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6.641 Electromagnetic Fields, Forces, and Motion, Spring 2005

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Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion
Quiz 1
March 9, 2005

You are allowed to use a one page (both sides) formula sheet that you have prepared for Quiz 1. In addition, the 6.641 Formula Sheet is attached in the study materials section.

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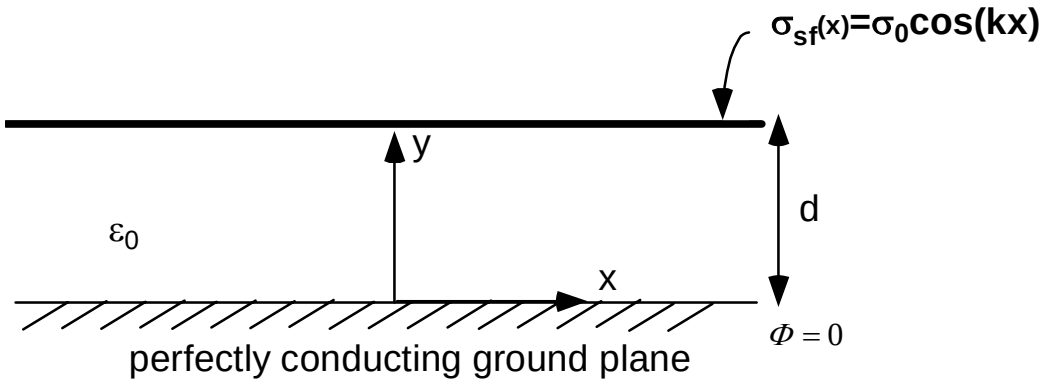
1. (30 points)

An isolated free surface charge distribution $\sigma_{sf}(x) = \sigma_0 \cos(kx)$ is located at y = 0 and has infinite extent in the x and z directions. The potential due to this surface charge distribution is given by:

$$\Phi(x, y) = \frac{\sigma_0 \cos(kx) e^{-k|y|}}{2\varepsilon_0 k}$$

Now the surface charge is moved to the plane $y = d$ and a zero potential ground plane ($\Phi = 0$) of infinite extent in the x and z directions and infinite conductivity fills the space for $y < 0$ as shown in the figure below.

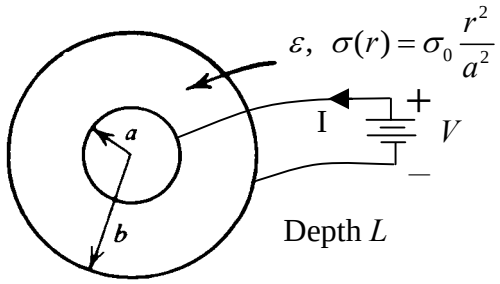


a) Find the potential $\Phi(x,y)$ in the region between the imposed surface charge distribution and the ground plane ($0 \le y \le d$).

b) Find the free surface charge density $\sigma_{sf}(x, y = 0)$ on the ground plane.

c) Find the force per unit area in the y direction, $f_y(x, y = d)$, on the imposed free surface charge distribution at $y = d$.

2. (35 points)



Concentric cylindrical electrodes with respective radii a and b and depth L enclose an Ohmic material whose constant permittivity is ε and whose conductivity σ varies quadratically with radial position as $\sigma(r)=\sigma_0\frac{r^2}{a^2}$. A DC voltage V is applied across the cylindrical electrodes which has been on for a long time so that all time transients have decayed and the system is in the DC steady state. Neglect fringing field effects.

a) Find the DC steady state electric field $E_r(r)$ between the electrodes.

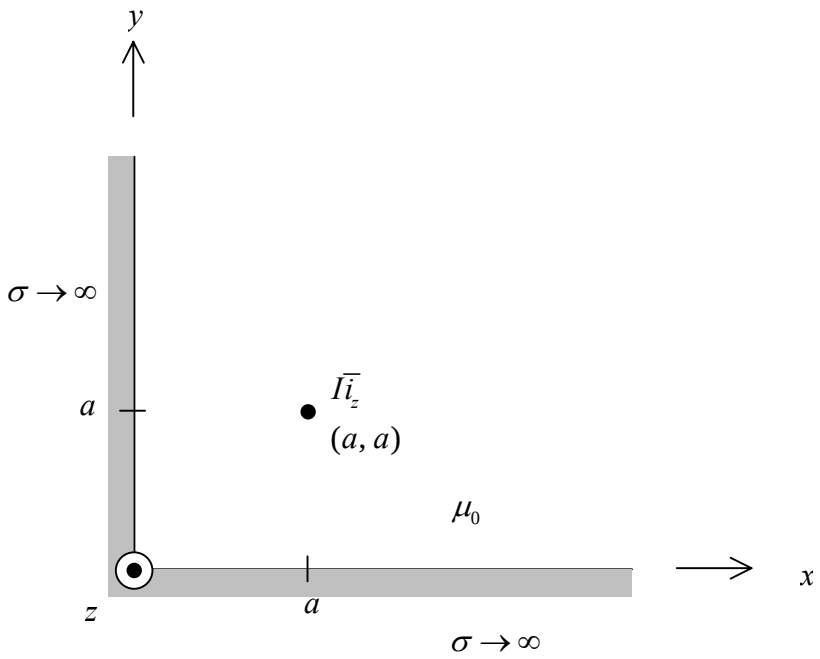
b) What are the DC steady state free volume charge density, $\rho_f(r)$, and the DC steady state free surface charge densities on the electrodes, $\sigma_{sf}(r=a)$ and $\sigma_{sf}(r=b)$?

c) What are the total DC steady state free volume charge in the dielectric and DC steady state total free surface charge on each electrode? What is the total DC steady state free charge in the system?

d) Find the DC steady state current I supplied by the DC voltage source.

e) What is the resistance R between the electrodes?

3. (35 points)



A line current $I\bar{i}_z$ is located in free space with magnetic permeability μ_0 at the coordinate ($x = a$, $y = a$) in the right angled corner formed by perfectly conducting planes at $x = 0$ and $y = 0$ for $x > 0$ and $y > 0$.

a) Find image currents that will satisfy the boundary conditions along the $x = 0$ and $y = 0$ surfaces for $x > 0$ and $y > 0$.

b) Find the vector potential magnitude and direction at any point in the *xy* plane for $x > 0$ and $y > 0$.

Hint: The vector potential for a single isolated z directed line current I of infinite extent and located in free space at $x = 0$, $y = 0$ with no conducting boundaries present is:

$$A_z[x, y] = -\frac{\mu_0 I}{4\pi} \ln[x^2 + y^2] + \text{constant}$$

c) Find the surface current density, $\bar{K}(x, y = 0)$, along the $y = 0$ surface for $x > 0$.