

# Lecture 37 - Bipolar Junction Transistor

(*cont.*)

May 7, 2007

## Contents:

1. Common-emitter short-circuit current-gain cut-off frequency,  $f_T$

## Reading material:

del Alamo, Ch. 11, §11.4.2

## Key questions

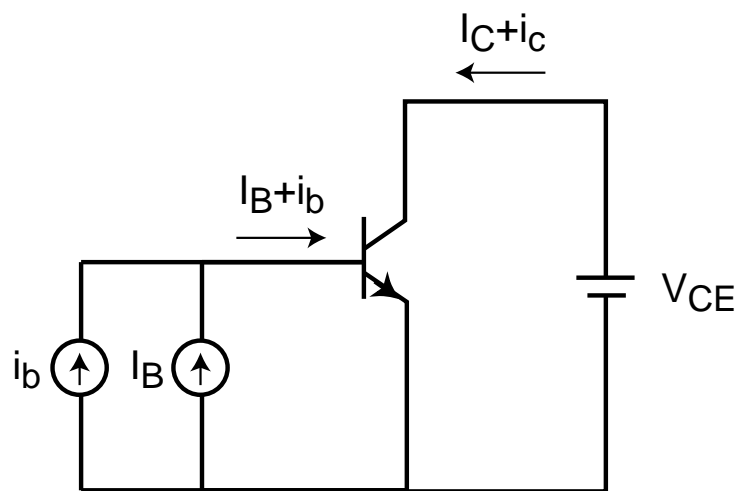
- How is the frequency response of a transistor assessed?
- What determines the frequency response of an ideal BJT?
- How can the frequency response of a BJT be engineered?

# 1. Common-emitter short-circuit current-gain cut-off frequency, $f_T$

$f_T$ : high-frequency figure of merit for transistors

Short-circuit means from the small-signal point of view.

BJT is biased in FAR.



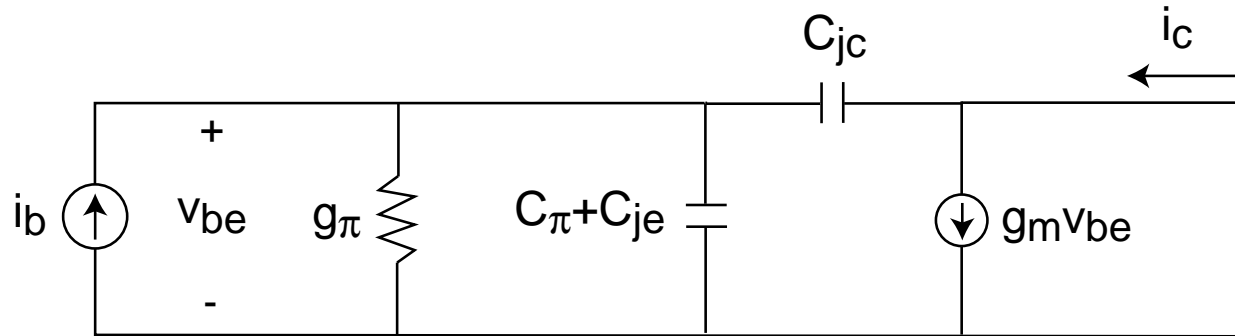
Focus on small-signal current gain:

$$h_{21} = \left. \frac{i_c}{i_b} \right|_{v_{ce}=0}$$

For low frequency,  $h_{21} \rightarrow \beta_F$ , for high frequency  $h_{21}$  rolls off due to capacitors.

Definition of  $f_T$ : frequency at which  $|h_{21}| = 1$ .

Small-signal equivalent circuit model:



$$i_c = g_m v_{be} - j\omega C_{jc} v_{be}$$

$$i_b = [g_\pi + j\omega(C_\pi + C_{je} + C_{jc})]v_{be}$$

Then:

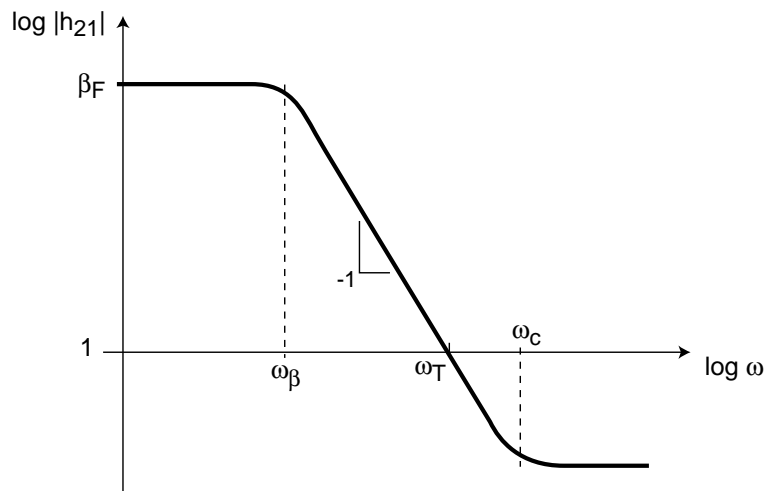
$$h_{21} = \frac{g_m - j\omega C_{jc}}{g_\pi + j\omega(C_\pi + C_{je} + C_{jc})}$$

Magnitude of  $h_{21}$ :

$$|h_{21}| = \frac{\sqrt{g_m^2 + \omega^2 C_{jc}^2}}{\sqrt{g_\pi^2 + \omega^2(C_\pi + C_{je} + C_{jc})^2}}$$

$$|h_{21}| = \frac{\sqrt{g_m^2 + \omega^2 C_{jc}^2}}{\sqrt{g_\pi^2 + \omega^2 (C_\pi + C_{je} + C_{jc})^2}}$$

Bode plot of  $|h_{21}|$ :



Three regimes in  $|h_{21}|$ :

- low frequency,  $\omega \ll \omega_\beta$ :

$$|h_{21}| \simeq \frac{g_m}{g_\pi} = \beta_F$$

- intermediate frequency,  $\omega_\beta \ll \omega \ll \omega_c$ :

$$|h_{21}| \simeq \frac{g_m}{\omega(C_\pi + C_{je} + C_{jc})}$$

- high frequency,  $\omega \gg \omega_c$ :

$$|h_{21}| \simeq \frac{C_{jc}}{C_\pi + C_{je} + C_{jc}}$$

Angular frequencies that separate three regimes:

$$\omega_{\beta} = \frac{g_{\pi}}{C_{\pi} + C_{je} + C_{jc}}$$

$$\omega_c = \frac{g_m}{C_{jc}}$$

Angular frequency at which  $|h_{21}| = 1$ :

$$\omega_T = \frac{g_m}{C_{\pi} + C_{je} + C_{jc}}$$

In terms of frequency:

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{je} + C_{jc})}$$

Note:

$$\omega_{\beta} = \frac{\omega_T}{\beta_F}$$

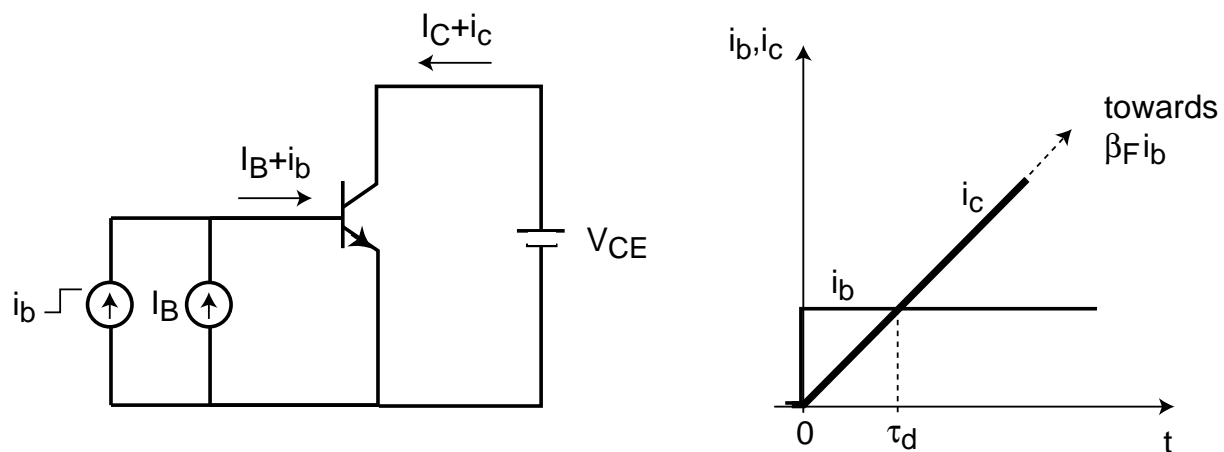
## □ Physical meaning of $f_T$

$1/2\pi f_T$  has units of time. Define *delay time*:

$$\tau_d = \frac{1}{2\pi f_T} = \frac{C_\pi}{g_m} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m} = \tau_{tB} + \frac{\tau_{tE}}{\beta_F} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m}$$

Four delay components in  $\tau_d$ .

Consider response of BJT to a step-input base current:



At  $t = 0$

$$I_B \rightarrow I_B + i_b$$

As  $t \rightarrow \infty$

$$V_{BE} \rightarrow V_{BE} + v_{be}$$

$$I_C \rightarrow I_C + i_c = I_C + \beta_F i_b.$$

*How much time does it take for  $i_C$  to reach its final value?*

Charge must be delivered to four regions in BJT:

- *Quasi-neutral emitter*

$$q_e = \tau_{tE} i_b$$

- *Quasi-neutral base*

$$q_b = \tau_{tB} i_c$$

- *Emitter-base depletion region*

$$q_{je} = C_{je} v_{be} = \frac{C_{je}}{g_m} i_c$$

- *Base-collector depletion region*

$$q_{jc} = C_{jc} v_{bc} = C_{jc} v_{be} = \frac{C_{jc}}{g_m} i_c$$

Charge delivered at constant rate to base. Time that it takes for all charge to be delivered:

$$\tau_\beta = \frac{q_e + q_b + q_{je} + q_{jc}}{i_b} = \tau_{tE} + \beta_F (\tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m}) = \frac{1}{2\pi f_\beta}$$



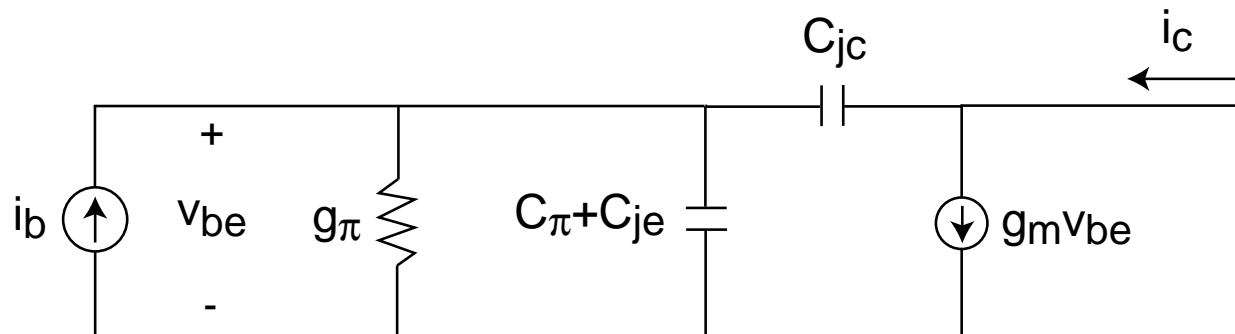
How much time does it take for  $i_C$  to build up to  $I_C + i_b$ ?

Since  $i_c = \beta_F i_b$ ,

$$\tau_d = \frac{\tau_\beta}{\beta_F} = \frac{\tau_{tE}}{\beta_F} + \tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m} = \frac{1}{2\pi f_T}$$

- $\tau_d = \frac{1}{2\pi f_T}$ : delay time before  $i_C$  increases to  $I_C + i_b$
- $\tau_\beta = \frac{1}{2\pi f_\beta}$ : delay time before  $i_C$  increases to  $I_C + \beta_F i_b$

With sinusoidal input:



$f \uparrow \Rightarrow$  fraction of  $i_b$  that goes into capacitors  $\uparrow \Rightarrow v_{be} \downarrow \Rightarrow i_c \downarrow$ .

At  $f_T$ :  $|i_c| = |i_b|$

## □ Key dependencies of $f_T$ in ideal BJT

★  $f_T$  dependence on  $I_C$ :

Rewrite  $f_T$ :

$$f_T = \frac{g_m}{2\pi(C_\pi + C_{je} + C_{jc})} = \frac{1}{2\pi\tau_F} \frac{1}{1 + \frac{kT}{q\tau_F} \frac{C_{je} + C_{jc}}{I_C}}$$

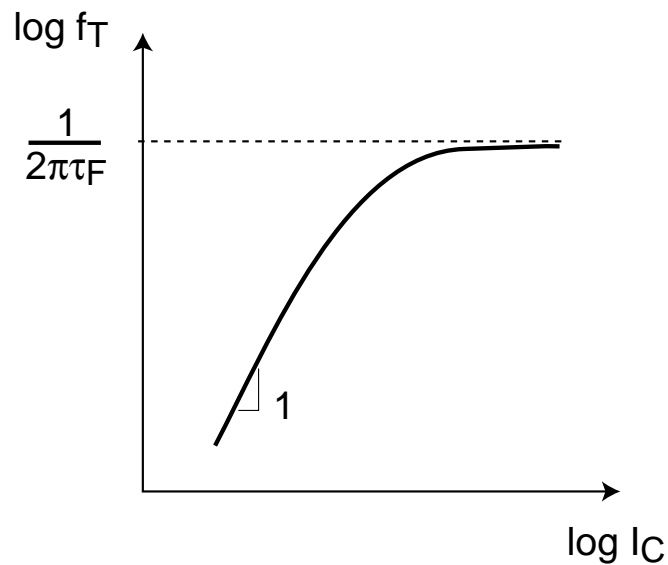
Two limits:

- Small  $I_C$ : limited by depletion capacitances

$$f_T \simeq \frac{q}{2\pi kT} \frac{I_C}{C_{je} + C_{jc}}$$

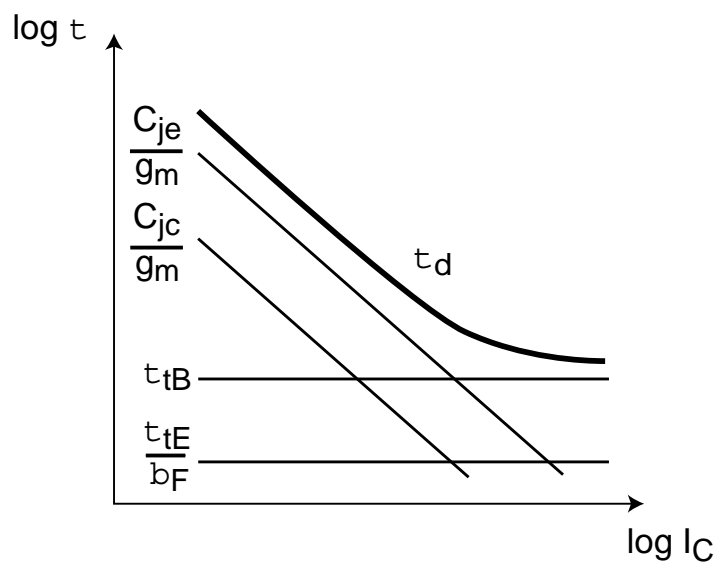
- Large  $I_C$ : limited by intrinsic delay (dominated by  $\tau_{tB}$ )

$$f_T \simeq \frac{1}{2\pi\tau_F}$$

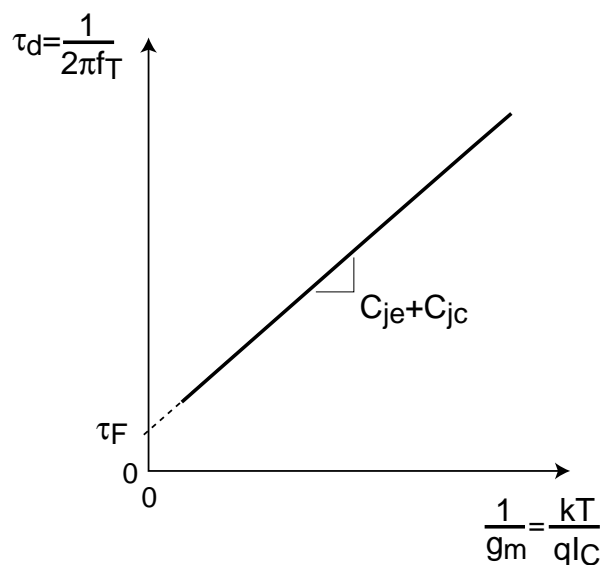


Alternative view of  $I_C$  dependence:

$$\tau_d = \frac{\tau_{tE}}{\beta_F} + \tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m} = \frac{1}{2\pi f_T}$$



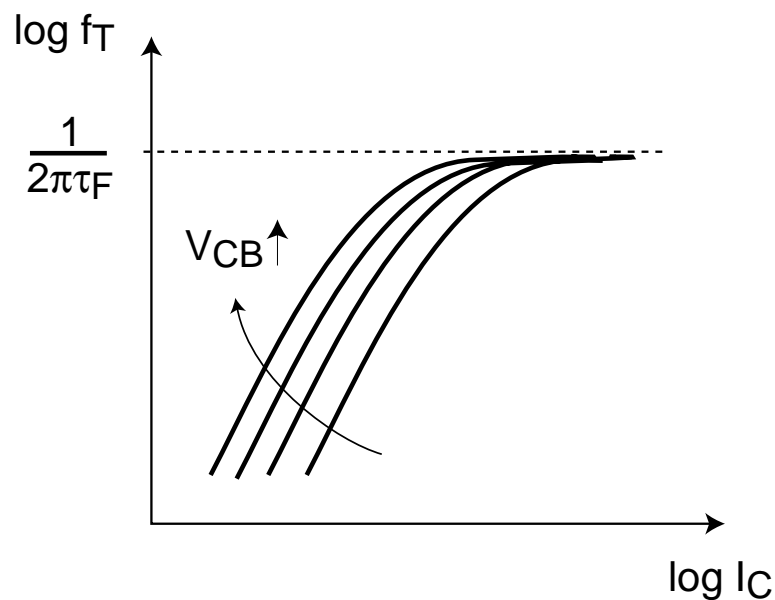
Standard experimental technique to extract  $\tau_F$  and  $C_{je} + C_{jc}$ :



★  $f_T$  dependence on  $V_{BC}$ :

$V_{CB} \uparrow$  (B-C junction is more reverse biased)  $\Rightarrow C_{jc} \downarrow \Rightarrow f_T \uparrow$

[but only in low  $I_C$  regime of  $f_T$ ]



★  $f_T$  dependence on device layout:

- For low  $I_C$ :  $f_T$  dominated by  $C_{je}$ ,  $C_{jc}$

$$\frac{C_{je}}{g_m} \propto \frac{A_E C_{jeo}}{I_C}$$

$$\frac{C_{jc}}{g_m} \propto \frac{A_C C_{jco}}{I_C}$$

If  $A_E \uparrow$  or  $A_C \uparrow$  (keeping  $I_C$  constant)  $\Rightarrow f_T \downarrow$

- For high  $I_C$ :  $f_T$  dominated by  $\tau_F$ ;  $f_T$  independent of  $A_E$  or  $A_C$

## □ Device design strategies for improving $f_T$

Four delay terms in  $f_T$ :

$$\tau_d = \frac{1}{2\pi f_T} = \frac{\tau_{tE}}{\beta_F} + \tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m}$$

Strategies to reduce each delay component:

★ *Emitter charging time*,  $\frac{\tau_{tE}}{\beta_F}$ , minimized by

- enhancing  $\beta_F$ ,
- having a shallow emitter ( $\tau_{tE} \sim W_E^2$ ),
- building steep doping profile in emitter.

$\frac{\tau_{tE}}{\beta_F}$  small contribution to  $\tau_d$ , not much payoff.

★ Base transit time,  $\tau_{tB}$ , minimized by

- reducing  $W_B$  ( $\tau_{tB} \sim W_B^2$ ),
- introducing drift field in base (through impurity gradient or SiGe composition gradient).

Significant device engineering towards minimizing  $\tau_{tB}$ .

Example 1 [Kasper 1993]:

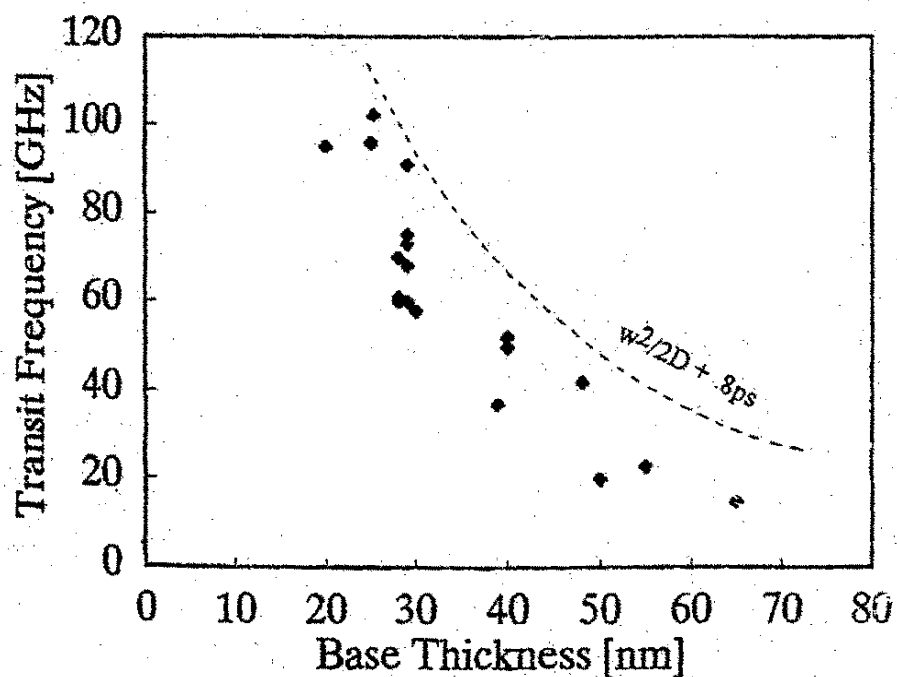


Fig. 2: Transit frequency  $f_T$  versus SiGe thickness (effective base width)

[Kasper, 1993]

Kasper, E., and A. Gruhle. "Silicon Germanium Heterobipolar Transistor for High Speed Operation." *Proceedings of the IEEE/Cornell Conference on Advanced Concepts in High Speed Semiconductor Devices and Circuits, August 2-4, 1993*. New York, NY: IEEE Electron Devices Society, 1993, pp. 23-30. ISBN: 9780780308954. Copyright 1993 IEEE. Used with permission.

Example 2 [Yamazaki, IEDM 1990, p. 309]:

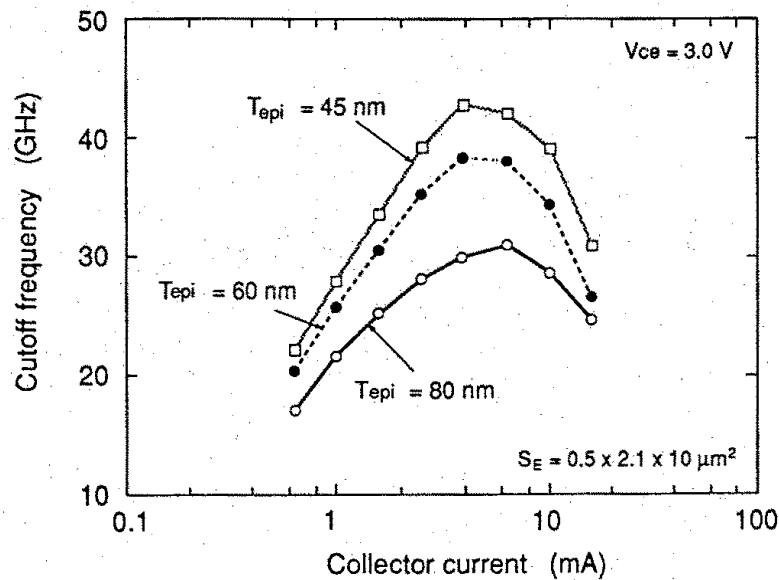


Fig. 5 Cutoff frequency as a function of collector current.

Yamazaki, T., et al. "A 11.7 GHz 1/8-divider Using 43 GHz Si High Speed Bipolar Transistor with Photoepitaxially Grown Ultra-thin Base." *Technical Digest of the International Electron Devices Meeting, San Francisco, CA, December 9-12, 1990*. New York, NY: Institute of Electrical and Electronics Engineers, 1990, pp. 309-312. Copyright 1990 IEEE. Used with permission.

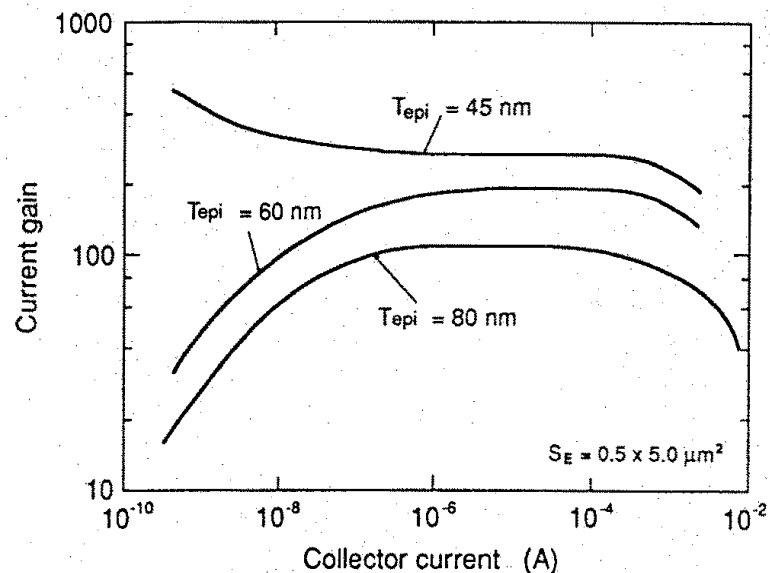


Fig. 4 Current gain versus collector current characteristics.

Yamazaki, T., et al. "A 11.7 GHz 1/8-divider Using 43 GHz Si High Speed Bipolar Transistor with Photoepitaxially Grown Ultra-thin Base." *Technical Digest of the International Electron Devices Meeting, San Francisco, CA, December 9-12, 1990*. New York, NY: Institute of Electrical and Electronics Engineers, 1990, pp. 309-312. Copyright 1990 IEEE. Used with permission.

Example 3 [Crabbé, IEDM 1990, p. 17]:

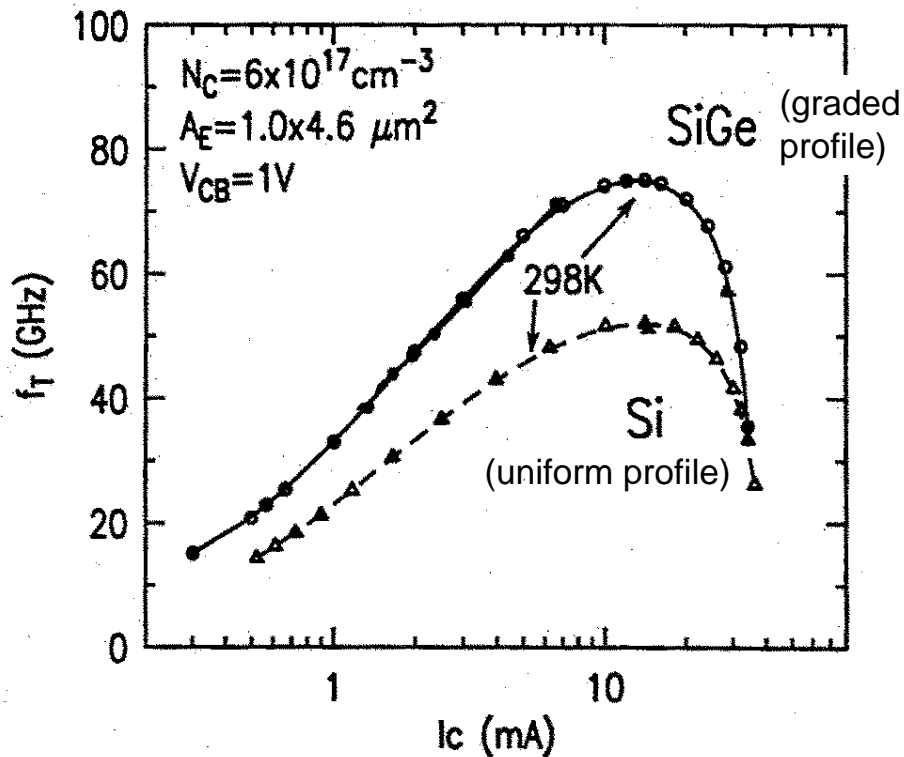


Fig. 10. Collector current dependence of  $f_T$  at 298K and 85K for Si and SiGe devices. In both cases, the peak  $f_T$  increases at lower temperature as well as the associated collector current.

Crabbe, E. F., et. al. "Low Temperature Operation of Si and SiGe Bipolar Transistors." *Technical Digest of the International Electron Devices Meeting, San Francisco, CA, December 9-12, 1990*. New York, NY: Institute of Electrical and Electronics Engineers, 1990, pp. 17-20. Copyright 1990 IEEE. Used with permission.



★ *E-B SCR charging time,  $C_{je}/g_m$ :*

$$\frac{C_{je}}{g_m} \propto \frac{A_E C_{jeo}}{I_C} = \frac{C_{jeo}}{J_C}$$

Minimized by:

- $N_B \downarrow$
- tailoring doping profiles at E-B junction

★ *B-C SCR charging time,  $C_{jc}/g_m$ :*

$$\frac{C_{jc}}{g_m} \propto \frac{A_C C_{jco}}{I_C} = \frac{A_C}{A_E} \frac{C_{jco}}{J_C}$$

Minimized by:

- $N_C \downarrow$
- tailoring doping profiles at B-C junction.
- tightening layout of transistor:  $\frac{A_C}{A_E} \rightarrow 1$

## Key conclusions

- $f_T$ : high-frequency figure of merit for transistors: frequency at which  $|h_{21}| = 1$ .
- $f_T$  of ideal BJT:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_{je} + C_{jc})}$$

- *Delay time*,  $\tau_d = \frac{1}{2\pi f_T}$ : time it takes for step increase in  $i_B$  to yield an identical step increase in  $i_C$ .
- Most effective ways to engineer  $f_T$ :
  - reduce  $W_B$
  - introduce drift field in base (through impurity gradient or SiGe composition gradient)
  - tighten layout:  $\frac{A_C}{A_E} \rightarrow 1$