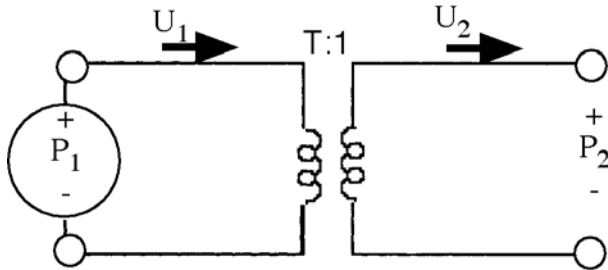## Lecture 11: Electro-Mechano-Acoustic Transformers

I. Ideal Transformers & Transducers

   1. <u>Ideal Transformers and Transformation of variables</u>

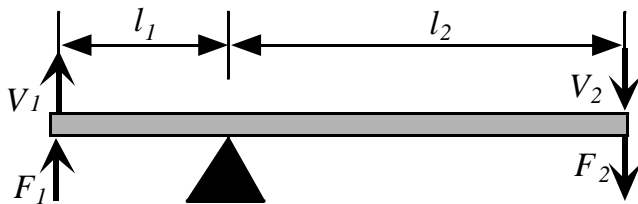*a. One example of a two-port is an Ideal Electrical transformer, where:*

$$\frac{U_2}{U_1} = T = \frac{P_1}{P_2}$$

where *T= "the turns ratio"*

and is dimensionless.
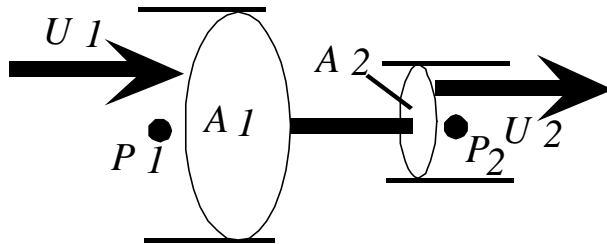
How does the transformer 'Transform' impedance?

b. An example of an ideal mechanical transformer is a massless rigid lever arm.

$$\frac{V_2}{V_1} = T = \frac{F_1}{F_2}$$

where *T=l_2/l_1*
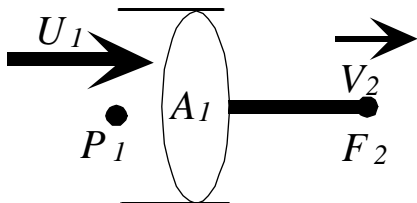*and is dimensionless.*

c. Ideal coupled pistons act as (dimensionless) acoustic transformers.

$$\frac{U_2}{U_1} = T = \frac{P_1}{P_2}$$

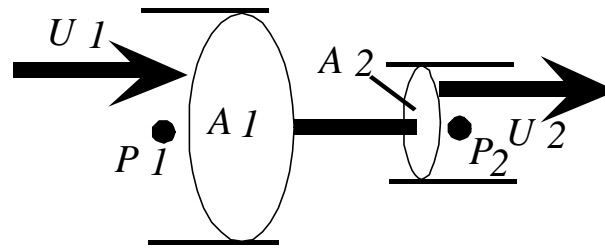where *T=A_2/A_1*
and is dimensionless.

d. An ideal piston and a coupled rod can act as an acoustico-mechanical transformer.

$$\frac{V_2}{U_1} = T = \frac{P_1}{F_2}$$

where *T=1/A_1* with units of 1/area

e. A first order model of the vertebrate middle ear is as an acoustic transformer that couples acoustic power from air to the fluid-filled inner ear.

$$\frac{U_2}{U_1} = T = \frac{P_1}{P_2}$$

A cross-section through half a lizard head	In the lizards the tympanic membrane is about 20 times larger in area than that of the boney footplate of the ossicle (the wide part that couples to one of the inner ear windows). This leads to a transformer ratio of 1/20 that would suggest.

A cross-section through half a lizard head

In the lizards the tympanic membrane is about 20 times larger in area than that of the boney footplate of the ossicle (the wide part that couples to one of the inner ear windows). This leads to a transformer ratio of 1/20 that would suggest.

$$\frac{U_2}{U_1} = \frac{\not{V}A_{FP}}{\not{V}A_{TM}} \approx \frac{1}{20}.$$

The Impedance transformation ratio is $T^2 = 1/400$:

$$\text{If} \quad \frac{P_2}{U_2} = 400; \quad \frac{P_1}{U_1} = 1$$

The acoustic transformer trades a change in volume velocity for a change in pressure of opposite magnitude. As we will see in the next two lectures, the ideal transformer is an idealization in the real world it's difficult to build a massless, rigid piston that is perfectly mobile.

2. The microphone. An acoustic – mechanical – electric transducer

a. The capacitive microphone

The signal flow through a capacitive microphone can be separated into three separate stages:
- An Acoustic stage that generates the acoustic input signal:
- A Mechanical stage that gathers the force produced by the acoustic signal
- An electric system that transforms mechanical force and motion into voltage and current
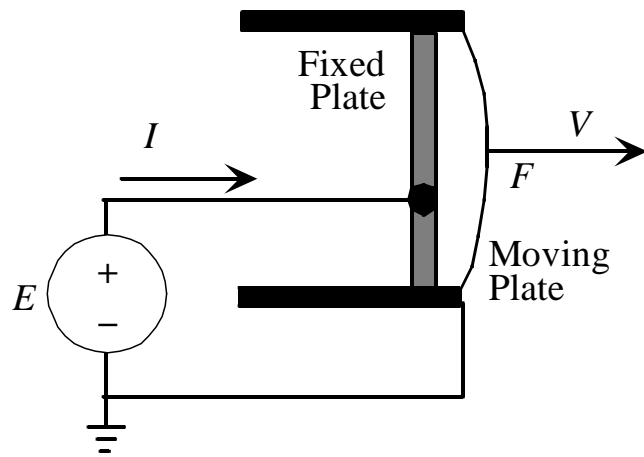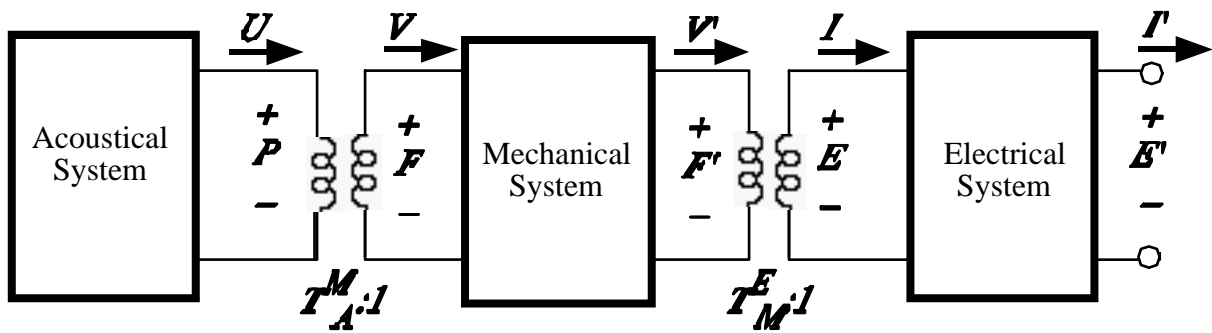
The heart of an *electro-static* transducer is a charged capacitor with a fixed internal plate and a moving external plate where the capacitance depends on the distance between the plates and the distance is a function of the voltage across the plates. Such a transducer acts as an *electro*mechanical transformer *such that -*

$$\frac{V}{I} = T_{ES} = \frac{E}{F}$$

where $T_{ES} = \dfrac{x_0(\text{the static separation of the plates})}{C_0(\text{static capaci}\tan ce)E_0(\text{static voltage})}$

when the voltage induced variations in *x, C and F* are small.   Input voltages produce a force on and velocity of the moving plate (or diaphragm) that when integrated over the surface of the moving plate produce a volume velocity and a sound pressure.

b. A Low-Frequency Model of the Electrostatic Microphone:

In the electrostatic microphone, the microphone acts simply as long as the capacitance of the diaphragm and backplate C_E controls the electrical stage and the compliance of the diaphragm C_M controls the mechanical stage.
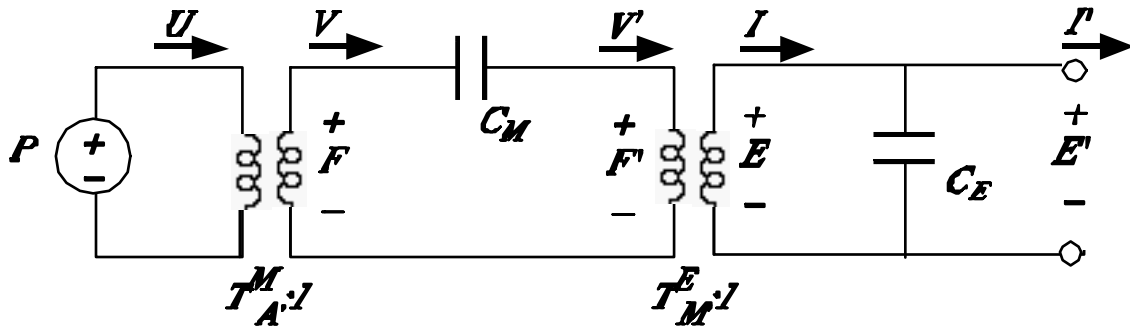
In our circuit, C_M is placed in series with the acoustic source, since the force and motion is gathered by motion of the diaphragm.

C_E is placed in parallel because it represents the static electrical capacitance which is their even when the diaphragm is fixed and V and $I = 0$.

Expensive 'capacitive' microphones with small (high-impedance) capacitance and stiff (low compliance) membranes are used as "STANDARD MICROPHONES" throughout the world.

What is the Thevenin impedance of the microphone measured at its output electrical port?   With the sound source turned off:

$$\underline{Z}_E^{TH}(\omega) = \frac{E'}{-I'} = \frac{1}{j\omega C_E} \text{ in parallel with } \frac{1}{j\omega C_M}\left(T_M^E\right)^2$$

$$= \frac{1}{j\omega\left(C_E + \dfrac{C_M}{\left(T_M^E\right)^2}\right)}$$

What is the Thevenin voltage equivalent to a sound pressure of P?

$$E^{TH}(\omega) = \frac{P}{T_A^M T_M^E} \; \frac{1/(j\omega C_E)}{1/(j\omega C_E) + \left(T_M^E\right)^2 / (j\omega C_M)}$$

$$= \frac{P}{T_A^M T_M^E} \; \frac{1/C_E}{1/C_E + \left(T_M^E\right)^2 / C_M}$$

3. Reciprocity Revisited

We have already given you a formal definition of reciprocal networks in terms of constraints placed on two-port network parameters, e.g. in the case of a two-port electrical network coupled to two voltage sources:

$$I_1 = Y_{11}E_1 + Y_{12}E_2$$
$$I_2 = Y_{21}E_1 + Y_{22}E_2$$

When E_1 is set to 0: $I_1 = Y_{12}E_2$; while when E_2 is set to 0: $I_2 = Y_{21}E_1$. If the system is made up of passive RLCs and transformers, then the system is reciprocal and $Y_{12} = Y_{21}$. This means you can couple up a known voltage source to one side of the network and measure the same current on the other side of the network, no matter which side you couple to the source.

Given the 'Admittance description of the two-port on the previous page:
$$I_1 = Y_{11}E_1 + Y_{12}E_2$$
$$I_2 = Y_{21}E_1 + Y_{22}E_2$$

Eqn 11.1

Let's fix I_1 with a current source while setting $I_2 = 0$, i.e. opening the circuit at port 2. The relationship between I_1 and E_2 can be defined from Eqns 11.1

Since $I_2 = 0$;
$$E_1\big|_{I_2=0} = -\frac{Y_{22}}{Y_{21}}E_2 \qquad (11.2)$$

Substituting into 11.1A yields
$$I_1\big|_{I_2=0} = E_2\left(Y_{12} - \frac{Y_{11}Y_{22}}{Y_{21}}\right). \qquad (11.3)$$

Now lets fix I_2 with a current source while setting $I_1 = 0$, i.e. opening the circuit at port 1. The relationship between I_2 and E_1 can be defined from Eqns 11.1 as

$$I_2\big|_{I_1=0} = E_1\left(Y_{21} - \frac{Y_{11}Y_{22}}{Y_{12}}\right). \tag{11.4}$$

If reciprocity holds: $Y_{21}=Y_{12}=Y_R$, and

$$\frac{E_2}{I_1}\bigg|_{I_2=0} = \frac{E_1}{I_2}\bigg|_{I_1=0} = \frac{1}{Y_R - \dfrac{Y_{11}Y_{22}}{Y_R}} \tag{11.5}$$

Note that this reciprocal relationship only works for specific combinations of sources and port measurements.

1. Acoustic Reciprocity

　　Acoustic Reciprocity, as strictly defined by Lord Rayleigh, says that in a given environment, if a spherical sound source of strength U is placed at position 1 (x_1,y_1,z_1) and the sound pressure at some position 2 (x_2,y_2,z_2) is P, then placement of the same sound source at position 2, will produce a sound pressure of P at position 1.

4. The Electro-Static Speaker
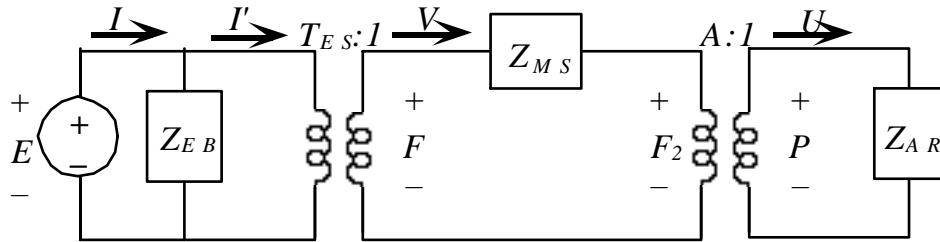
The diagram above is a schematic representation of all of the processes involved in the production of sound by the electrostatic speaker.

1. The right most segment of the circuit describes the relationship between the volume velocity U and sound pressure P produced by the source and the <u>acoustic radiation impedance</u> Z_{AR}.

$$Z_{AR} = \frac{P}{U}$$

2. The right most transformer describes the transformation of the force and the velocity produced by the moving plate of the transducer to volume-velocity and sound pressure and has units of area.

$$\frac{U}{V} = A = \frac{F_2}{P}$$

3. The middle mechanical-impedance describes the force necessary to move the outer plate with a given velocity.

$$Z_{MS} = \frac{F - F_2}{V}$$

4. The left hand transformer describes the transformation between voltage and force and current and velocity by the electro-static mechanism.

$$\frac{V}{I'} = T_{ES} = \frac{E}{F}$$

5. The left-most impedance block describes the electrical impedance when the outer plate is fixed and cannot move.

$$Z_{EB} = \frac{E}{I}\bigg|_{V=0}$$

Note that the total electrical impedance seen at the voltage source varies as the outer plate moves and depends on the impedances of the mechanical and acoustic components

$$Z_E = \frac{V}{I} = Z_{EB} // T^2 \left(Z_{MS} + A^2 Z_{AR} \right)$$

$$= \frac{Z_{EB} T^2 \left(Z_{MS} + A^2 Z_{AR} \right)}{Z_{EB} + T^2 \left(Z_{MS} + A^2 Z_{AR} \right)} \quad .$$

If we assume:

-the electrical impedance is primarily capacitive
-the mechanical impedance is primarily determined by the compliance of the diaphragm, and
-the acoustic impedance is a radiation impedance,

We can model the electrostatic speaker with this relatively simple circuit.
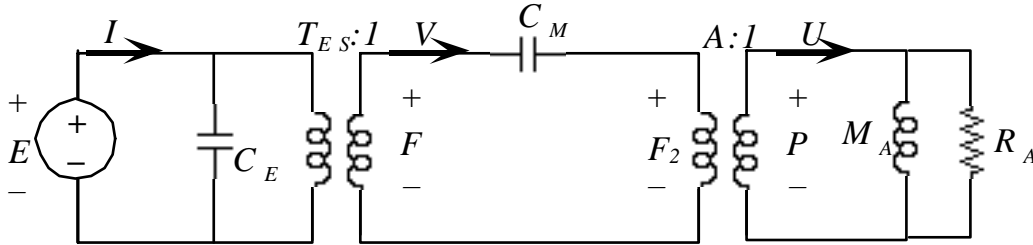
At low frequencies the Ratio of transfer ratio of *P/E* is determined by the compliance of the diaphragm C_M, and the acoustic mass. At high frequencies the pressure-to-voltage ratio depends on the area of the diaphragm *A*, and the electro-static transducer constant T_{ES}.

The Radiation Impedance Radiation

$$\frac{P}{U} = \frac{j\omega M_A R_A}{j\omega M_A + R_A}$$

The Mechanical Impedance with Radiation

$$\frac{F}{V} = \frac{1}{j\omega C_M} + A^2 \left(\frac{j\omega M_A R_A}{j\omega M_A + R_A} \right)$$

Sound Pressure / Force

$$\frac{P}{E} = \frac{1}{A} \left(\frac{A^2 \left(\dfrac{j\omega M_A R_A}{j\omega M_A + R_A} \right)}{\dfrac{1}{j\omega C_M} + A^2 \left(\dfrac{j\omega M_A R_A}{j\omega M_A + R_A} \right)} \right)$$

Sound Pressure / Volt

$$\frac{P}{E} = \frac{1}{T_{ES}} \frac{1}{A} \left(\frac{A^2 \left(\dfrac{j\omega M_A R_A}{j\omega M_A + R_A} \right)}{\dfrac{1}{j\omega C_M} + A^2 \left(\dfrac{j\omega M_A R_A}{j\omega M_A + R_A} \right)} \right)$$
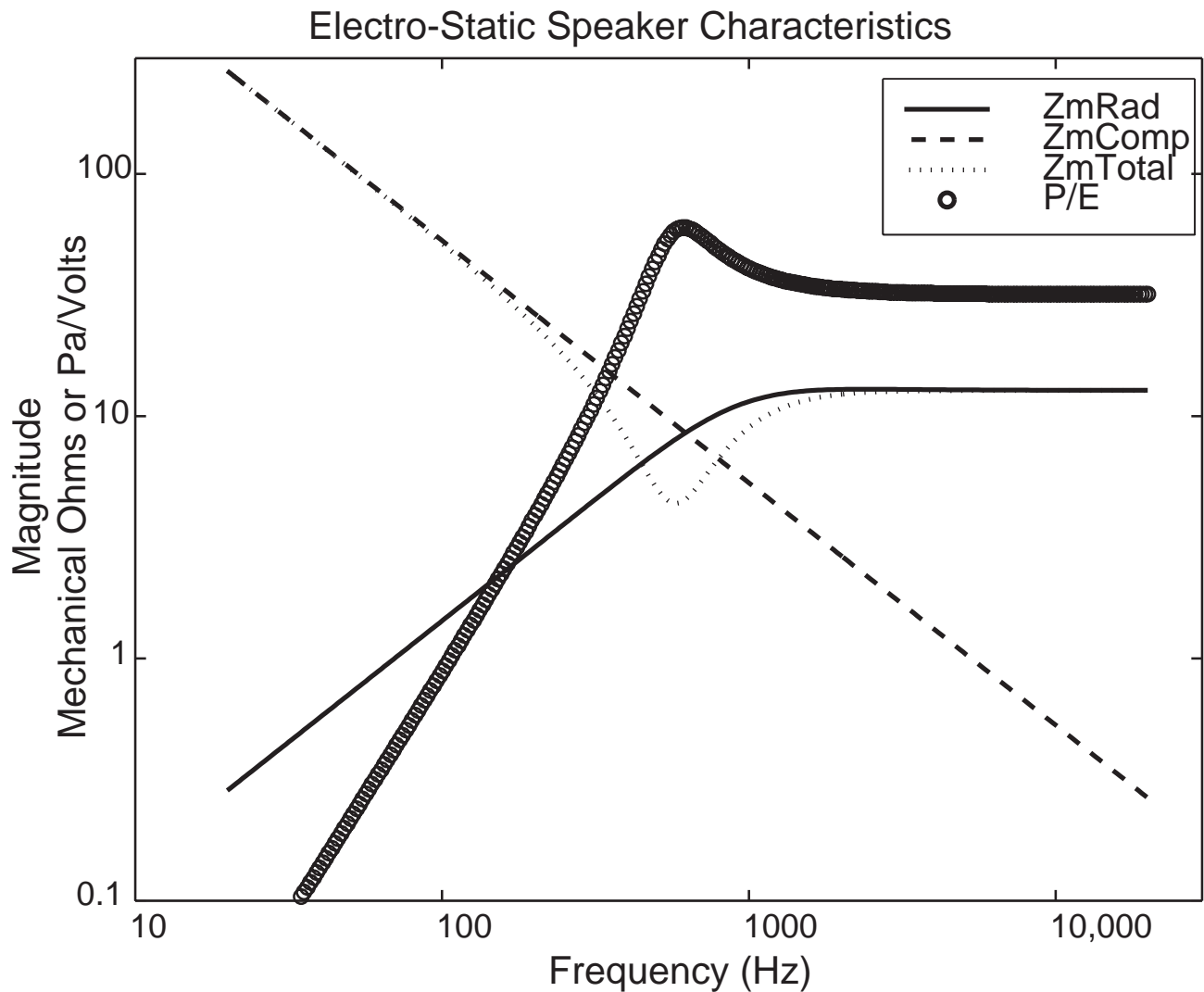
or $$\frac{P}{E} = \frac{1}{T_{ES}} \frac{1}{A} \left(\frac{-A^2 \omega^2 M_A R_A C_M}{-A^2 \omega^2 M_A R_A C_M + j\omega M_A + R_A} \right)$$

Sound Pressure / Volt (Approximates)

$$\left.\frac{P}{E}\right|_{small\ \omega} = \frac{1}{A\,T_{ES}}\left(-\omega^2 C_M\left(A^2 M_A\right)\right) \qquad\qquad \left.\frac{P}{E}\right|_{large\ \omega} = \frac{1}{A\,T_{ES}} \qquad .$$

Electro-Static Speaker Characteristics

Assumes:

$T_{ES}=1$
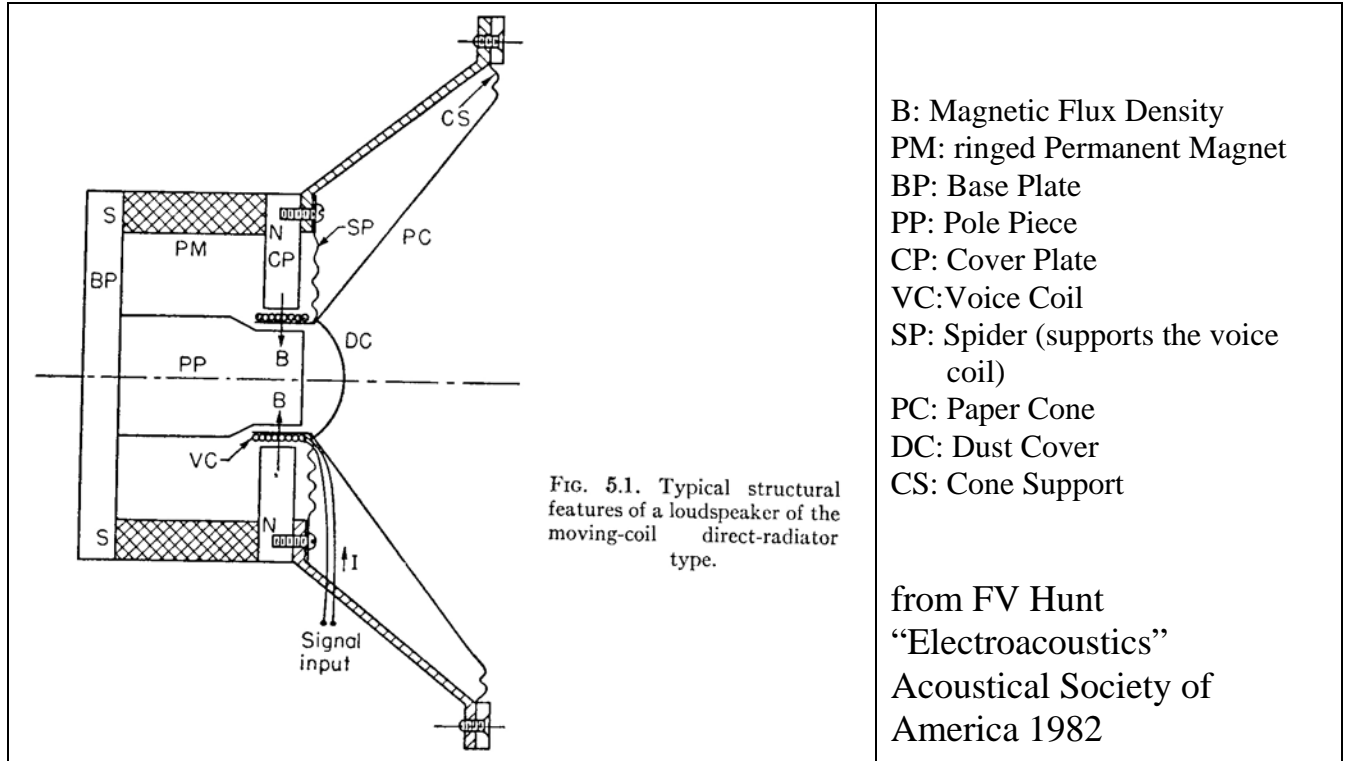
$A = \pi\,(0.1)^2\ \text{m}^2$

R_A and M_A defined by radius

$C_M = 3\times10^{-5}$ m/N

5. The Electro-Dynamic Speaker

The Electrodynamic speaker usually uses magnetic force to move a light-weight diaphragm.   The diagram below shows the magnet (PM), the current input to the 'voice coil' (VC) in the magnetic field, and the attachment of the paper cone PC to the coil.

FIG. 5.1. Typical structural features of a loudspeaker of the moving-coil   direct-radiator type.

B: Magnetic Flux Density
PM: ringed Permanent Magnet
BP: Base Plate
PP: Pole Piece
CP: Cover Plate
VC: Voice Coil
SP: Spider (supports the voice coil)
PC: Paper Cone
DC: Dust Cover
CS: Cone Support

from FV Hunt
"Electroacoustics"
Acoustical Society of America 1982

Courtesy of Acoustical Society of America. Used with permission.

In generating an electrical analog of electro-acoustic transduction in the electro-dynamic speaker, it is still convenient to break the system into electrical; mechanical and acoustical sections that are connected by 'ideal' transducers modeled by transformers with dimensions. Something new in the model below is that alternating current in a magnetic field produces an electromotive Force. This proportionality between current and force leads to the use of the electrical-mechanical admittance analogy where force and sound pressure are analogous to current, and velocity and volume velocity are analogous to voltage.
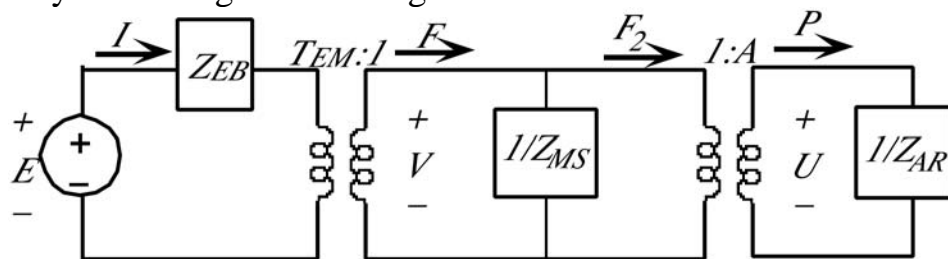
Because of this change in analogies, the acoustical and mechanical boxes are defined by impedance elements with values of 1 over the mechanical and acoustical impedances, and the electrical impedance Z_{EB} (measured when the speaker cone is fixed in place) is in series with the voltage source.

1. The Effect of Speaker Enclosures
     The box around a speakers serves to increase the output power of the sound by making the moving cone a monopole source. However, in doing so it places some acoustic-mechanical constraints on the motion of the diaphragm.
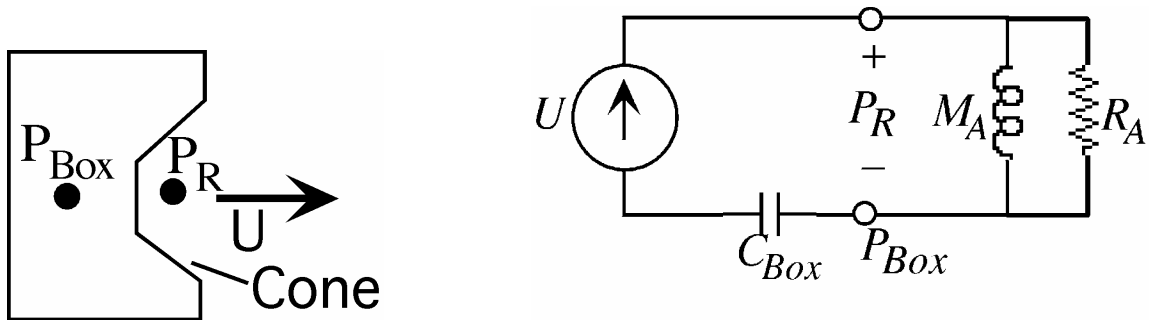
$$\frac{P}{U} = \frac{j\omega M_A R_A}{j\omega M_A + R_A} + \frac{1}{j\omega C_{Box}}$$

These constraints can be modified by use of openings in the speaker cabinet, as in bass-reflex speakers.