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## 6.453 *Quantum Optical Communication* Lecture 2

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### 6.453 *Quantum Optical Communication* — Lecture 2

- Handouts
  - Lecture notes, slides
  - if you missed Lecture 1, see instructor after class
- Fundamentals of Dirac-Notation Quantum Mechanics
  - Quantum systems
  - States as ket vectors
  - State evolution via Schrödinger's equation
  - Quantum measurements — observables

## Quantum Systems and Quantum States

- **Definition 1:**

A quantum-mechanical system  $\mathcal{S}$  is a physical system governed by the laws of quantum mechanics.

- **Definition 2:**

The state of a quantum mechanical system at a particular time  $t$  is the sum total of all information that can be known about the system at time  $t$ . It is a ket vector  $|\psi(t)\rangle$  in an appropriate Hilbert space  $\mathcal{H}_{\mathcal{S}}$  of possible states. Finite energy states have unit length ket vectors, i.e.,  $\langle\psi(t)|\psi(t)\rangle = 1$ .

## Time Evolution via the Schrödinger Equation

- **Axiom 1:**

For  $t \geq 0$ , an isolated system with initial state  $|\psi(0)\rangle$  will reach state

$$|\psi(t)\rangle = \hat{U}(t, 0)|\psi(0)\rangle$$

where  $\hat{U}(t, 0)$  is the unitary time-evolution operator for the system  $\mathcal{S}$ .  $\hat{U}(t, 0)$  is obtained by solving

$$j\hbar \frac{d\hat{U}(t, 0)}{dt} = \hat{H}\hat{U}(t, 0), \quad \text{for } t \geq 0, \text{ with } \hat{U}(0, 0) = \hat{I}$$

where  $\hat{H}$  is the Hamiltonian (energy) operator for  $\mathcal{S}$ . Equivalently, we have the Schrödinger equation

$$j\hbar \frac{d|\psi(t)\rangle}{dt} = \hat{H}|\psi(t)\rangle, \quad \text{for } t \geq 0, \text{ with } |\psi(0)\rangle \text{ initial condition}$$

## Quantum Measurements: Observables

- *Axiom 2:*

An observable is a measurable dynamical variable of the quantum system  $\mathcal{S}$ . It is represented by an Hermitian operator which has a complete set of eigenkets.

- *Axiom 3:*

For a quantum system  $\mathcal{S}$  that is in state  $|\psi(t)\rangle$  at time  $t$ , measurement of the observable

$$\hat{O} \equiv \sum_n o_n |o_n\rangle\langle o_n|$$

yields an outcome that is one of the eigenvalues,  $\{o_n\}$ , with

$$\Pr(\text{outcome} = o_n) = |\langle o_n | \psi(t) \rangle|^2$$

## Quantum Measurements: Observables

- *Projection postulate:*

Immediately after a measurement of an observable  $\hat{O}$ , with distinct eigenvalues, yields outcome  $o_n$  the state of the system becomes  $|o_n\rangle$ .

- *Axiom 3a:*

For a quantum system  $\mathcal{S}$  that is in state  $|\psi(t)\rangle$  at time  $t$ , measurement of the observable

$$\hat{O} = \int_{-\infty}^{\infty} do o |o\rangle\langle o|$$

yields an outcome that is one of the eigenvalues,  $o$ , with

$$p(o) = |\langle o | \psi(t) \rangle|^2$$

## Coming Attractions: Lectures 3 and 4

- Lecture 3:  
Fundamentals of Dirac-Notation Quantum Mechanics
  - Quantum measurements — statistics
  - Schrödinger picture versus Heisenberg picture
  - Heisenberg uncertainty principle
  
- Lecture 4:  
Quantum Harmonic Oscillator
  - Quantization of a classical  $LC$  circuit
  - Annihilation and creation operators
  - Energy eigenstates — number-state kets

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