MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

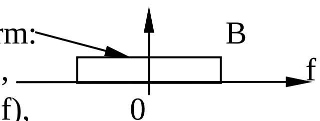
Solutions -- Problem Set No. 3 December 22, 2003

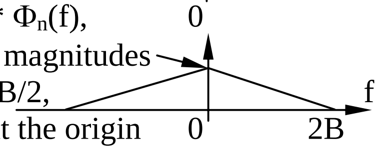
Problem 3.1

a) The solution follows the course notes. Part (a) is (2.2.33) where we compute $E[wxyz] = E[wx]\,E[yz] + E[wy]\,E[xz] + E[wz]\,E[xy]$, where w, x, y, and z correspond to $n_{a1} + S_1/2^{0.5}$, $n_{b1} + S_1/2^{0.5}$, $n_{a2} + S_2/2^{0.5}$, and $n_{b2} + S_2/2^{0.5}$, respectively.

b) The solution, (2.2.34), follows directly from (2.2.33) by noting the Fourier transform of multiplication is convolution (*).

c) The signal and noise power density spectra have the same form: where the magnitude is $kT_A/2$ and $kT_R/2$ for signal and noise, respectively. All three graphs for $\Phi_s(f) * \Phi_s(f)/2$, $\Phi_s(f) * \Phi_n(f)$, and $\Phi_n(f) * \Phi_n(f)$ have the same form, but with different magnitudes at the origin: they are $(kT_A)^2B/4$, $k^2T_AT_BB/2$, and $(kT_R)^2B/2$, respectively. In addition, the s x s term has an impulse at the origin of magnitude $(kT_sB)^2/4$.





d) The output power spectral density $\Phi_o(f) = |H(f)|^2\Phi_m(f)$, where the components of $\Phi_m(f)$ are sketched in part (c).
The output DC power is in the s x s impulse at f = 0, times $|\underline{H}(f=0)|^2$, where $|\underline{H}(f=0)|^2 = (\int h(t)dt)^2 = (A\tau)^2$ by Parseval's theorem. Thus

$$\boxed{P_{DC} = (kT_AB)^2(A\tau)^2/4}$$

The AC power is limited to a very narrow bandwidth (~$1/\tau$ Hz) determined by H(f), so only the power spectral densities near the origin are relevant here; these are given in part (c) above. To integrate over frequency to obtain the total AC power we sum the powers near zero frequency and multiply by $\int|\underline{H}(f)|^2df = \int h^2(t)dt = A^2\tau$ by Parseval's theorem. Thus

$$\boxed{P_{AC} = A^2\tau[(kT_A)^2B/4 + k^2T_AT_BB/2 + (kT_R)^2B/2]}$$

e) $\Delta T_{rms} = (P_{AC})^{0.5}/[\partial(P_{DC})^{0.5}/\partial T_A]$ where the AC and DC powers were found in (d).
$(P_{AC})^{0.5} = kT_{eff}A(B\tau)^{0.5}/2$ where $T_{eff}^2 = T_A^2 + 2T_AT_R + 2T_R^2$
$(P_{DC})^{0.5} = kT_ABA\tau/2$, so

$$\boxed{\Delta T_{rms} = T_{eff}/(B\tau)^{0.5}}$$

Problem 3.2

a) We wish to use only the spectrum below 10 MHz, so the 1000 spectral samples are each 10 kHz wide (nominally). If there were no aliasing (implying perfect boxcar filtering) and no apodization, then 1000 sample delays in the shift register are necessary to yield 1000 independent power spectral densities. However, the aliasing resulting from the given non-ideal 10-MHz filter requires us to increase the sampling frequency so the 1-MHz sidelobe region (see Figure in problem) does not fall inside the desired aliasing-free central 10-MHz region. If we set the sampling frequency = $\boxed{\text{21 MHz,}}$ then the result is as illustrated and as desired.

b) If we further apodize the observed autocorrelation function of the signal, then proportionally still more taps are required to preserve 1000 independent spectral intervals; the number of taps must therefore be further increased by a factor of 1.4 (given in the problem). Thus the total required number of taps, or stages in the shift register, is: 1000 x 1.05 x 1.4 = $\boxed{1470.}$

Problem 3.3

a) Assume the Thevenin source has voltage $\underline{V}_s$ in series with $R_s + jX_s$, and the load voltage and current are $\underline{V}$ and $\underline{I}$, respectively. The average power P dissipated in the load resistor R is:

$$P = 0.5\, R_e\{\underline{V}\underline{I}^*\}$$
$$= 0.5\, R_e\,\{[\underline{V}_s(R + jX)]/[(R + R_s) + j(X + X_s)] \times \underline{V}_s^*/[(R + R_s) - j(X + X_s)]\}$$
$$= 0.5|V_s|^2\, R_e\{(R + jX)/[(R + R_s)^2 + (X + X_s)^2]\}$$

$\partial P/\partial \underline{\bar{x}} = 0 \Rightarrow \boxed{X = -X_s}$ and

$\partial P/\partial R = 0 = 0.5|V_s|^2\, \partial[R/(R + R_s)^2]/\partial R \to \partial[R/(R + R_s)^2]/\partial R = 0 \Rightarrow$

$(R + R_s)^{-2} + R(-2)(R + R_s)^{-3} = 0 \Rightarrow$

$R + R_s - 2R = 0 \Rightarrow \boxed{R = R_s}$ Q.E.D.

b) $G_e = |\underline{\Gamma}|^2 = |(Z_{Ln} - 1)/(Z_{Ln} + 1)|^2 = \boxed{|(-R_L - R)/(-R_L + R)|^2}$ where $Z_{Ln} = -R_L/R$

Therefore $\boxed{G_e \to \infty \text{ as } R_L \to R,\ G_e \to 1 \text{ as } R_L \to 0, \infty, \text{ and } G \cong 100 \text{ for } R_L \cong 1.11R}$

The combination of a negative resistance amplifier and a 3-port circulator behaves like a normal amplifier with positive resistance.