Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.551J / HST 712J
Problem Set 4

Issued: October 7, 2004 Due: October 14, 2004

Problem 1.

Assume that the volume velocity through an acoustic mass M_A is $u(t) = U \cos 2\pi f t$.

a. Determine an expression for the pressure $p(t)$ across the acoustic mass.

b. Sketch graphs of $u(t)$ and $p(t)$ over the time interval $0 \le t < 1/f$. Use the axes provided in Fig. 5. Provide scales for the axes. You may assume $f = 1$.

c. Determine an expression for the acoustic power $w(t)$ supplied to the acoustic mass in terms of M_A, U, and f.

d. What is the average power over one time period (e.g., $0 \le t < 1/f$) supplied to the acoustic mass?

e. Determine an expression for the energy $E_M(t)$ stored in the acoustic mass in terms of M_A, U, and f.

f. What is the average value of $E_M(t)$ over one time period (e.g., $0 \le t < 1/f$)?

g. Sketch graphs of $w(t)$ and $E_M(t)$ over one time period (e.g., $0 \le t < 1/f$). Use the axes provided in Fig. 5.

You may find the following trigonometric identities helpful:

$$
\begin{aligned}
\cos x \times \cos y &= \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y) \\
\sin x \times \cos y &= \frac{1}{2}\sin(x+y) + \frac{1}{2}\sin(x-y) \\
\sin x \times \sin y &= \frac{1}{2}\cos(x-y) - \frac{1}{2}\cos(x+y)
\end{aligned}
$$

Problem 2.

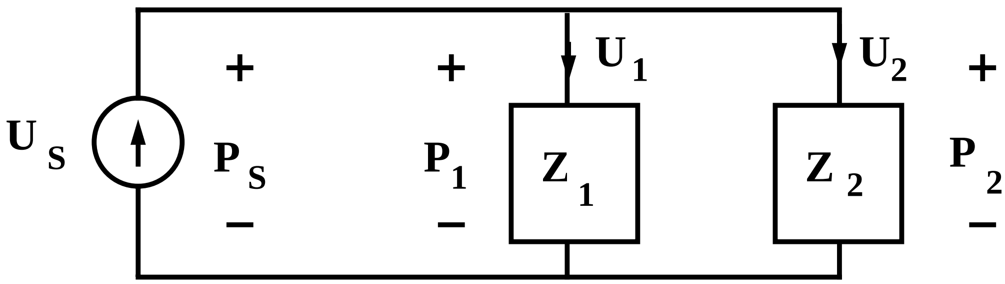

Figure 1: Example of an acoustic circuit consisting of a volume velocity source and two elements connected *in parallel*. All pressures and volume velocities are assumed to have the same e^{st} time dependence.

The acoustic circuit in Fig. 1 is an example of a *parallel* connection of acoustic elements.

a. Identify the nodes in this circuit. Write equations expressing Kirchoff's Volume Velocity Law at all nodes. Show that one of the equations can be derived from the others.

b. Write equations expressing Kirchoff's Pressure Law for the loops of the circuit. How many independent equations can be written.

c. Use the equations expressing Kirchoff's Pressure Law to specify a general principle for circuit elements connected in parallel.

d. Combine this result with the equations expressing Kirchoff's Volume Velocity Law to determine an expression for the impedance $Z = P/U_S$.

e. Generalize this result to the case in which more than two impedances are connected in parallel.

f. Determine expressions for the volume velocities U_1 and U_2 and for the volume velocity ratio U_1/U_2. What property of the circuit elements determines this ratio?

Problem 3.

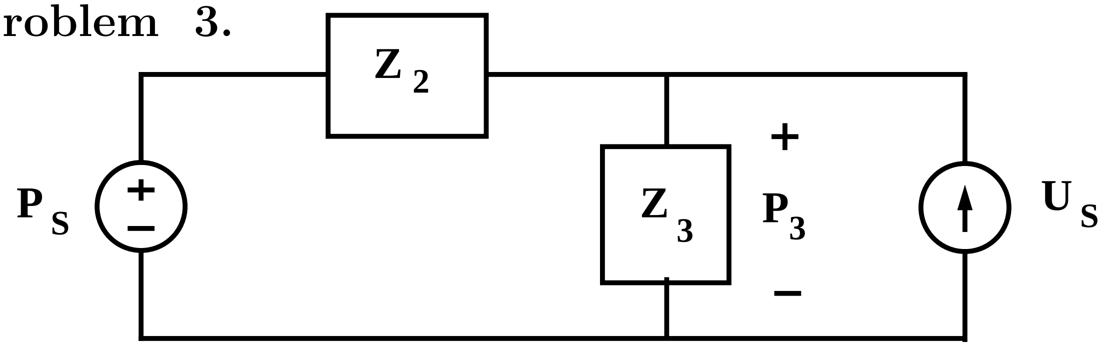

Figure 2: An acoustic circuit consisting of a pressure source, a volume velocity source and two impedances. All pressures and volume velocities are assumed to have the same e^{st} time dependence.

For the circuit of Fig. 2

a. Write equations for the pressure differences "across" and the volume velocities "through" each of the elements in Figure 2.

b. Write equations that express Kirchoff's pressure and volume velocity laws for the nodes and loops of this circuit.

c. Solve the equations formulated in parts (a) and (b) of this problem to determine an expression for P_3 in terms of U_S, P_S, Z_2 and Z_3.

d. Determine P_3 in terms of P_S, Z_2 and Z_3 when $U_S = 0$.

e. Determine P_3 in terms of U_S, Z_2 and Z_3 when $P_S = 0$.

f. Use of the Superposition Principle to determine P_3 in terms of U_S, P_S, Z_2 and Z_3. Make use of your results to parts (d) and (e) of this problem.

Problem 4.

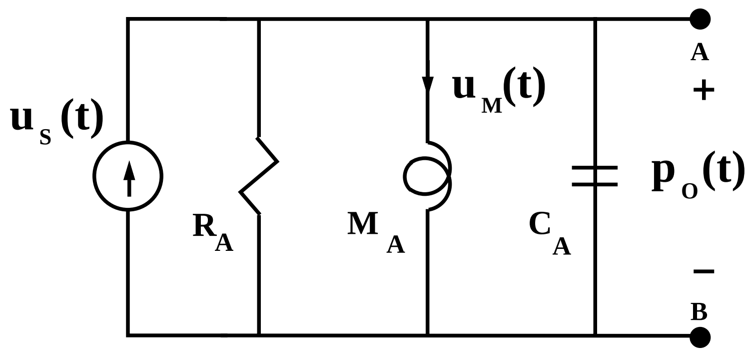

Figure 3: Example of an acoustic circuit consisting of a volume velocity source, an acoustic mass, an acoustic resistance, and an acoustic compliance connected in parallel. All pressures and volume velocities are assumed to have the same e^{st} time dependence, i.e. $u_S(t) = U_S e^{st}$, $u_M(t) = U_M e^{st}$, and $p_O(t) = P_O e^{st}$.

For the acoustic circuit of Fig. 3 the system function that relates the amplitude of the pressure P_O across the volume velocity source to the amplitude of the volume velocity source is $Z(s) = P_O/U_S$ and the system function that relates the amplitude of the volume velocity through the acoustic mass to the amplitude of the volume velocity source is $G(s) = U_M/U_S$. When $s = j\omega$

$$\mathbf{Z}(j\omega) = |\mathbf{Z}| e^{j\theta_Z} \quad (1)$$
$$\mathbf{G}(j\omega) = |\mathbf{G}| e^{j\theta_G} \quad (2)$$

a. Determine an expression (in terms of s, M_A, R_A, and C_A) for the impedance Z.

b. Determine an expression (in terms of s, M_A, R_A, and C_A) for the volume velocity transfer ratio G

c. Identify the poles and zeroes of Z and G. Do these two system functions have the same poles? How do the poles and zeroes of Z and G differ from the poles and zeroes of Y and H in Example 2 of the class notes?

d. Determine approximate expressions for the dependence of $Z(s)$ and $G(s)$ on s in the two limiting cases: i) $s \to 0$, and ii) $s \to \infty$.

e. Plot graphs of $|\mathbf{G}(j\omega)|$, $|\mathbf{Z}(j\omega)|\ \theta_G(j\omega)$, and $\theta_H(j\omega)|$, $|\mathbf{Z}(j\omega)|\ \theta_G(j\omega)$ as functions of ω on the axes provided in Fig. 6 and 7. As in the Class Notes, assume $R/\sqrt{M/C} = 0.1$.

Problem 5.

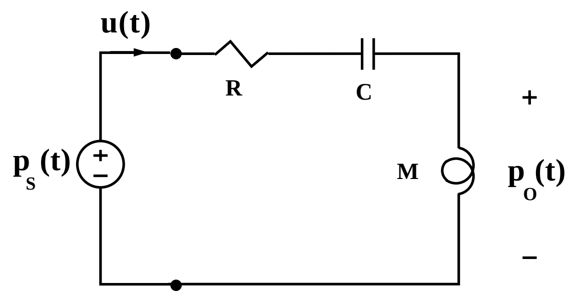

Figure 4: Example of an acoustic circuit consisting of a pressure source, an acoustic mass, an acoustic resistance, and an acoustic compliance, all connected in series. All pressures and volume velocities are assumed to have the same e^{st} time dependence.

Assume that all pressures and volume velocities in the circuit of Fig. 4 are sinusoidal with frequency $\omega = 1/\sqrt{MC}$.

a. Determine an expression for the vector power

$$\mathbf{W} = \frac{1}{2}\mathbf{P}\mathbf{U}_\mathbf{S}^*$$

supplied to the portion of the circuit to the right of the terminals in terms of U_S, R, M, and C.

b. At the frequency $\omega = 1/\sqrt{MC}$, what is the relation between the time average[1] energy stored in the acoustic mass and the time average energy stored in the acoustic compliance?

[1] Over one period of $p_S(t)$.

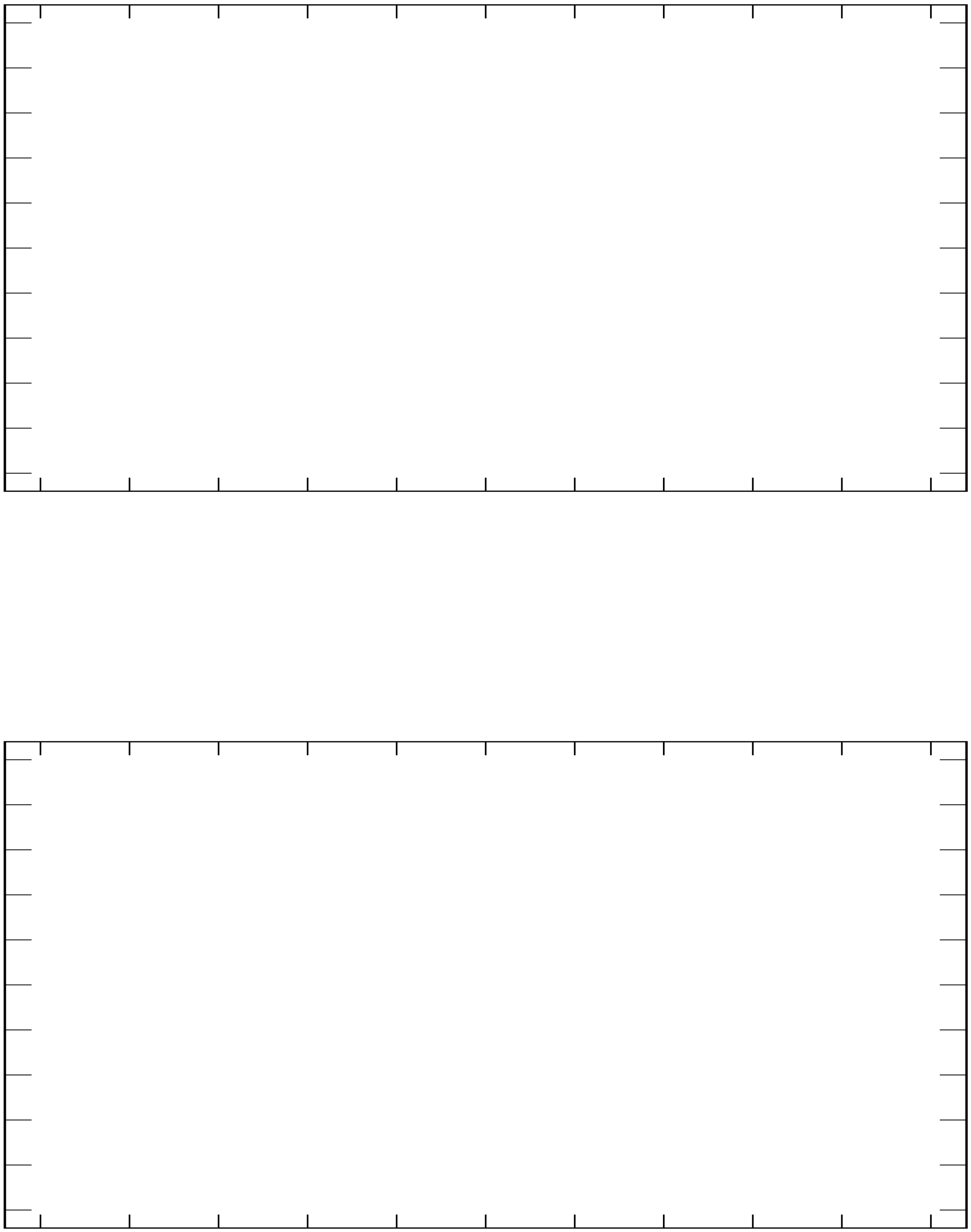

Figure 5: Graphs for Problem 1, parts b (upper panel), and g (lower panel).

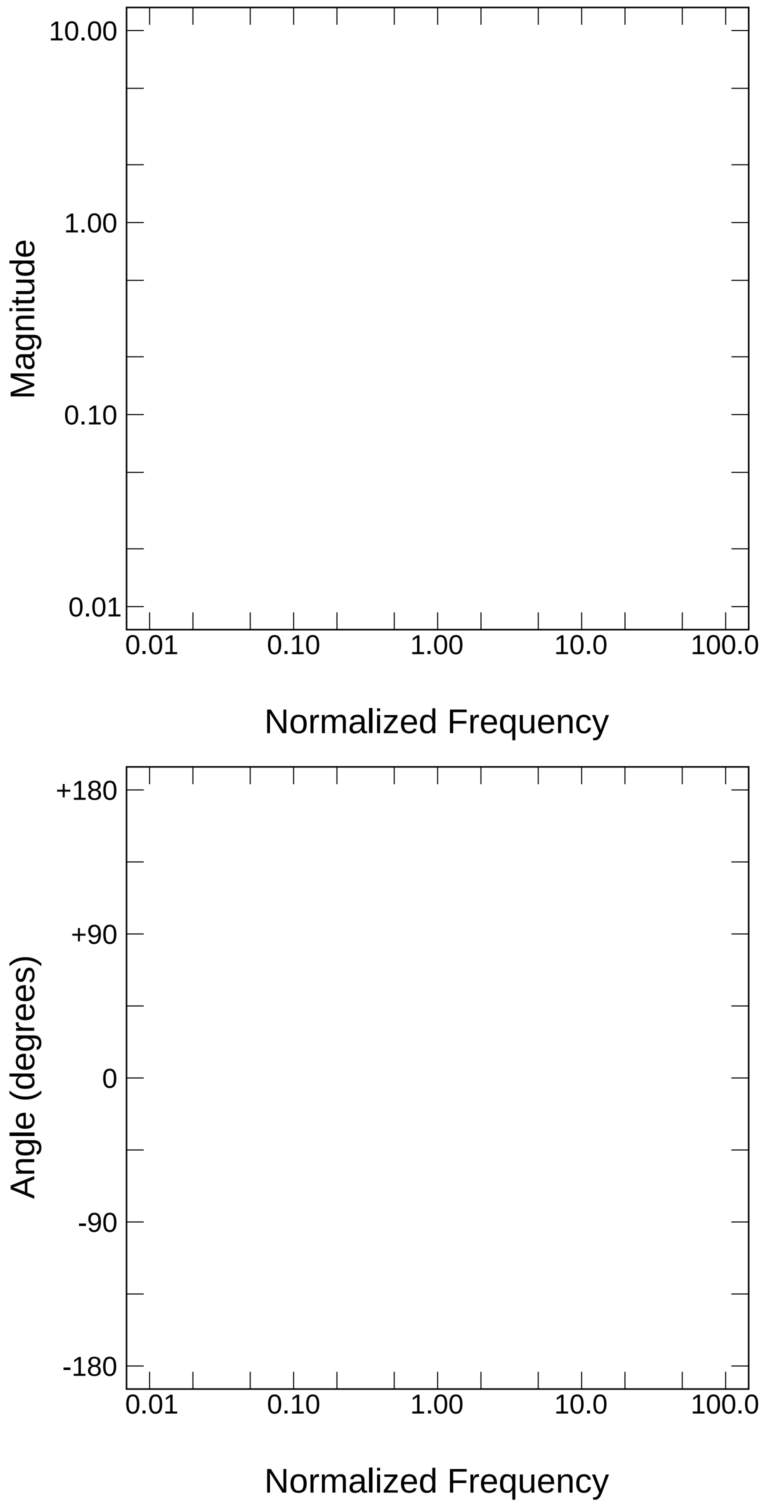

Figure 6: Graphs for Problem 4 part (e).

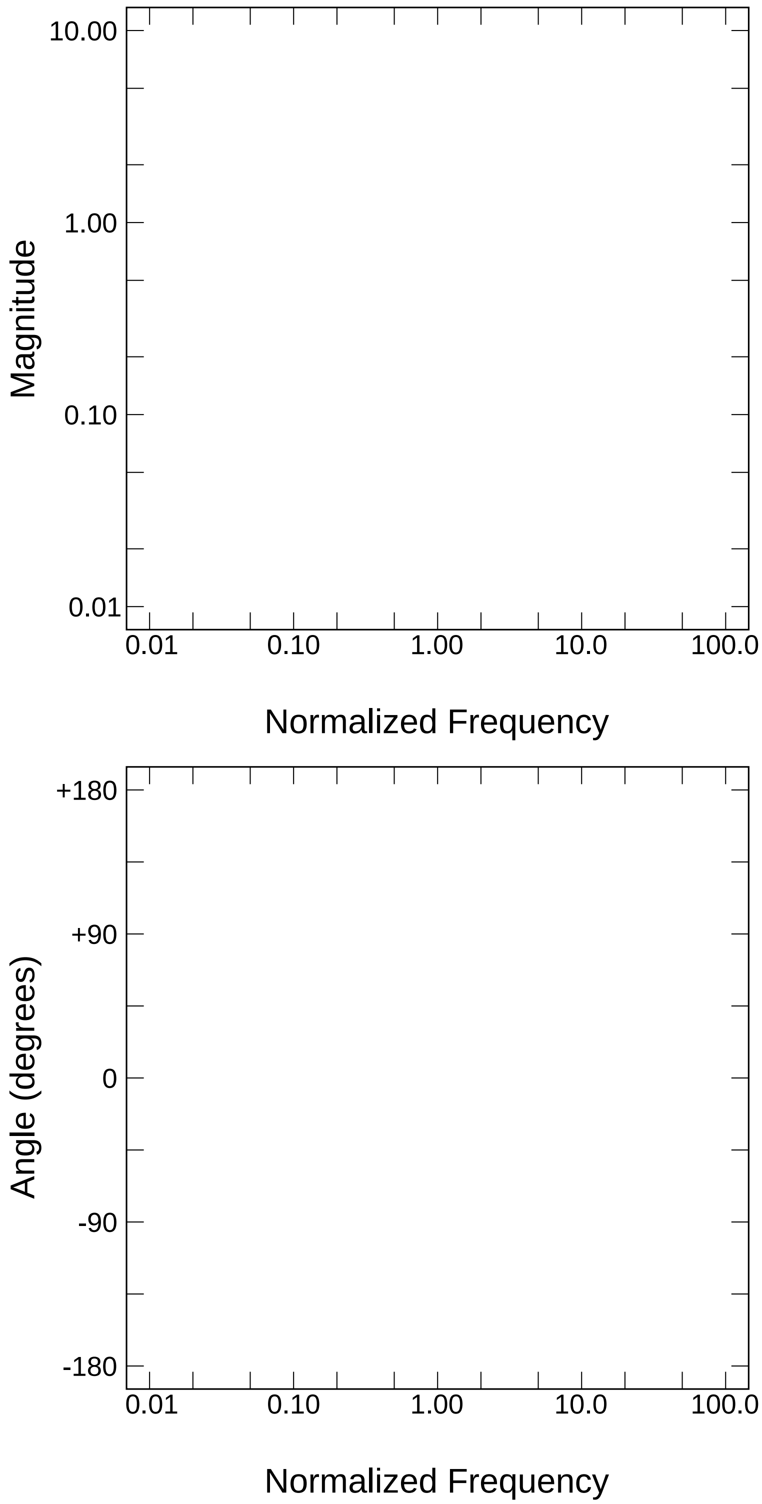

Figure 7: Graphs for Problem 4 part (e).