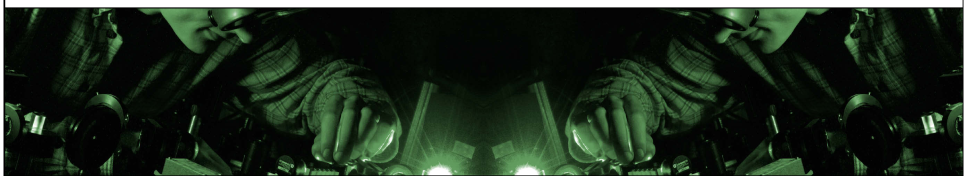


October 25, 2016

6.453 *Quantum Optical Communication*
Lecture 13

Jeffrey H. Shapiro

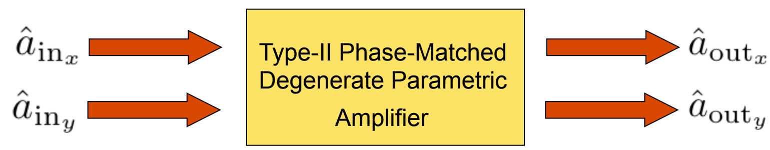
Optical and Quantum Communications Group

RESEARCH LABORATORY OF ELECTRONICS
Massachusetts Institute of Technology

6.453 *Quantum Optical Communication* - Lecture 13

- Announcements
  - Pick up lecture notes, slides
- Two-Mode Linear Systems
  - Parametric amplifier statistics
  - Entanglement
- Four-Mode Quantum Systems
  - Polarization entanglement
  - Qubit teleportation

Parametric Amplifier with Gain *G*



- Two-Mode Bogoliubov Transformation:

$$\begin{aligned}\hat{a}_{\text{out}_x} &= \sqrt{G}\,\hat{a}_{\text{in}_x} + \sqrt{G-1}\,\hat{a}^\dagger_{\text{in}_y}\\ \hat{a}_{\text{out}_y} &= \sqrt{G}\,\hat{a}_{\text{in}_y} + \sqrt{G-1}\,\hat{a}^\dagger_{\text{in}_x}\end{aligned}$$

Output State of the Parametric Amplifier

- Quantum Characteristic-Function Analysis:

$$\begin{aligned}\chi_W^{\rho_{\text{out}}}(\zeta_x^*, \zeta_y^*, \zeta_x, \zeta_y) &\equiv \langle e^{-\zeta_x^*\hat{a}_{\text{out}_x} - \zeta_y^*\hat{a}_{\text{out}_y} + \zeta_x\hat{a}^\dagger_{\text{out}_x} + \zeta_y\hat{a}^\dagger_{\text{out}_y}}\rangle\\ &= \chi_W^{\rho_{\text{in}}}(\xi_x^*, \xi_y^*, \xi_x, \xi_y)\end{aligned}$$

$$\xi_x \equiv \sqrt{G}\zeta_x - \sqrt{G-1}\zeta_y^* \text{ and } \xi_y \equiv \sqrt{G}\zeta_y - \sqrt{G-1}\zeta_x^*$$

- Important Special Case: Vacuum-State Inputs

$$\begin{aligned}\chi_A^{\rho_{\text{out}}}(\zeta_x^*, \zeta_y^*, \zeta_x, \zeta_y) &= e^{-G(|\zeta_x|^2 + |\zeta_y|^2) + 2\sqrt{G(G-1)}\text{Re}(\zeta_x\zeta_y)}\\ &\neq \chi_A^{\rho_{\text{out}_x}}(\zeta_x^*, \zeta_x)\chi_A^{\rho_{\text{out}_y}}(\zeta_y^*, \zeta_y)\end{aligned}$$

output state is *entangled*

Parametric Amplifier Output with Vacuum Inputs

- Individual Output Modes are in Classical States:

$$\chi_A^{\rho_{\text{out}_x}}(\zeta_x^*, \zeta_x) = \chi_A^{\rho_{\text{out}}}(\zeta_x^*, 0, \zeta_x, 0) = e^{-G|\zeta_x|^2}$$
$$= \langle 0|e^{-\zeta_x^* \hat{a}_{\text{out}_x}} e^{\zeta_x \hat{a}_{\text{out}_x}^\dagger}|0\rangle e^{-(G-1)|\zeta_x|^2}$$

$$P_{\text{out}_x}(\alpha, \alpha^*) = P_{\text{out}_y}(\alpha, \alpha^*) = \frac{e^{-|\alpha|^2/(G-1)}}{\pi(G-1)}$$

  - Each output is a phase-insensitive-amplified zero-input field

- Joint Output State is Non-Classical:
  - The $\pm 45^\circ$ (diagonal) basis modes are in squeezed vacuum states

Parametric Amplifier Output with Vacuum Inputs

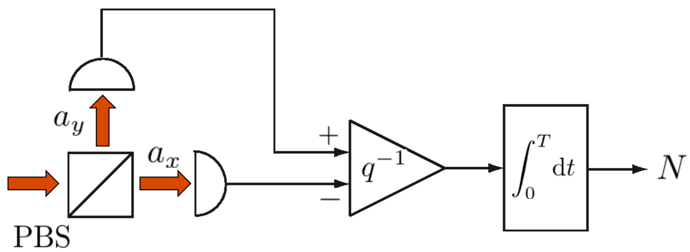
- Squeezed-Vacuum Representation:

$$|\psi\rangle_{\text{out}} = |\psi\rangle_{\text{out}_{45}} |\psi\rangle_{\text{out}_{-45}}$$
$$= |0; \sqrt{G}, -\sqrt{G-1}\rangle_{45}\, |0; \sqrt{G}, \sqrt{G-1}\rangle_{-45}$$

- Photon-Twins Representation:

$$|\psi\rangle_{\text{out}} = \sum_{n=0}^{\infty} \sqrt{\frac{(G-1)^n}{G^{n+1}}}\, |n\rangle_x |n\rangle_y$$

Photon-Twins Behavior



- Semiclassical Theory for Deterministic $\boldsymbol{a}$:

$$\langle \Delta N^2 \rangle = \langle N_x \rangle + \langle N_y \rangle$$

- Quantum Theory for Paramp Outputs $\hat{\boldsymbol{a}}$ from Vacuum Inputs:

$$\langle \Delta \hat{N}^2 \rangle = 0$$

Reduced Density Operators for Paramp Outputs

- Measure Observable $\hat{O}_x = \sum_n o_n |o_n\rangle_{x\,x}\langle o_n|$:

$$\begin{aligned} \Pr(\,\hat{O}_x = o_n \mid |\psi\rangle_{\text{out}}\,) &= \text{tr}[\hat{\rho}_{\text{out}}(|o_n\rangle_{x\,x}\langle o_n| \otimes \hat{I}_y)] \\ &= \text{tr}(\hat{\rho}_{\text{out}_x}|o_n\rangle_{x\,x}\langle o_n|) \end{aligned}$$

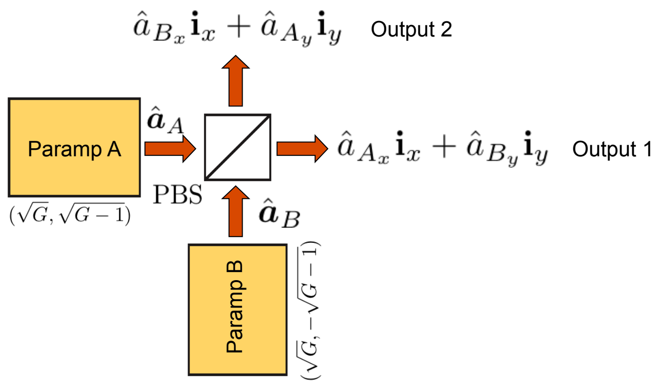
implies $\hat{\rho}_{\text{out}_x} = \text{tr}_y(\hat{\rho}_{\text{out}})$

- For the Parametric Amplifier with Vacuum-State Inputs

$$\hat{\rho}_{\text{out}_k} = \sum_{n=0}^{\infty} \frac{(G-1)^n}{G^{n+1}} |n\rangle_{k\,k}\langle n|, \quad \text{for } k = x, y$$

Creating Polarization-Entangled Photon Pairs

- Polarization-Combined Outputs from Anti-Phased Paramps



- Low-gain operation: $0 < G - 1 = \Delta G \ll 1$

Creating Polarization-Entangled Photon Pairs

- Dual-Paramp Output State in Low-Gain Limit:

$$
\begin{aligned}
|\psi\rangle_{\text{out}} &= |\psi\rangle_A \otimes |\psi\rangle_B \\
&= \sum_{n=0}^{\infty} \sqrt{\frac{\Delta G^n}{G^{n+1}}}\, |n\rangle_{A_x}|n\rangle_{A_y} \otimes \sum_{m=0}^{\infty} (-1)^m \sqrt{\frac{\Delta G^m}{G^{m+1}}}\, |m\rangle_{B_x}|m\rangle_{B_y} \\
&\approx (|0\rangle_{A_x}|0\rangle_{B_y}) \otimes (|0\rangle_{B_x}|0\rangle_{A_y}) \\
&+ \sqrt{\Delta G}\,[(|1\rangle_{A_x}|0\rangle_{B_y}) \otimes (|0\rangle_{B_x}|1\rangle_{A_y}) - (|0\rangle_{A_x}|1\rangle_{B_y}) \otimes (|1\rangle_{B_x}|0\rangle_{A_y})]
\end{aligned}
$$

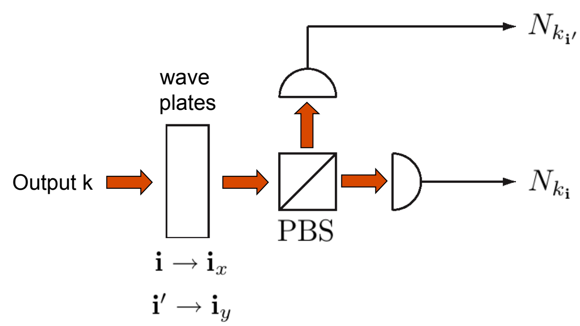
- Entangled Bi-Photon State Realized by Post-Selection:

$$\frac{1}{\sqrt{2}}(|x\rangle_1|y\rangle_2 - |y\rangle_1|x\rangle_2)$$

Polarization-Entangled Photon Pairs

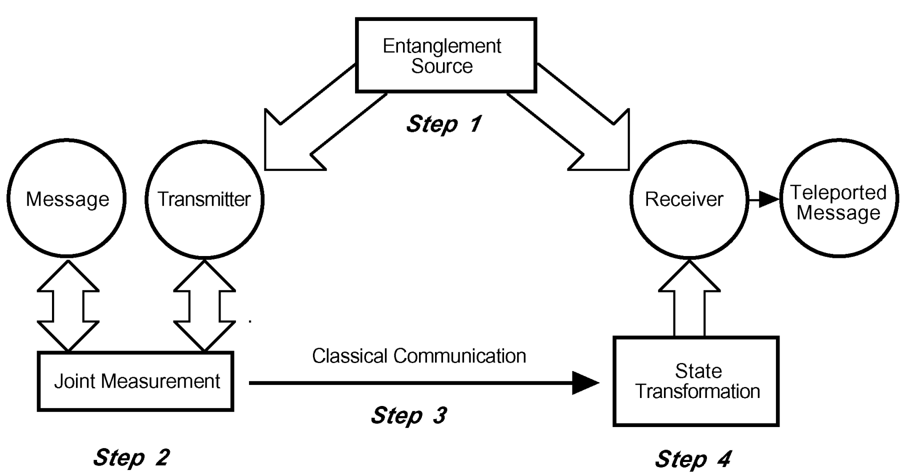
- Measure Outputs 1 and 2 in Conjugate-Pair Polarizations:

$$|\mathbf{i}\rangle_k = \alpha|x\rangle_k + \beta|y\rangle_k \text{ and } |\mathbf{i}'\rangle_k = \beta^*|x\rangle_k - \alpha^*|y\rangle_k, \text{ for } k = 1, 2$$



- Polarization Entanglement: $\Pr(\, N_{2_{\mathbf{i}'}} = 1 \mid N_{1_{\mathbf{i}}} = 1\,) = 1$

The Four Steps of Qubit Teleportation



Coming Attractions: Lecture 14

- Lecture 14:
  Teleportation
  - Polarization entanglement and qubit teleportation
  - Quadrature entanglement and continuous-variable teleportation