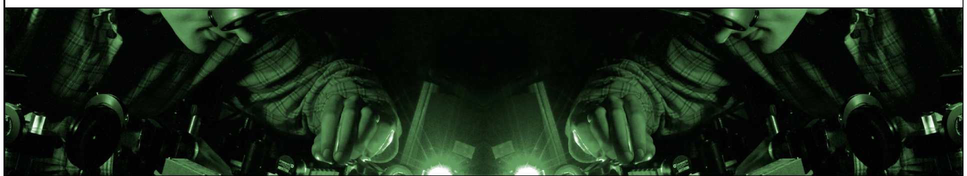


September 29, 2016

6.453 *Quantum Optical Communication* Lecture 7

Jeffrey H. Shapiro

Optical and Quantum Communications Group

RESEARCH LABORATORY OF ELECTRONICS
Massachusetts Institute of Technology

www.rle.mit.edu/qoptics

6.453 *Quantum Optical Communication* — Lecture 7

- Announcements
  - Turn in problem set 3
  - Pick up problem set 3 solutions, problem set 4, lecture notes, slides
- Quantum Harmonic Oscillator
  - Quadrature-measurement statistics and phase space
  - Characteristics functions and the Wigner distribution
  - Positive operator-valued measurement of $\hat{a}$

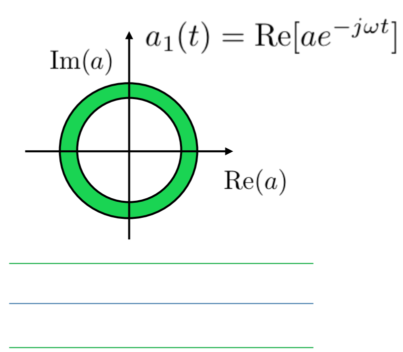
Quadrature-Measurement Statistics: Summary

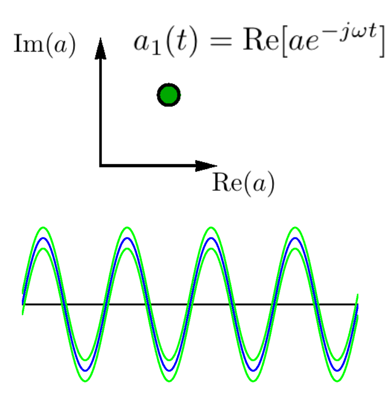
State	$\langle \hat{a}(t) \rangle$
$\vert n\rangle$	0
$\vert\alpha\rangle$	$\alpha e^{-j\omega t}$
$\vert\beta;\mu,\nu\rangle$	$(\mu^*\beta - \nu\beta^*)e^{-j\omega t}$

State	$\langle \Delta\hat{a}_1^2(t) \rangle$	$\langle \Delta\hat{a}_2^2(t) \rangle$
$\vert n\rangle$	$(2n+1)/4$	$(2n+1)/4$
$\vert\alpha\rangle$	$1/4$	$1/4$
$\vert\beta;\mu,\nu\rangle$	$\vert\mu - \nu e^{-2j\omega t}\vert^2/4$	$\vert\mu + \nu e^{-2j\omega t}\vert^2/4$

Quadrature-Measurement Statistics of $|n\rangle$ and $|\alpha\rangle$

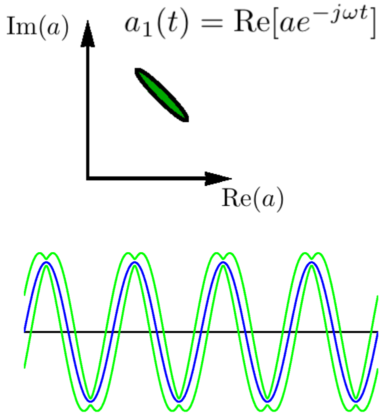
- Number State
- Coherent State



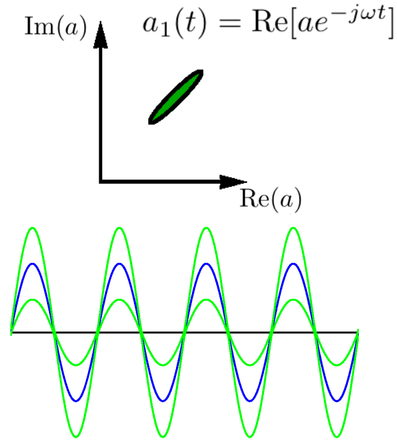


Quadrature-Measurement Statistics of $|\beta;\mu,\nu\rangle$

- Amplitude-Squeezed State



- Phase-Squeezed State



Classical Random Variable Review

- Probability Density Function: $p_x(X)$
- Characteristic Function: $M_x(jv)$
- Fourier Relationship

$$M_x(jv) \equiv \langle e^{jvx} \rangle = \int_{-\infty}^{\infty} \mathrm{d}X\, e^{jvX} p_x(X)$$

$$p_x(X) = \int_{-\infty}^{\infty} \frac{\mathrm{d}v}{2\pi}\, e^{-jvX} M_x(jv)$$

Quadrature-Measurement Statistics

- Characteristic Function for the Quadrature Measurement

$$M_{a_1(t)}(jv) \equiv \langle e^{jv\hat{a}_1(t)}\rangle = \langle e^{jv[\hat{a}_1\cos(\omega t))+\hat{a}_2\sin(\omega t)]}\rangle$$

$$\hat{a}(t) = \hat{a}e^{-j\omega t} \longrightarrow M_{a_1}(jv) = \chi_W(\zeta^*,\zeta)|_{\zeta=jv/2}$$

- Wigner Characteristic Function

$$\chi_W(\zeta^*,\zeta) \equiv \langle e^{-\zeta^*\hat{a}+\zeta\hat{a}^\dagger}\rangle = \langle e^{\zeta\hat{a}^\dagger}e^{-\zeta^*\hat{a}}\rangle e^{-|\zeta|^2/2}$$

Quadrature-Measurement Statistics of $|n\rangle$

- Wigner Characteristic Function of the Number State

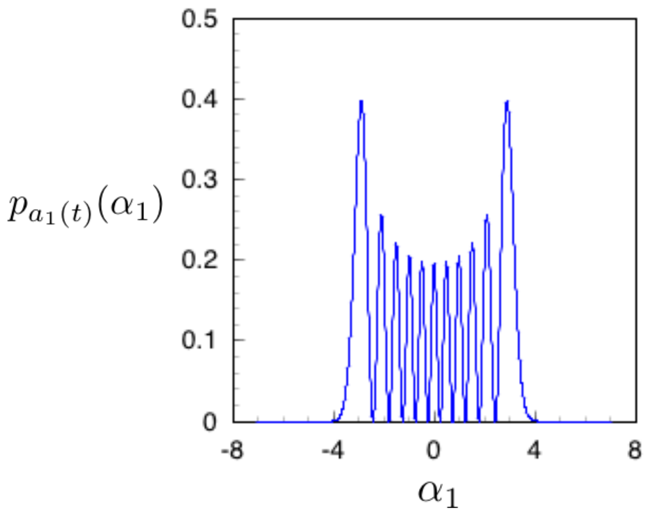
$$\chi_W(\zeta^*,\zeta) = \langle n|e^{\zeta\hat{a}^\dagger}e^{-\zeta^*\hat{a}}|n\rangle e^{-|\zeta|^2/2} = L_n(|\zeta|^2)e^{-|\zeta|^2/2}$$

- Quadrature-Measurement Probability Density Function

$$\begin{aligned} p_{a_1}(\alpha_1) &= \int_{-\infty}^{\infty}\frac{\mathrm{d}v}{2\pi}L_n(v^2/4)e^{-v^2/8}e^{-jv\alpha_1} \\ &= \sqrt{\frac{2}{\pi}}\frac{e^{-2\alpha_1^2}}{2^n n!}[H_n(\sqrt{2}\alpha_1)]^2 \end{aligned}$$

Quadrature-Measurement Statistics of $|n\rangle$

- Example: $|n\rangle, \quad \text{with } n = 10$



Wigner Distribution in (α_1, α_2) Space

- Inverse Transform of the Wigner Characteristic Function

$$W(\alpha, \alpha^*) \equiv \int \frac{\mathrm{d}^2\zeta}{\pi^2} \chi_W(\zeta^*, \zeta) e^{\zeta^*\alpha - \zeta\alpha^*}$$

  - Vacuum-State Wigner Distribution:

$$W(\alpha, \alpha^*) = \frac{2}{\pi} e^{-2|\alpha|^2} = \text{ 2-D Gaussian pdf}$$

  - One-Photon Wigner Distribution:

$$W(\alpha, \alpha^*) = \frac{2}{\pi} e^{-2|\alpha|^2}(4|\alpha|^2 - 1) \neq \text{ valid 2-D pdf}$$

Measuring the $\hat{a}$ Operator

- Definition: Measurement of the $\hat{a}$ Operator
  - yields an outcome that is a complex number $\alpha = \alpha_1 + j\alpha_2$
  - joint probability density for getting this outcome is

$$p(\alpha) = \frac{|\langle\alpha|\psi\rangle|^2}{\pi}$$

- Consistency Checks:

$$p(\alpha) \geq 0$$

$$\int \mathrm{d}^2\alpha\, p(\alpha) = \langle\psi|\left(\int \frac{\mathrm{d}^2\alpha}{\pi}\,|\alpha\rangle\langle\alpha|\right)|\psi\rangle = 1$$

Measuring the $\hat{a}$ Operator: Summary

State	$\langle\alpha\rangle$
$\lvert n\rangle$	0
$\lvert\beta\rangle$	β
$\lvert\beta;\mu,\nu\rangle$	$\mu^*\beta - \nu\beta^*$

State	$\langle\Delta\alpha_1^2\rangle$	$\langle\Delta\alpha_2^2\rangle$
$\lvert n\rangle$	$(n+1)/2$	$(n+1)/2$
$\lvert\beta\rangle$	$1/2$	$1/2$
$\lvert\beta;\mu,\nu\rangle$	$(\lvert\mu-\nu\rvert^2+1)/4$	$(\lvert\mu+\nu\rvert^2+1)/4$

Coming Attractions: Lectures 8 and 9

- Lecture 8:
  Quantum Harmonic Oscillator
  - Reconciling the $\hat{a}$ measurement with the notion of observables

  Single-Mode Photodetection
  - Direct Detection — semiclassical versus quantum
  - Homodyne Detection — semiclassical versus quantum

- Lecture 9:
  Single-Mode Photodetection
  - Heterodyne Detection — semiclassical versus quantum
  - Realizing the $\hat{a}$ measurement