set Due: 9/13/06 at the beginning of class

## READINGS

Chapters 1—3 of the text.

## PROBLEMS

**Problem 1.1** Text Problem 1.1

A collimated laser beam with wavelength $\lambda$ is incident upon a slit of width $w$. This beam is modeled as an ideal scalar plane wave.

(a) What is the width of the emerging beam at the detector, which is at distance $R$ from the slit? Determine the width, in this case, as the distance between the first pair of nodes on either side of the central maximum.

(b) If a second slit is added that is displaced by $s$ (between slit centers), what is the distance between nodes at the detector?

(c) What happens if the incident beam comes in at a small angle relative to the slits?

**Problem 1.2** The Fourier transform and its inverse are given by

$$A(x) = \int_{-\infty}^{\infty} A(q) e^{iqx} \frac{dq}{2\pi}$$

$$A(q) = \int_{-\infty}^{\infty} A(x) e^{-iqx} dx$$

(a) Prove the following properties from the above definition:

(i) $A(bx)$ has the Fourier transform $\frac{1}{|b|} A(q/b)$.

(ii) $A(x - x_o)$ has the Fourier Transform $e^{-iqx_o} A(q)$.

(iii) $\frac{d}{dx} A(x)$ has the Fourier Transform $iqA(q)$.

(b) Find the Fourier Transform $A(q)$ of the following by using the tables in the Formula Sheet, which is listed on the class website under Materials. If you are using another set of Fourier Transform tables, remember that we use the physics convention, and that those tables are typically published in terms of $t$ and $\omega$ rather than in terms of $x$ and $q$.

(i) $A(x) = xe^{-x^2}$.

(ii) $A(x) = e^{ikx}$.

(iii) $A(x) = [\sin kx]e^{-(x/b)^2}$

(iv) Make a rough sketch of $A(q)$ from part (iii).

**Problem 1.3** We will encounter a gaussian wave function in quantum mechanics many times. The normalized expression is given by

$$\Psi(x) = \left(\frac{1}{\pi L^2}\right)^{1/4} e^{-x^2/(2L^2)}$$

(a) In `Matlab` define a column vector $\vec{\Psi}_L$ which is evaluated on the interval $-20\text{nm} \leq x \leq 20\text{nm}$ with increments of $\delta x = 0.02\text{nm}$. Let L=0.5 nm, 1 nm, and 2 nm.
(b) Plot $|\vec{\Psi}_{0.5}|^2$, $|\vec{\Psi}_1|^2$, $|\vec{\Psi}_2|^2$ over the range $-6\text{nm} \leq x \leq 6\text{nm}$. Make sure the plot is in a single graph. If you evaluate each function separately see the `hold` function in `Matlab`. But you can be more clever and define a matrix with 3 columns, one for each $L$ and then plot the matrix,
(c) Show that the wave function is normalized to $\int_{-\infty}^{\infty} |\Psi_L(x)|^2\, dx = 1$ by numerically calculating for each $L$

$$\sum_i \Psi_L^*(x_i)\Psi_L(x_i)\,\delta x = 1.$$

Note that this sum is best calculated in `Matlab` by doing an inner product of the vector and its adjoint, namely by noting that $\sum_i \Psi_L^*(x_i)\Psi_L(x_i) = \vec{\Psi}_L^\dagger \vec{\Psi}_L$. (The inner product of two vectors `A` and `B` is `A' * B`).
(d) Numerically evaluate

$$\langle x \rangle = \int_{-\infty}^{\infty} \Psi_L^*(x)\, x\, \Psi_L(x)\, dx \approx \sum_i \Psi_L^*(x_i)\, x_i\, \Psi_L(x_i)\, \delta x$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \Psi_L^*(x)\, x^2\, \Psi_L(x)\, dx \approx \sum_i \Psi_L^*(x_i)\, x_i^2\, \Psi_L(x_i)\, \delta x$$

and hence find $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$. Do the sums as a vector products. Note what `A'*diag(x)*B)` does.