LECTURE 14

Last time:

- Fano's Lemma revisited
- Fano's inequality for codewords
- Converse to the coding theorem

Lecture outline

- Feedback channel: setting up the problem
- Perfect feedback
- Feedback capacity

Reading: Sct. 8.12.

Feedback channel

What types of channels do we think if as being feedback channels?

Byzantine general problem

Error in forward and feedback channel

But does that mean that we cannot transmit without error?

Perfect feedback

as good as we are going to get in terms of feedback

Feedback code: $(2^{nR}, n)$ is a sequence of mappings $x_i(M, \underline{Y}^{i-1})$ where each x_i is a function of the message M and the previous received signals

Question: why only the past received signals?

Feedback capacity

The probability of error is:

$P_e = P(\widehat{M} \neq M)$

where $g(\underline{Y}^n) = \widehat{M}$

C_{FB} is the supremum of all achievable rates

Clearly $C_{FB} \geq C$

How about the reverse direction?

Let us reconsider the hapless intra-army messengers.

But do we suffer in rate?

Feedback capacity

Let us try to show that $C_{FB} \leq C$

Try Fano's inequality for code words

However, the channel is no longer a DMC

For DMC, we used:

Assume that the message M is drawn with uniform PMF from $\{1, 2, \ldots, 2^{nR}\}$

Then $nR = H(M)$

Also

$$\begin{aligned} & H(M) = H(M|\underline{Y}) + I(M;\underline{Y}) \\ = \; & H(M|\underline{Y}) + H(\underline{Y}) - H(\underline{Y}|M) \\ = \; & H(M|\underline{Y}) + H(\underline{Y}) - H(\underline{Y}|\underline{X}) \\ = \; & H(M|\underline{Y}) + I(\underline{X};\underline{Y}) \\ \leq \; & 1 + P_e nR + nC \end{aligned}$$

no longer applicable!

Feedback capacity

We still have

$nR = H(M)$

Need to relate $I(M;\underline{Y})$ to $I(\underline{X};\underline{Y})$, which will give then a relation to C as before

$$
\begin{aligned}
& I(M;\underline{Y}^n) \\
= \;& H(\underline{Y}^n) - \sum_{i=1}^{n} H(Y[i]|\underline{Y}^{i-1}, M) \\
= \;& H(\underline{Y}^n) - \sum_{i=1}^{n} H(Y[i]|\underline{Y}^{i-1}, M, X[i]) \\
= \;& H(\underline{Y}^n) - \sum_{i=1}^{n} H(Y[i]|X[i]) \\
\leq \;& \sum_{i=1}^{n} H(Y[i]) - \sum_{i=1}^{n} H(Y[i]|X[i]) \\
= \;& \sum_{i=1}^{n} I(X[i];Y[i]) \\
\leq \;& nC
\end{aligned}
$$

Feedback capacity

Thus, we still have that

$$nR = H(M) \leq 1 + P_e nR + nC$$

which as for the DMC implies

$R \leq \frac{1}{n} + P_e R + C$

so $R \leq C$

Hence $C = C_{FB}$

FEEDBACK DOES NOT HELP!

Discussion

DMC does not benefit from feedback

What other things might happen:

- There is an unknown part of the channel

- There is memory in the channel