# Massachusetts Institute of Technology
## Department of Electrical Engineering and Computer Science
### 6.691 Seminar in Advanced Electric Power Systems

Problem Set 6 Solutions April 24, 2004

The whole story is told in the attached MATLAB script, which is a straightforward adaptation of the script we wrote for Problem Set 2. Note we have adjusted power levels and added cost data. Note that, within the ground rules of this problem set, there is, very likely, an unambiguous 'optimum' solution. However, I won't pretend that what you see here is that optimum. What we have found is two *pretty good* solutions which meet the basic requirements.

Note that the incremental costs of the power plants are widely different, even after adding the penalty functions, so that loading is strictly in 'merit order'. The two large power plants, with lower incremental cost, are run at base load, with the smaller, higher incremental cost, plant running as needed.

I have made provision for capacitive reactive compensation at buses 10 and 13. If capacitors are used their capital cost must be added to that of the power plants (for cost based pricing).

Note that it was most convenient to use the *decoupled* Newton-Raphson method for load flow, as this yields one of the matrices needed for calculation of 'penalty functions' for the two large power plants.

Total cost is simply the evaluated capital cost plus incremental fuel cost, being careful to keep the units right ($/MWh).

Two solutions were tried out. First, I kept generation as seemed reasonable, with each of the two major plants generating 256 MW and 70 MVAR and holding the 'swing bus' at a voltage of 1.0 per unit. To maintain voltage at buses 10 and 13 I used some reactive compensation (6 MVAR at Bus 10, 12.5 MVAR at Bus 13). The result looks like this:

```
Abnormal Bus Voltages are:
Bus   Per-Unit
Real and Reactive Power at Plants
Plant    1  P =    82.66 MW  Q =    38.85 MVAR (   91.33 MVA)
Plant    2  P =   256.00 MW  Q =    70.00 MVAR (  265.40 MVA)
Plant    3  P =   256.00 MW  Q =    70.00 MVAR (  265.40 MVA)
Reactive Compensation at Bus 10 =       6.00 MVAR
Reactive Compensation at Bus 13 =      12.50 MVAR
Penalty Function Plant    2 =    1.01538
Penalty Function Plant    3 =    1.01064
System Lambda = $     25.00/MWh
Penalized at Plant 2 = $     15.23/MWh
Penalized at Plant 3 = $     15.16/MWh
Cost per Hour at Basis Point = $   9746.37
Power Delivered       =    585 MW
Average Fuel Cost     = $16.66/MWh
Plant Capital         = $   232000000
```

```
Reactive Comp Capital = $      277500
Total Capital         = $   424277500
Revenue Required/year = $    51466399
Captial Cost/MWh        = $     14.34
Generation Cost/MWh     = $     31.00
EDU>> V

V =

    1.0000
    1.0116
    1.0042
    0.9660
    0.9671
    0.9705
    0.9652
    0.9761
    0.9761
    0.9512
    0.9833
    0.9868
    0.9513
    1.0001
    0.9771
    0.9762
    0.9742
```

Note that in the foregoing output, right at the top, we find no buses with abnormal voltages.

The second solution to the problem was achieved by adjusting the swing bus voltage (so that it injects more reactive power into the system) and increasing reactive output from Generator 3, which is close to bus 13. Note the swing bus is close to Bus 10. This yielded a satisfactory answer as well:

```
That took          9 iterations
Abnormal Bus Voltages are:
Bus   Per-Unit
Real and Reactive Power at Plants
Plant    1  P =    82.34 MW  Q =    30.33 MVAR (   87.75 MVA)
Plant    2  P =   256.00 MW  Q =    70.00 MVAR (  265.40 MVA)
Plant    3  P =   256.00 MW  Q =    95.00 MVAR (  273.06 MVA)
Reactive Compensation at Bus 10 =         0 MVAR
Reactive Compensation at Bus 13 =         0 MVAR
Penalty Function Plant    2 =    1.04435
Penalty Function Plant    3 =    1.03901
System Lambda = $     25.00/MWh
Penalized at Plant 2 = $     15.67/MWh
```

```
Penalized at Plant 3 = $     15.59/MWh
Cost per Hour at Basis Point = $   9738.59
Power Delivered       =    585 MW
Average Fuel Cost     = $16.65/MWh
Plant Capital         = $   232000000
Reactive Comp Capital = $           0
Total Capital         = $   424000000
Revenue Required/year = $    51432737
Captial Cost/MWh        = $     14.33
Generation Cost/MWh     = $     30.98
EDU>> V

V =

    1.0300
    1.0467
    1.0476
    1.0014
    1.0045
    1.0100
    1.0028
    1.0142
    1.0067
    0.9559
    1.0185
    1.0254
    0.9501
    1.0331
    1.0146
    1.0083
    1.0001
```

In this case, because capacitors cost money and reactive power from the plants does not the second case appears to be slightly less expensive. In practice this might not be the case as we have completely discounted the cost of distribution and have considered only one generation condition (and no contingencies).

The MATLAB script that implements this is commented:

```
% Solution to Problem Set 6
% This is the same system as Problem Set 2
% with some economics built in
% Newton-Raphson Solution..Decoupled.

tol=.000000001;
% Inputs are plant behavior
V_1 = 1.0;                       % set point voltage at slack bus
P_2 = 2.56+j*0.7;                % plant at bus 2
P_3 = 2.56+j*0.7;                % plant at bus 3
Q_10 = 0.06;                        % add reactive power at bus 10
Q_13 = 0.125;                        % add reactive power at bus 13

Vk_1 = 40e6;                     % capital cost of plant 1 (dollars)
Vk_2 = 192e6;                    % of plant 2
Vk_3 = 192e6;                    % of plant 3
CI_1 = 25;                       % Incremental cost (dollars/MWh)
CI_2 = 15;
CI_3 = 15;

% global
r = .12;                         % interest rate
Lt = 40;                         % lifetime
lf = .7;                         % load factor for capital stuff
P_q = 15000;                     % cost of capacitors ($/MVAR)

% first, here are the line impedances
Z_l = [3.629+j*20.53 4.718+j*26.7 3.085+j*17.47 2.774+j*15.66 3.085+j*17.47...
       2.411+j*13.69 2.514+j*14.18 2.618+j*14.78...
       1.996+j*8.17 1.529+j*6.30 1.089+j*4.46 1.97+j*8.09 3.551+j*20.09...
       3.551+j*20.09 1.886+j*10.63 3.003+j*17.16 3.433+j*11.49 3.033+j*10.15...
       4.462+j*15.54 1.270+j*7.13];
% and here are the connections for each line
C_l = [1 14;
       1 11;
       14 2;
       11 2;
       1 9;
       9 4;
       11 5;
       2 12;
       5 8;
       5 4;
       5 7;
```

```
       5 6;
       12 3;
       6 3;
       7 15;
       3 15;
       17 10;
       10 13;
       13 16;
       8 12;
       9 17;
       15 16];

Pb = 100e6;       % we are going to use this base power
Vb1 = 161e3;      % and these base voltages
Vb2 = 69e3;

Zb1 = Vb1^2/Pb;
Zb2 = Vb2^2/Pb;
Ib1 = Pb/(sqrt(3)*Vb1);
Ib2 = Pb/(sqrt(3)*Vb2);

z_l = zeros(size(Z_l));
% unfortunately we have to cobble together the impedance vector
z_l(1:16) = Z_l(1:16) ./ Zb1;
z_l(20) = Z_l(20) / Zb1;
z_l(17:19) = Z_l(17:19) ./ Zb2;

% the last two lines are transformers
z_line = [z_l j*.08/1.5 j*.08/1.5];

%fprintf('Line Impedances \n')
%fprintf('Buses           Ohms                   Per-Unit\n')
%for i = 1:length(Z_l)
%fprintf('%3.0f %3.0f %10.3f + j %10.3f   %10.4f + j %10.4f\n',...
%         C_l(i, 1), C_l(i, 2), real(Z_l(i)), imag(Z_l(i)),...
%         real(z_l(i)), imag(z_l(i)))
%end

Nl = length(z_line);       % number of lines
Nb = 17;                   % number of buses
% Now construct the node-incidence matrix:
NI = zeros(Nb, Nl);        % to start: now we fill it in
for i = 1:Nl
  NI(C_l(i, 1), i) = 1;
  NI(C_l(i, 2), i) = -1;
```

```
end

y_line = zeros(Nl);
% now the line admittance matrix is:
for i = 1:Nl
y_line(i, i) = 1 / z_line(i);
end

% and the bus admittance matrix is:
y_bus = NI * y_line * NI';

% Here are the bus power flows:

S_bus = [2.2+j*.7;       % bus 1 is irrelevant as it turns out
         P_2;
         P_3;
         -.6-j*.1;
         -1-j*.3;
         -.8-j*.15;
         -.9-j*.2;
         -.4-j*.05;
         -.1-j*.05;
         -.15-j*.1+j*Q_10;
         -.75-j*.15;
         -.4-j*.15;
         -.3-j*.1+j*Q_13;
         -.35-j*.1;
         -.1;
         0; 0];

G = real(y_bus);
B = imag(y_bus);

P_r = real(S_bus);
Q_r = imag(S_bus);

% now here is an initial guess about voltages:
V = [ V_1; ones(Nb-1, 1)];
th = zeros(Nb, 1);

% now we go into a loop
n_iter = 0;
not_done = 1;
while (not_done == 1)
P = zeros(Nb, 1);
```

```
Q = zeros(Nb, 1);
for i = 1:Nb,
    for k = 1:Nb,
        P(i) = P(i) + V(i)*V(k)*(G(i, k)*cos(th(i)-th(k))+B(i,k)*sin(th(i)-th(k)));
        Q(i) = Q(i) + V(i)*V(k)*(G(i, k)*sin(th(i)-th(k))-B(i,k)*cos(th(i)-th(k)));
    end
end

X = [th(2:Nb); V(2:Nb)];                     % to be found
J11 = zeros(Nb-1);                 % components of the Jacobian
J12 = zeros(Nb-1);
J21 = zeros(Nb-1);
J22 = zeros(Nb-1);

for i = 2:Nb
 for k = 2:Nb
  ii= i-1;
  kk = k-1;
  if k ~= i
   J11(ii, kk) = V(i)*V(k)*(G(i,k)*sin(th(i)-th(k))-B(i,k)*cos(th(i)-th(k)));
   J12(ii, kk) = V(i)*(G(i,k)*cos(th(i)-th(k))+ B(i,k)*sin(th(i)-th(k)));
   J21(ii, kk) = -V(i)*V(k)*(G(i,k)*cos(th(i)-th(k))+B(i,k)*sin(th(i)-th(k)));
   J22(ii, kk) = V(i)*(G(i,k)*sin(th(i)-th(k))-B(i,k)*cos(th(i)-th(k)));
  else
   J11(ii, ii) = -V(i)^2*B(i,i)-Q(i);
   J12(ii, ii) = P(i)/V(i)+V(i)*G(i,i);
   J21(ii, ii) = P(i)-V(i)^2*G(i,i);
   J22(ii, ii) = Q(i)/V(i)-V(i)*B(i,i);
%   pause
  end  % end of if k~= i
 end   % end of index k loop
end    % end of index i loop: Jacobian is constructed

J = [J11 zeros(Nb-1);zeros(Nb-1) J22];

% now find real and reactive power
PE = P-P_r;
QE = Q-Q_r;
E = [PE(2:Nb);QE(2:Nb)];
SE = sum(E .^2);            % this is absolute, per-unit error
if SE < tol
  not_done = 0;
end

X = X - inv(J)*E;
```

```
th =[0;X(1:Nb-1)];
V = [V_1; X(Nb:2*Nb-2)];

n_iter = n_iter + 1;

%fprintf('Error = %g\n', SE);
%pause
end

fprintf('That took %10.0f iterations\n', n_iter)

% now generate per-unit line flows

v_bus = V .* exp(j .* th);
v_line = NI' * v_bus;
i_line = y_line * v_line;

I_line = zeros(size(i_line));
I_line(1:16) = Ib1 .* i_line(1:16);
I_line(17:19) = Ib2 .* i_line(17:19);
I_line(20:22) = Ib1 .* i_line(20:22);

% now generate a report:

fprintf('Abnormal Bus Voltages are:\n')
fprintf('Bus   Per-Unit\n');
for i = 1:Nb
    if (V(i) > 1.05) | (V(i) < 0.95)
  fprintf('%3.0f  %3.4f\n',i, V(i))
    end
end

fprintf('Real and Reactive Power at Plants\n')
for i = 1:3
    fprintf('Plant %4.0f  P = %8.2f MW  Q = %8.2f MVAR (%8.2f MVA)\n',...
        i, 100*P(i), 100*Q(i), 100*sqrt(P(i)^2+Q(i)^2))
end
fprintf('Reactive Compensation at Bus 10 = %10.2f MVAR\n',100*Q_10);
fprintf('Reactive Compensation at Bus 13 = %10.2f MVAR\n',100*Q_13);

%fprintf('Line Current Flows\n')
%fprintf('Line    Amperes\n')
%for i = 1:22
%  fprintf('%3.0f   %3.1f\n',i, abs(I_line(i)));
%end
```

```
% now we calculate the 'penalty factors'
% first is the vector of power dependence at bus 1 with angles:
% and we are calculating the negative of what is in the book
dp1dth = zeros(Nb-1,1);
for k = 2:Nb
    dp1dth(k-1) = V(k)*(G(1,k)*sin(th(k))+B(1,k)*cos(th(k)));
end
Jti = inv(J11');             % this is the vector of variations
PV = Jti*dp1dth;             % this is the result of the operation
for i = 1:2
    pf(i) = 1/PV(i);
    fprintf('Penalty Function Plant %4.0f = %10.5f\n',i+1,pf(i))
end

% System Lambda is incremental cost at Plant 1
% and Penalized Lambda is at the other two plants
fprintf('System Lambda = $%10.2f/MWh\n',CI_1)
fprintf('Penalized at Plant 2 = $%10.2f/MWh\n',pf(1)*CI_2);
fprintf('Penalized at Plant 3 = $%10.2f/MWh\n',pf(2)*CI_3);

% That is nice, but we need to compute a total cost
CG = 100*(P(1)*CI_1 + P(2)*CI_2 + P(3)*CI_3);
fprintf('Cost per Hour at Basis Point = $%10.2f\n',CG)
% Now we need a denominator, which is delivered power
PD = -100*sum(P(4:Nb));
Cf = CG/PD;
fprintf('Power Delivered       = %6.0f MW\n', PD)
fprintf('Average Fuel Cost     = $%4.2f/MWh\n', Cf);
% now get capital cost
Vk_c = P_q*100*(Q_10+Q_13);        % we spent this much for caps
R = r/(1-(r+1)^-Lt);               % this is annual cost factor
V_k = Vk_1+Vk_2+Vk_3+Vk_c;
CK = R*V_k;                        % gotta make this much per year
                                   % (this is 'revenue requirement')
% now assume that we deliver 70% of the base case power
E = PD*24*365.25*.7;               % units are MWH/year
Ck = CK/E;                         % this turns into revenue per MWH
C_t = Ck+Cf;                       % and this is generation cost
fprintf('Plant Capital         = $%12.0f\n',Vk_1+Vk_2);
fprintf('Reactive Comp Capital = $%12.0f\n',Vk_c);
fprintf('Total Capital         = $%12.0f\n',V_k);
fprintf('Revenue Required/year = $%12.0f\n',CK);
fprintf('Captial Cost/MWh        = $%10.2f\n',Ck);
fprintf('Generation Cost/MWh       = $%10.2f\n',C_t);
```

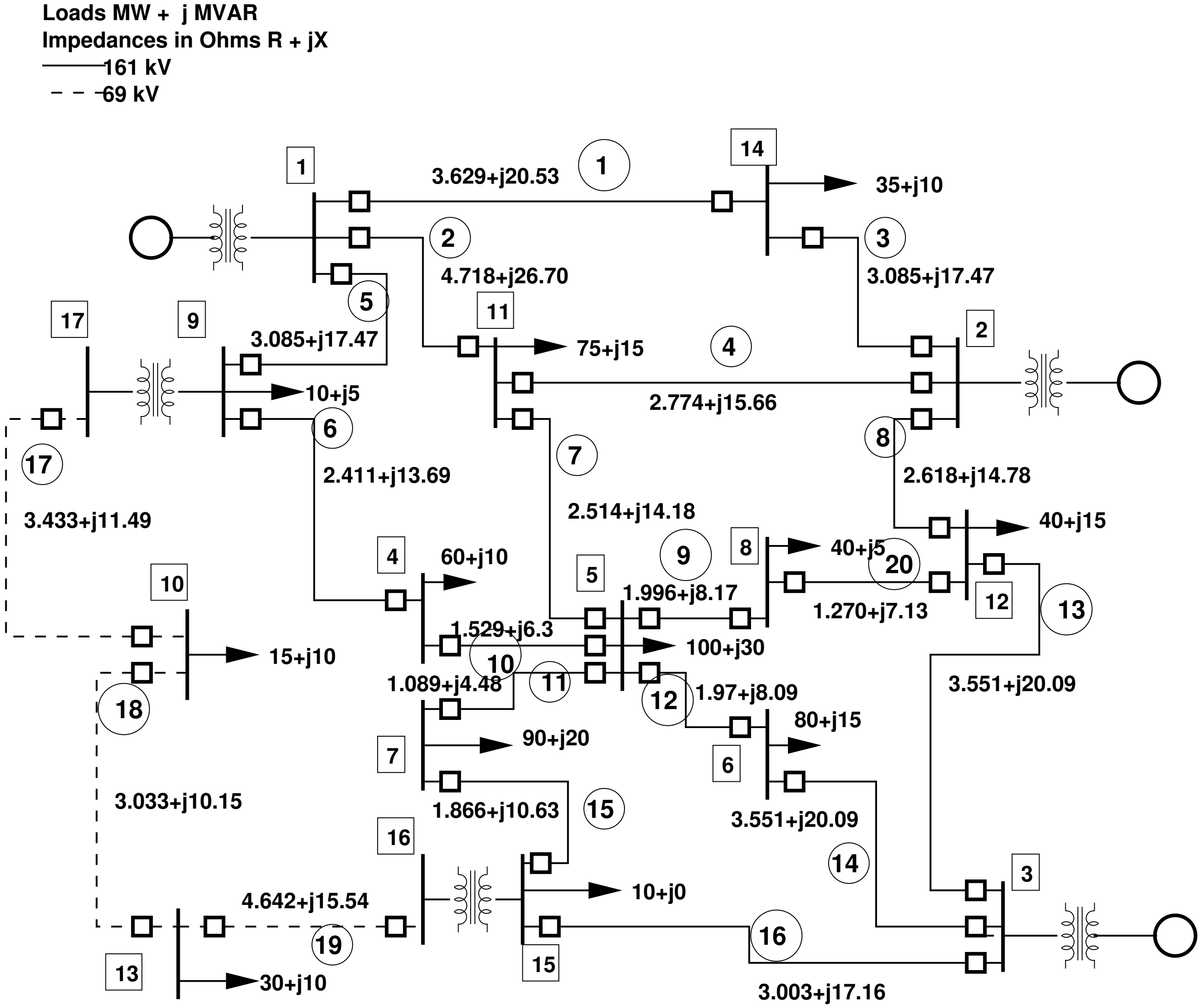

Figure 1: Example Power System