6.334 Power Electronics
Spring 2007

Power Electronics Notes - D. Perreault

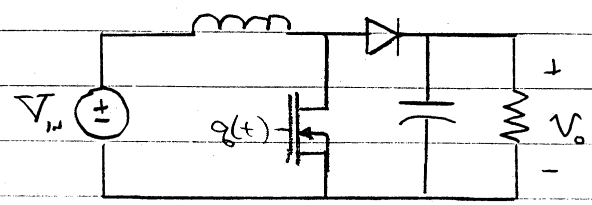
★★ Modeling And Control

★ Direct Circuit Averaging

READ KSV 11.1-11.3.4

Consider a Boost Converter

$$V_o = \frac{V_{IN}}{1-D}$$



Desire to regulate the output voltage V_o in the face of:

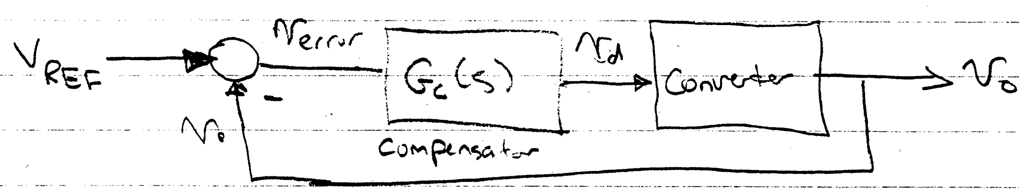
1. Load disturbances → $R_{min} \le R \le R_{max}$
2. Input voltage variations. → $V_{min} \le V \le V_{max}$

→ Feedforward has problems

1. control depends on idealized modeling assumptions
2. doesn't let us control response to load variations

⇒ Use Feedback!! change d depending on output voltage.

(check % that has had 6.302 ...)

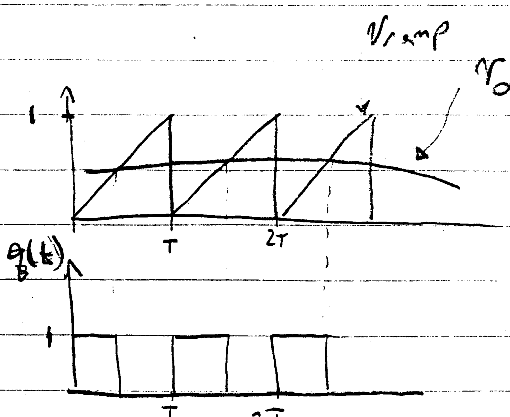


v_d is a voltage $0 < V_d < 1$ representing duty ratio

⇒ generate $q(t)$ switching fn.

over 1 cycle $\langle q(t) \rangle = \langle v_d \rangle$

→ PWM generation



★ ⇒ We still need a model for the converter!!

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We need a dynamic model for the converter.
Switched models are not easy to use
→ they carry too much information about the waveforms
→ we want to know about low frequency variations, not switching

Ex 1

(see example simulation of boost to illustrate this : Ex 1)

To study low-frequency "averaged" behavior, we can look at the local average value of the waveforms.

Define Local Average operator:

$$\boxed{\bar{x}(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\,d\tau}$$

(moving avg. over 1 cycle)

→ local average tracks low freq. variations, suppresses switching ripple info.

Properties of this operator

differentiation $\overline{\left(\frac{dx}{dt}\right)} = \frac{d}{dt}(\bar{x})$ (proof trivial)

Note: In general $\overline{x(t)\,y(t)} \neq \overline{x(t)}\;\overline{y(t)}$

★ But if $x(t)$ or $y(t)$ has both 1.) small ripple
2.) slow variation wrt

then $\overline{x(t)y(t)} \approx \overline{x(t)}\;\overline{y(t)}$

linearity $\overline{(ax+by)} = a\bar{x} + b\bar{y}$

(proof trivial)

time invariance $\overline{(x(t-t_0))} = \bar{x}(t-t_0)$ (proof trivial)

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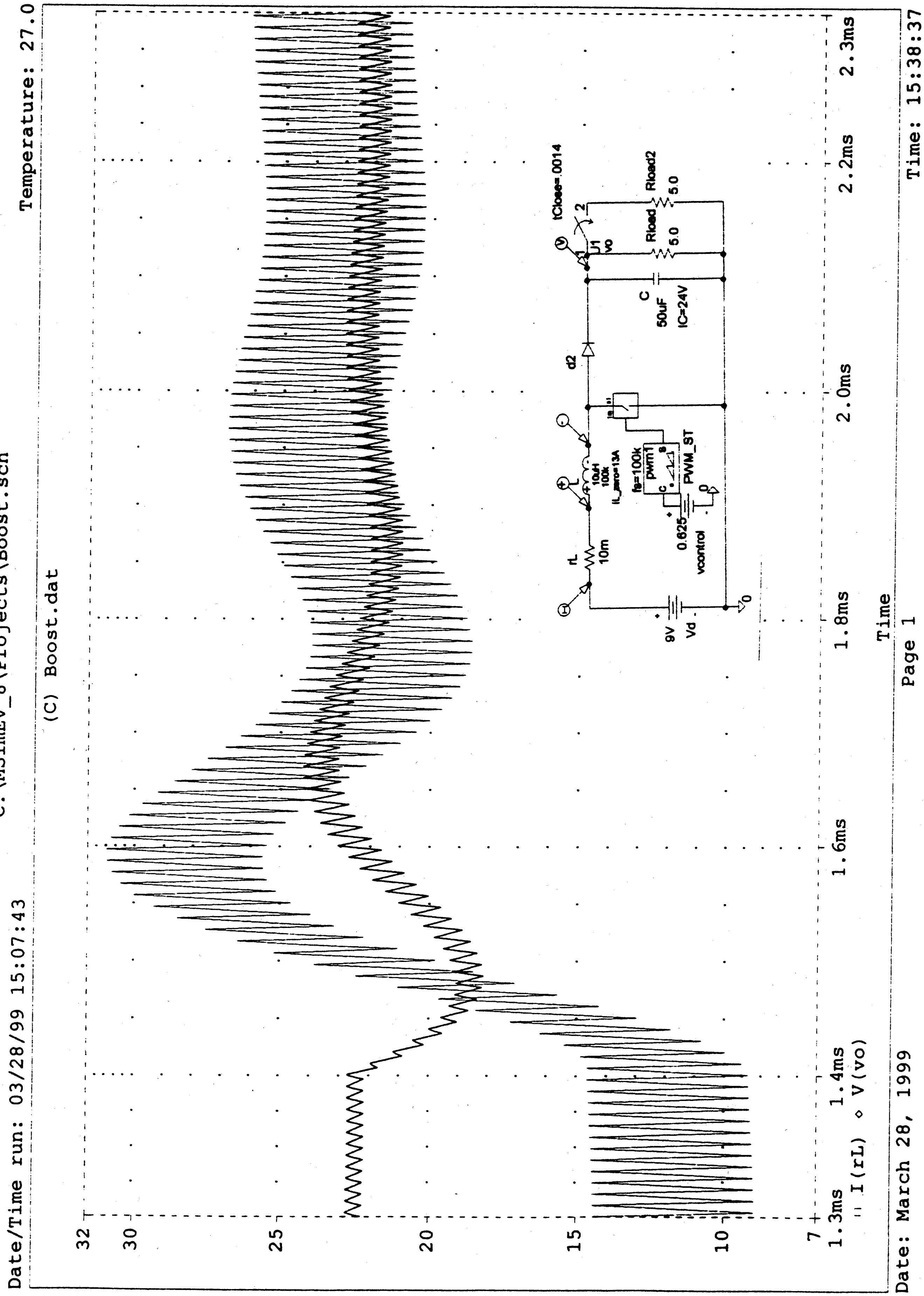
Boost Converter Switched Simulation

Ex 1



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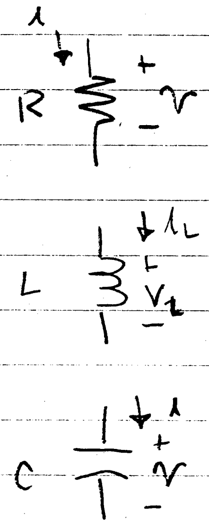
Consider the use of this operator on a circuit:

⇒ ☆ **Because LTI, KVL + KCL are satisfied for averaged vars.**

$$\text{KVL}: \sum v_{ij} = 0 \;\longrightarrow\; \sum \bar{v}_{ij} = 0$$
$$\text{KCL}: \sum i_j = 0 \;\longrightarrow\; \sum \bar{i}_j = 0$$

We can apply avg'ing in circuits

Consider constitutive laws for averaged vars:



$$v(t) = i(t)R$$
$$\bar{v}(t) = \bar{i}(t)R$$

$$v = L\frac{di}{dt}$$
$$\bar{v} = L\frac{d\bar{i}}{dt}$$

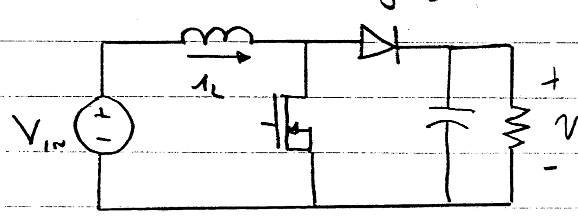
$$i = C\frac{dv}{dt}$$
$$\bar{i} = C\frac{d\bar{v}}{dt}$$

LTI circuit elements

Constitutive relationships do not change!

Nonlinear or time varying elements do change!

Boost ckt

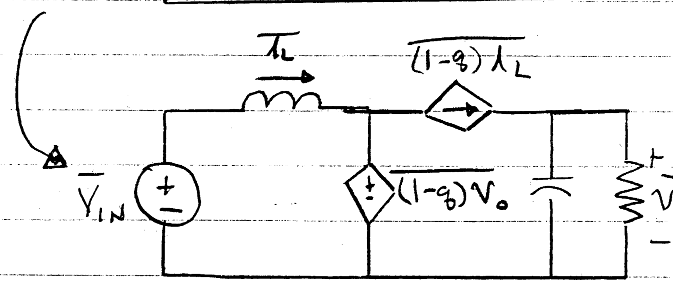


$$x(t)\,y(t) \neq \bar{x}(t)\,\bar{y}(t)$$

but IF $x(t)$ or $y(t)$ has
1. Small ripple
and 2. slow variation wrt T

$$\Rightarrow \overline{x(t)y(t)} \approx \overline{x(t)}\;\overline{y(t)}$$

model w/switching function



to average, place ‾ over all variables.
(LTI elements do not change)

If i_L has small ripple, slow variation $\therefore \overline{(1-q)i_L} \approx \overline{1-q}\cdot\bar{i}_L = d'\bar{i}_L$

v_C has small ripple, slow variation $\therefore \overline{(1-q)v_o} \approx \overline{1-q}\cdot\bar{v}_o = d'\bar{v}_o$

↳ i.e. ≈ const over a cycle

where $\{ d(t) = \overline{q(t)}$

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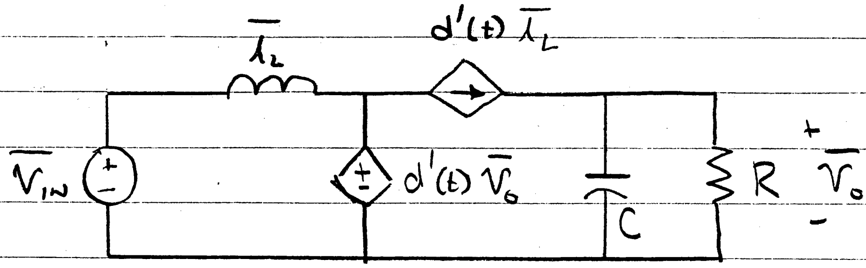
Averaged Circuit model



★ → in this circuit model we have no more switching (only depends on averaged duty cycle $d(t) = \overline{q(t)}$)

★ → model is not linear in our control variable $d(t)$ (because of $d'\hat{v}_o$, $d' i_L$ terms)

★ → model is very simple, + should be accurate for averaged variables if our assumptions are valid ($i.e.$ $\overline{q i_L} = \bar{q}\,\bar{i}_L$, etc.)

<u>Ex 2</u> (show simulation w/ both switched + avg'd models.)

→ good results (small offset due to device mods.)

★ → will be very useful for control!!

Note: not useful for some things. Ex switch power dissipation

$$P(t) = v(t)\,i(t)$$

$$\overline{P(t)} = \overline{v(t)\,i(t)} \neq \bar{v}(t)\,\bar{i}(t)$$

↳ small ripple assumption not met ...

→ Review how we got here + address questions!

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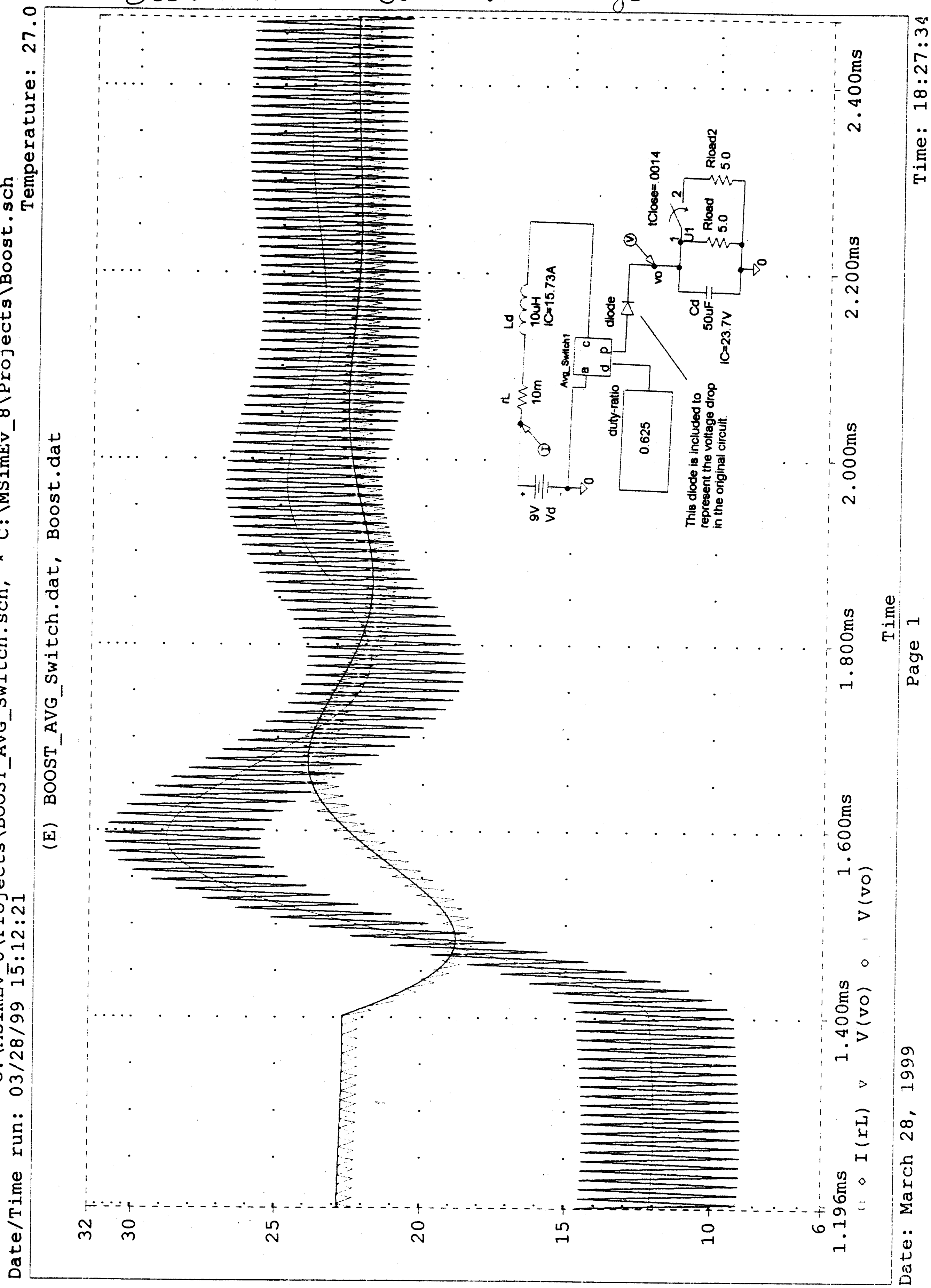
Boost Converter Switched + Averaged simulation

Ex2



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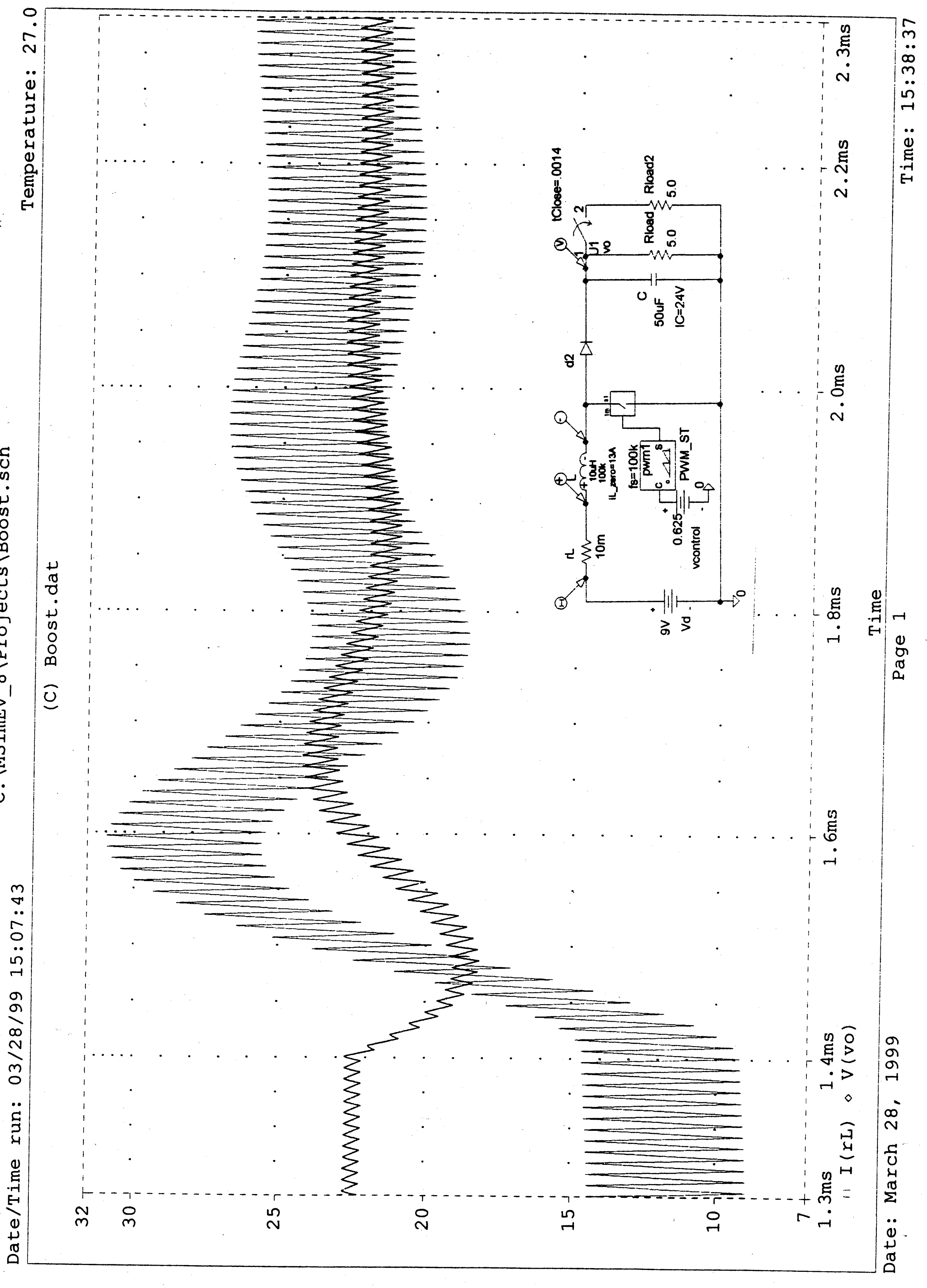
6.334 Lecture Notes

Control Lecture #1

Ex 1



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★ State-Space Averaging, Linearization

Reading: KS+V 12.1-12.4, 13.1-13.2

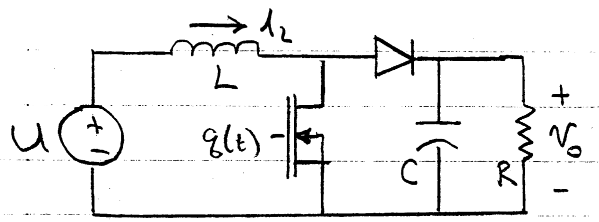
Intro: State-space averaging: different (more methodical) approach to the same type of model we built last time.

→ Review:

$$\boxed{\text{Local Average} \quad \bar{x}(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\,d\tau}$$

1. Linear
2. Time Invariant
3. $\overline{\left(\frac{dx}{dt}\right)} = \frac{d\bar{x}}{dt}$
4. $\overline{x(t)y(t)} \approx \bar{x}(t)\bar{y}(t)$ if x or y has slow variation & small ripple

Boost Converter



$q(t) = \begin{cases} 1 & \text{switch on} \\ 0 & \text{switch off} \end{cases}$ $\quad q' = 1 - q \quad$ $d(t) = \bar{q}$, $d'(t) = \bar{q}'$

State equations: i_L, v_o are state variables

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{u}{L}q(t) + \dfrac{(u - v_o)}{L}(1 - q(t)) \\[2ex] \dfrac{dv_o}{dt} = -\dfrac{1}{RC}v\,q(t) + \left[\dfrac{1}{C}i_L - \dfrac{1}{RC}v\right](1 - q(t)) \end{cases}$$

regroup & average

$$\boxed{\begin{cases} \dfrac{d\bar{i}_L}{dt} = \dfrac{\bar{u}}{L} - \dfrac{\overline{v_o q'(t)}}{L} \approx \dfrac{1}{L}\bar{u} - \dfrac{1}{L}\bar{v}d' \\[2ex] \dfrac{d\bar{v}_o}{dt} = -\dfrac{1}{RC}\bar{v}_o + \dfrac{1}{C}\overline{i_L q'(t)} \approx -\dfrac{1}{RC}\bar{v}_o + \dfrac{1}{C}\bar{i}_L d' \end{cases}}$$ ★

because $\overline{xy} \approx \bar{x}\bar{y}$

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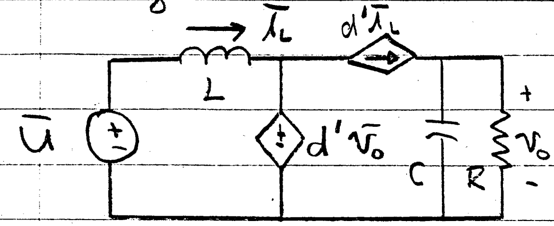
Averaged ckt model from direct ckt. averaging



$$\Rightarrow \begin{cases} \dfrac{d\bar{i}_L}{dt} = \dfrac{1}{L}\bar{u} - \dfrac{1}{L}\bar{v}_o d' \\[2ex] \dfrac{d\bar{v}_o}{dt} = -\dfrac{1}{RC}\bar{v}_o + \dfrac{1}{C}\bar{i}_L d' \end{cases}$$

Same state eqns: Circuit avg + state space avg are the same!
(circuit view vs. eqn. view.)

★ **Linearization**

To do linear control design, Linearize system about operating point.
→ explain what linearized dynamics mean. $x = \underset{\text{s.s.}}{X} + \underset{\text{pert}}{\hat{x}} \Rightarrow \hat{x} = x - X$ deviation from s.s

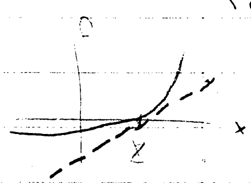
$$\boxed{\bar{u} = U + \hat{u} \quad \bar{i}_L = I_L + \hat{i}_L \quad \bar{v}_o = V_o + \hat{v}_o \quad d = D + \hat{d}}$$

formal def: given $\dfrac{dx}{dt} = f(x(t), r(t), t)$, $f(X, R, t) = 0$ op. point

$$\Rightarrow \cancel{\frac{dX}{dt}}^{0} + \frac{d\hat{x}}{dt} = \left.\frac{\partial f}{\partial x}\right|_{X,R} \hat{x} + \left.\frac{\partial f}{\partial r}\right|_{X,R} \hat{r} + \underbrace{f(X,R)}_{0}$$

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial r}$ must be continuous!!

1st order Taylor Expansion of f about X, R



approx f with a line near $x = X$

intuitive approach: substitute expanded variables in + simplify
(all purely s.s. terms **must** go away by definition of s.s.)

$$\begin{cases} \cancel{\dfrac{dI_L}{dt}}^{0} + \dfrac{d\hat{i}_L}{dt} = \dfrac{1}{L}U + \dfrac{1}{L}\hat{u} - \dfrac{1}{L}(V_o + \hat{v}_o)(1 - D - \hat{d}) \\[2ex] \cancel{\dfrac{dV_o}{dt}}^{0} + \dfrac{d\hat{v}_o}{dt} = -\dfrac{1}{RC}V_o - \dfrac{1}{RC}\hat{v}_o + \dfrac{1}{C}(I_L + \hat{i}_L)(1 - D - \hat{d}) \end{cases}$$

$$\frac{d\hat{i}_L}{dt} = \underbrace{\cancel{\frac{1}{L}U - \frac{1}{L}V_o D'}}_{\to 0} + \frac{1}{L}\hat{u} - \frac{D'}{L}\hat{v}_o + \frac{V_o}{L}\hat{d} + \underbrace{\cancel{\frac{1}{L}\hat{v}_o\hat{d}}}_{\to 0}$$

$$\frac{d\hat{v}_o}{dt} = \underbrace{\cancel{-\frac{1}{RC}V_o + \frac{D'I_L}{C}}}_{0} - \frac{1}{RC}\hat{v}_o + \frac{D'}{C}\hat{i}_L - \frac{I_L}{C}\hat{d} - \underbrace{\cancel{\frac{1}{C}\hat{i}_L\hat{d}}}_{\to 0}$$

$$\boxed{\begin{aligned} \frac{d\hat{i}_L}{dt} &= \frac{1}{L}\hat{u} - \frac{D'}{L}\hat{v}_o + \frac{V_o}{L}\hat{d} \\ \frac{d\hat{v}_o}{dt} &= -\frac{1}{RC}\hat{v}_o + \frac{D'}{C}\hat{i}_L - \frac{I_L}{C}\hat{d} \end{aligned}}$$

Linearized model at
← op pt. U, D, V_o, I_L

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Assume $u = \bar{U}$ (no perturbation in U); Laplace transform:

$$\begin{cases} s\hat{i}_L = -\frac{D'}{L}\hat{v}_o + \frac{V_o}{L}\hat{d} \\ s\hat{v}_o = -\frac{1}{RC}\hat{v}_o + \frac{D'}{C}\hat{i}_L - \frac{I_L}{C}\hat{d} \end{cases}$$

$$s\hat{v}_o = -\frac{1}{RC}\hat{v}_o + \frac{D'}{sC}\left(-\frac{D'}{L}v_o + \frac{V_o}{L}\hat{d}\right) - \frac{I_L}{C}\hat{d}$$

$$\left(s + \frac{D'^2}{sCL} + \frac{1}{RC}\right)v_o = \left(\frac{V_o D'}{sLC} - \frac{I_L}{C}\right)\hat{d}$$

$$\boxed{\frac{\hat{v}_o}{\hat{d}} = \frac{-s\frac{I_L}{C} + \frac{V_o D'}{LC}}{s^2 + \frac{1}{RC}s + \frac{D'^2}{LC}}}$$

→ 2nd order system
→ 2 LHP poles (underdamped)
→ 1 RHP zero (yuck!)

☆ → poles move w/ operating point !!

one op pt. only! → Ex/ $\bar{U} = 9V$, $\bar{V} = 24V$, $D = 0.625$, $I_L \approx 25.6A$
$L = 10\mu H$, $C = 50\mu F$, $R = 2.5\Omega$

$$\frac{\hat{v}_o}{\hat{d}} = \frac{-512000s + 1.8\times10^{10}}{s^2 + 8000s + 2.81\times10^8}$$

$$\begin{cases} \text{zero @ } s = 35{,}156 \text{ rad/sec} \\ \text{poles @ } s = -4{,}000 \pm j16{,}279 \text{ rad/sec} \end{cases}$$

≈ .4 msec osc. period

example from simulation (Ex 1/2)
∴ compare to it.!!

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★ <u>Control Design Example</u>

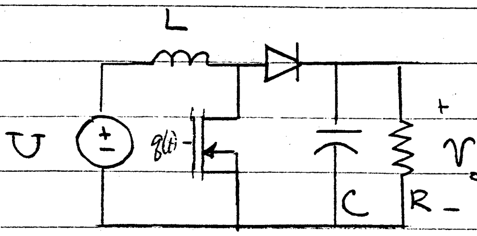
Boost Converter

$L = 10\mu H$ $C = 50\mu F$

$f_{sw} = 100KHz$ $V_{o,REF} = 24V$

$U_{Nom} = 9V$, $8V < U < 10V$

$2\Omega < R < 10\Omega$



Start with switched equations of state:

$$\begin{cases} \dfrac{d\lambda_L}{dt} = \dfrac{U}{L} q(t) + \dfrac{(u - v_o)}{L}(1 - q(t)) \\ \dfrac{dv_o}{dt} = -\dfrac{1}{RC} v_o q(t) + \left[\dfrac{1}{C}\lambda_L - \dfrac{1}{RC} v_o\right](1 - q(t)) \end{cases}$$

AVG.

State-space averaging with $\bar{x} = \frac{1}{T}\int_{t-T}^{t} x(\tau)\,d\tau$

results in nonlinear averaged model

$$\begin{cases} \dfrac{d\bar{\lambda}_L}{dt} \approx \dfrac{1}{L}\bar{u} - \dfrac{1}{L}\bar{v}\,d' \\ \dfrac{d\bar{v}_o}{dt} \approx -\dfrac{1}{RC}\bar{v}_o + \dfrac{1}{C}\bar{\lambda}_L d' \end{cases}$$

Linearize

Linearization about op. point U, I_L, V_o, D yields LTI linearized model of incremental dynamics

$$\begin{cases} \dfrac{d\tilde{\lambda}_L}{dt} = \dfrac{1}{L}\tilde{u} - \dfrac{D'}{L}\tilde{v}_o + \dfrac{V_o}{L}\tilde{d} \\ \dfrac{d\tilde{v}_o}{dt} = -\dfrac{1}{RC}\tilde{v}_o + \dfrac{D'}{C}\tilde{\lambda}_L - \dfrac{I_L}{C}\tilde{d} \end{cases}$$

LTI analysis

Using Laplace transform & Identity $\frac{V_o}{R} = D' I_L$

$$\boxed{H(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)} = \frac{-s\frac{V_o}{RCD'} + \frac{V_o D'}{LC}}{s^2 + \frac{1}{RC}s + \frac{D'^2}{LC}}}$$

Transfer function depends on op. pt.

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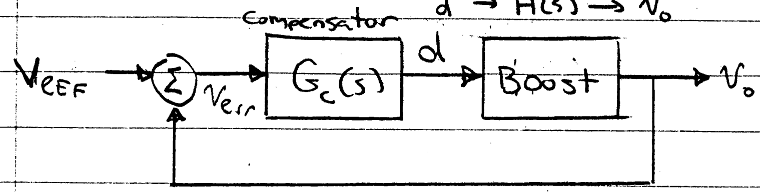
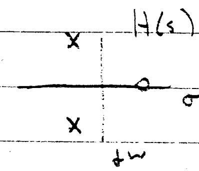
Back to original goal: Control!

$\tilde{d} \rightarrow H(s) \rightarrow \tilde{v}_o$



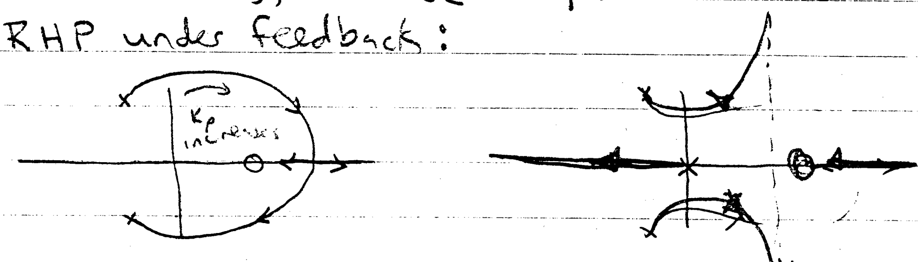
Looking at our transfer function, we have

1. 2 Lightly-damped poles in LHP
2. 1 RHP zero



Note sign in numerator of H(s)

This is tricky, because the poles tend to move to the RHP under feedback:



Proportional $G_c(s) = K_p$ Integral $G_c(s) = \frac{K_I}{s}$

→ So we must pick control gains that are not too high for stability..

Also, the poles move with operating point! (Variations in U, R for example)

→ look at slide for variations of poles with R.

(problem becomes more difficult at light load...)

★ → Could use a damping leg $\overline{v_o}$!! (neat trick)

must design controller valid over all operating points

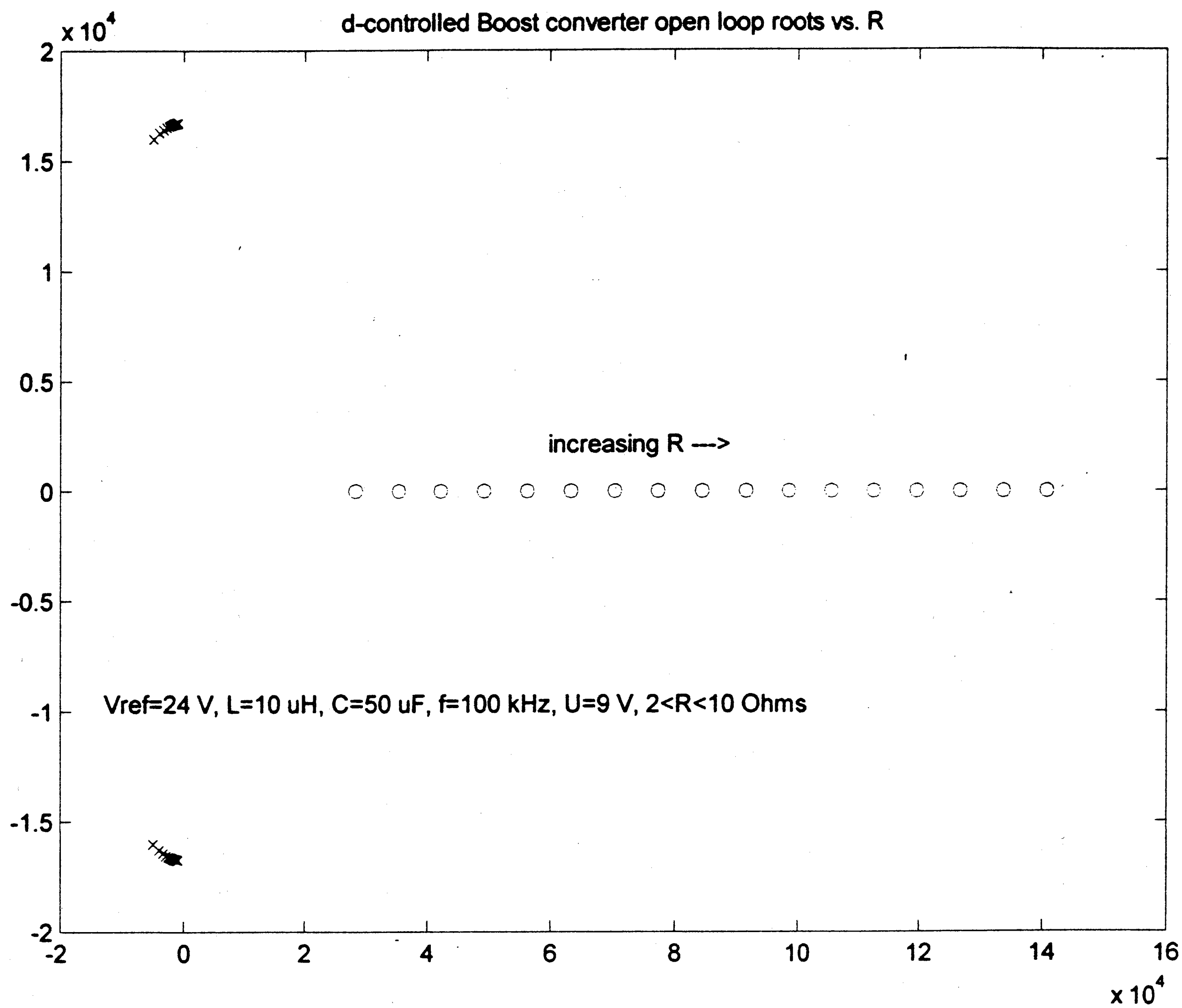
→ look at simple integral controller design

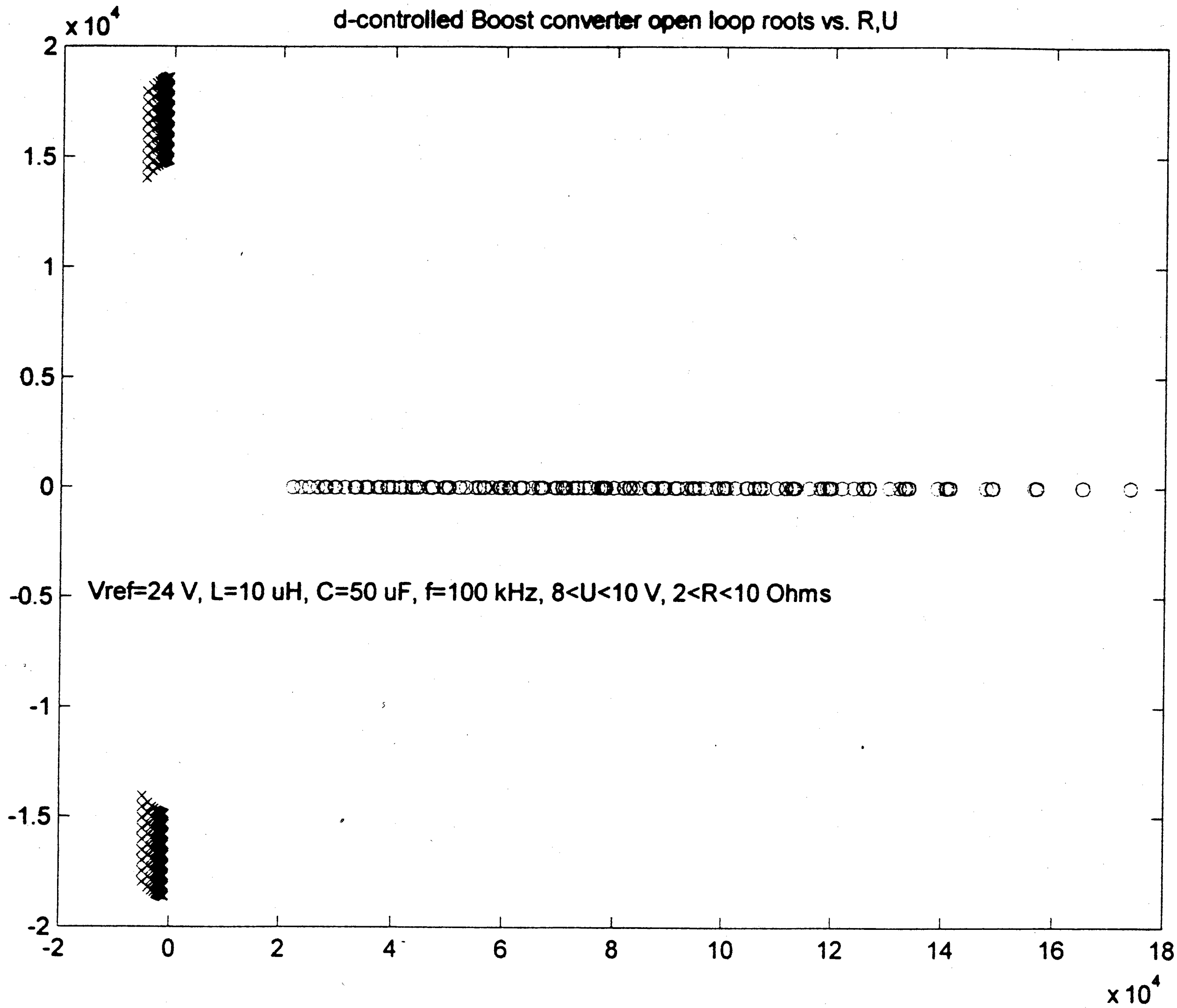
→ note: technically only small signal dynamics are determined, but power converters are forgiving in this respect.

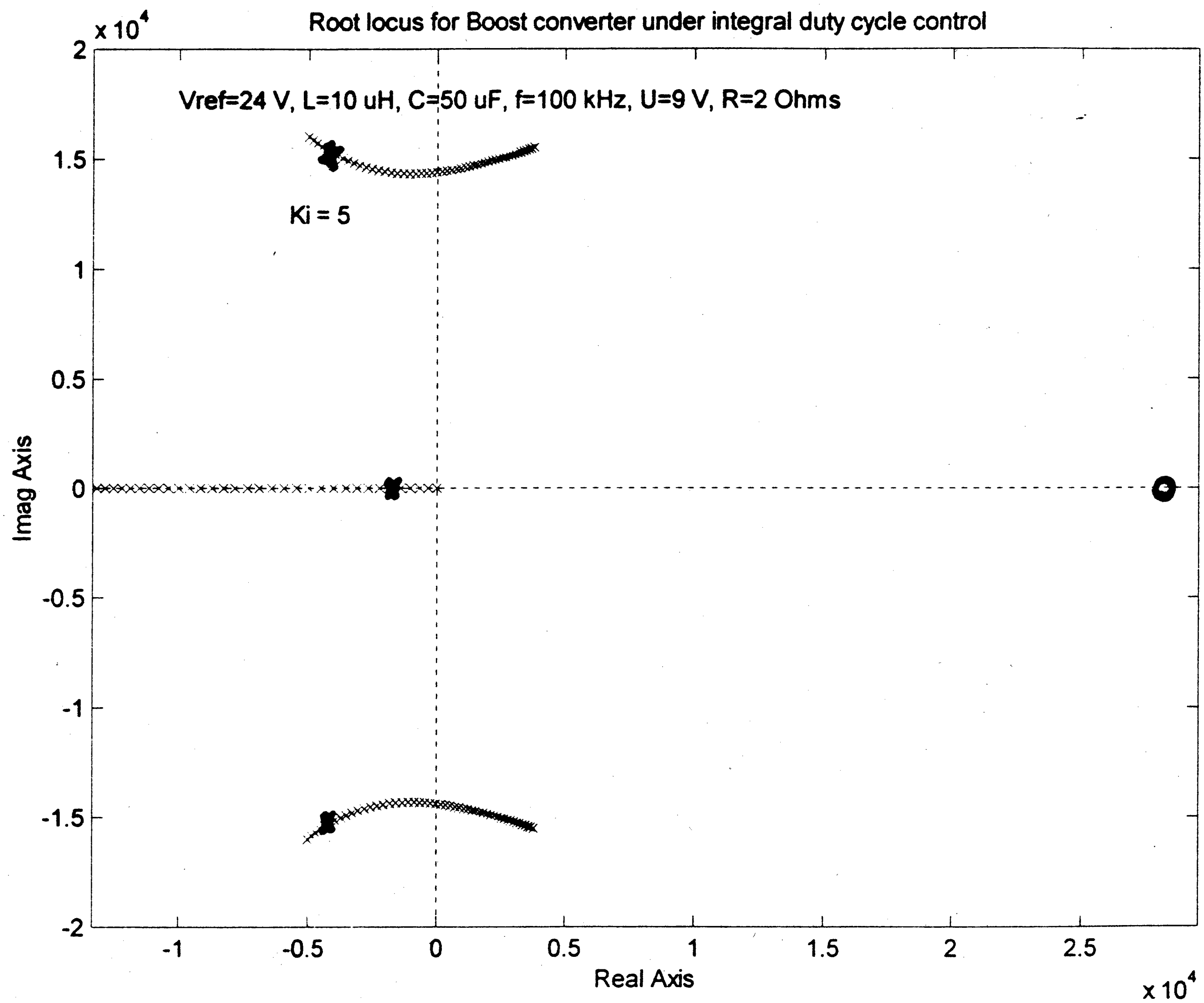
Note: This controller may not be ok in practice due to lightly-damped poles (noise sensitive, transient perf. etc.)

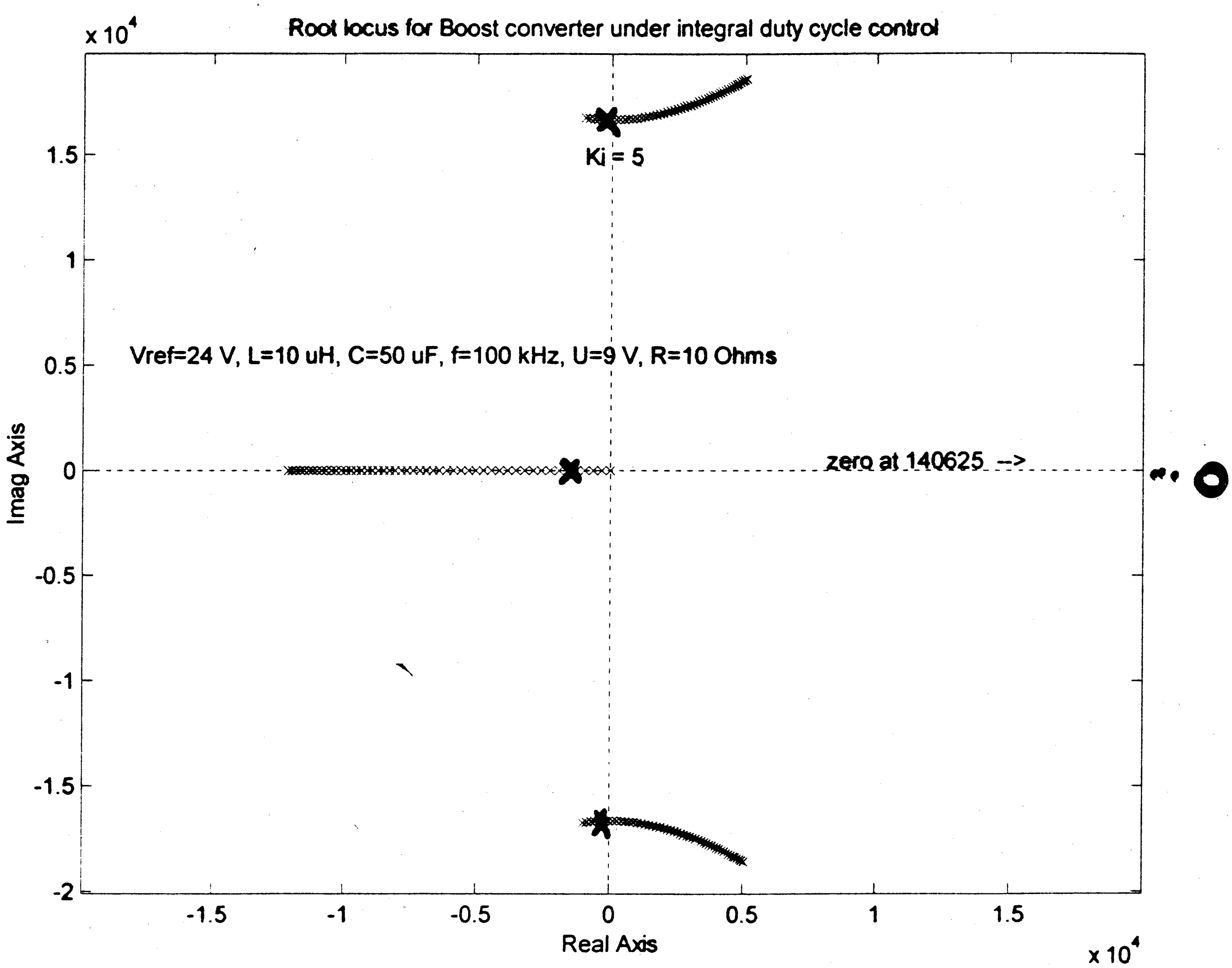
→ go to current-mode control!!

main prob is RHP zero!!

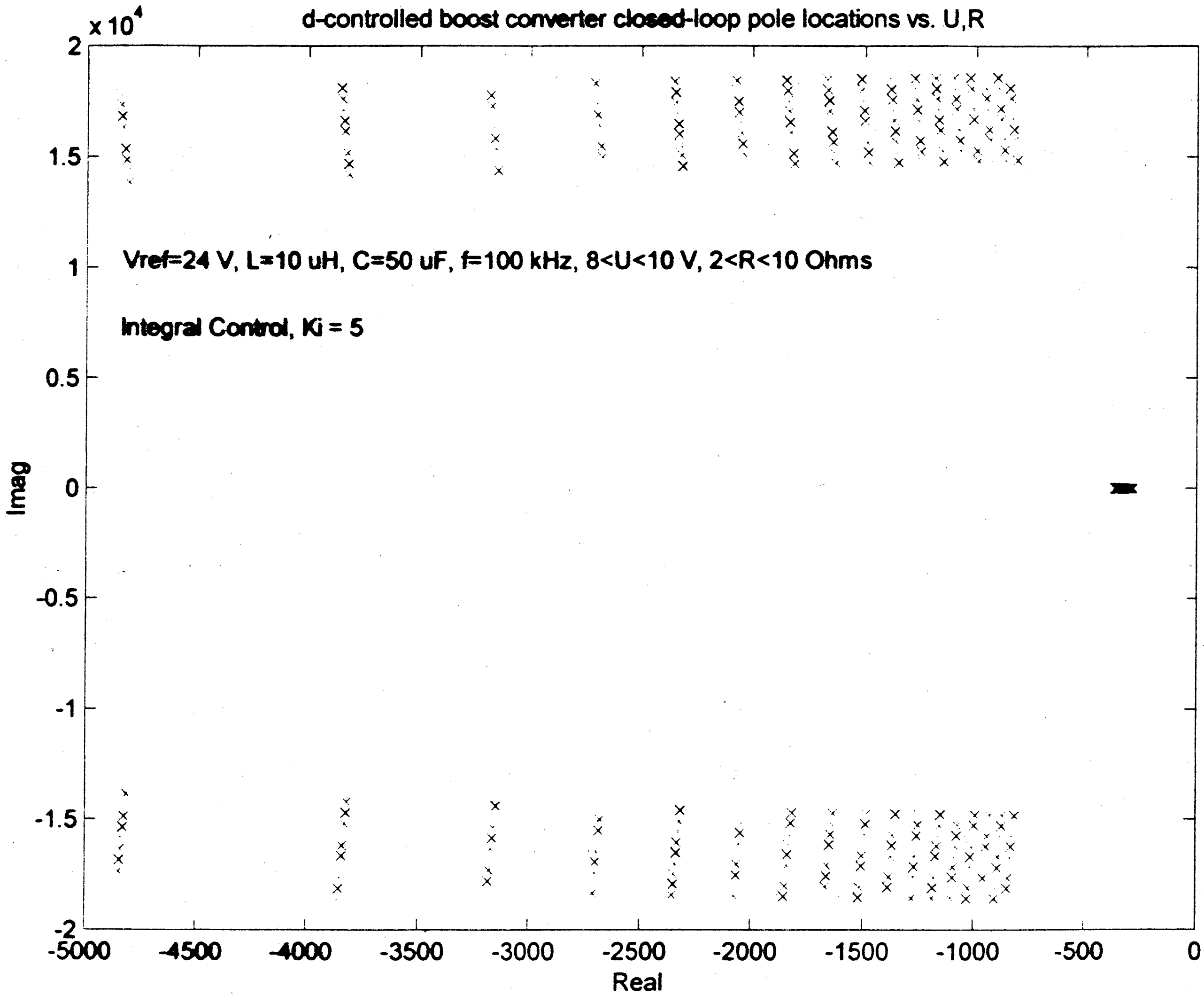

d-controlled Boost converter open loop roots vs. R
x 10^4
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
increasing R --->
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, U=9 V, 2<R<10 Ohms
-2
0
2
4
6
8
10
12
14
16
x 10^4


d-controlled Boost converter open loop roots vs. R,U
x 10^4
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
0
2
4
6
8
10
12
14
16
18
x 10^4
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, 8<U<10 V, 2<R<10 Ohms

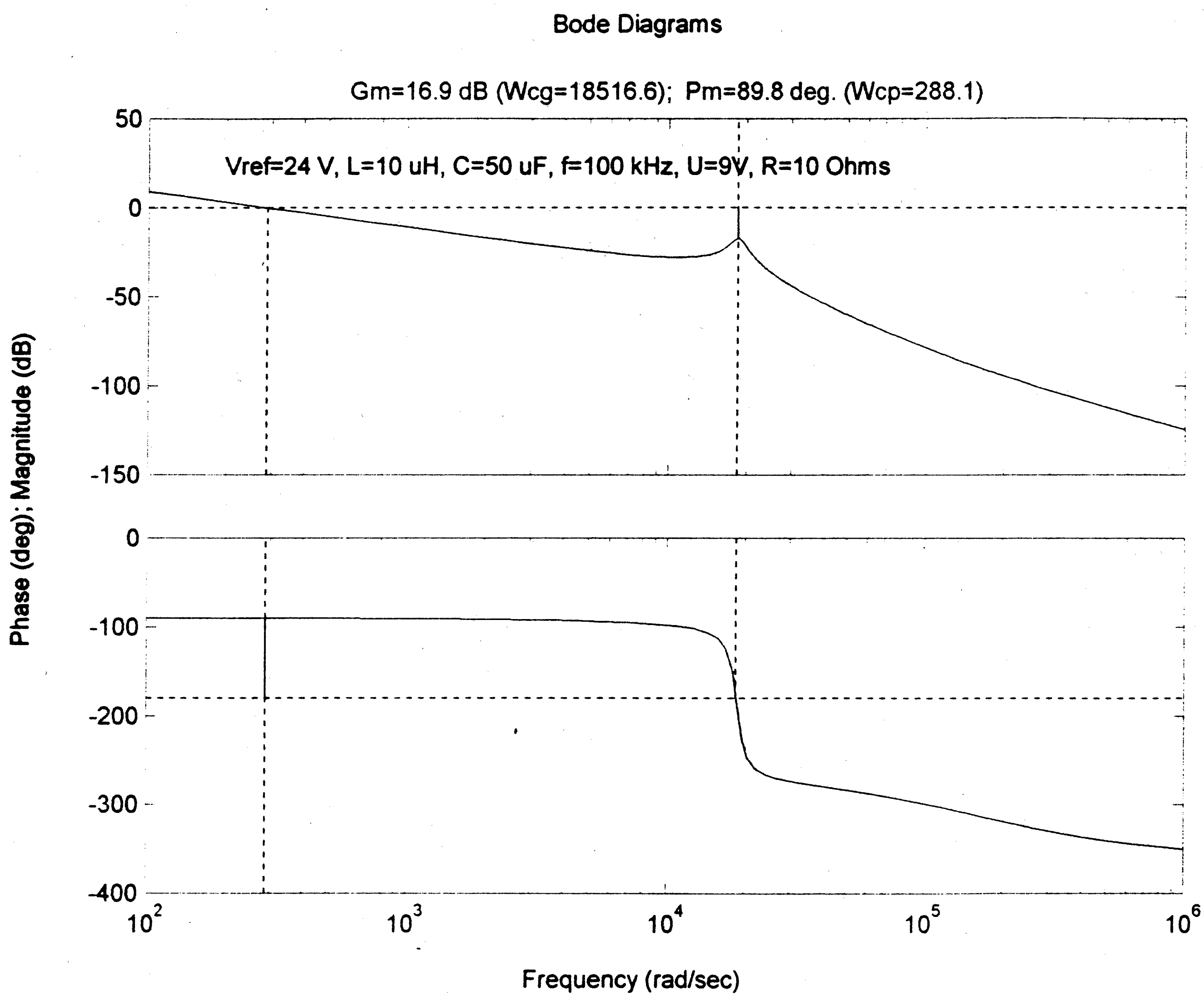

Root locus for Boost converter under integral duty cycle control
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, U=9 V, R=2 Ohms
Ki = 5
x 10^4
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Imag Axis
-1
-0.5
0
0.5
1
1.5
2
2.5
Real Axis
x 10^4


Root locus for Boost converter under integral duty cycle control
x 10^4
Ki = 5
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, U=9 V, R=10 Ohms
zero at 140625 -->
Imag Axis
Real Axis
x 10^4
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5

(67)



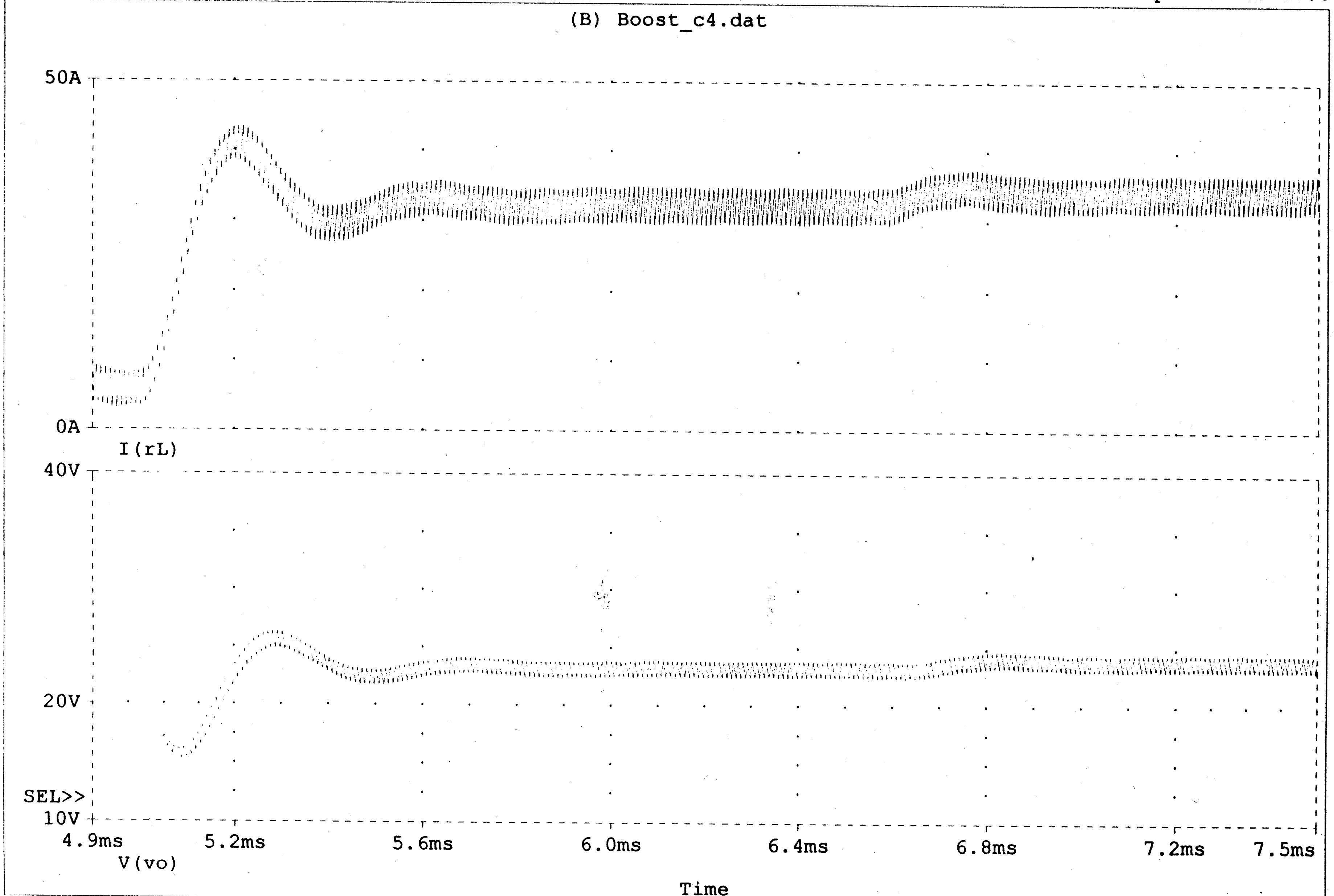
d-controlled boost converter closed-loop pole locations vs. U,R
x 10^4
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, 8<U<10 V, 2<R<10 Ohms
Integral Control, Ki = 5
Imag
Real
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5000
-4500
-4000
-3500
-3000
-2500
-2000
-1500
-1000
-500
0


Bode Diagrams
Gm=16.9 dB (Wcg=18516.6); Pm=89.8 deg. (Wcp=288.1)
Vref=24 V, L=10 uH, C=50 uF, f=100 kHz, U=9V, R=10 Ohms
50
0
-50
-100
-150
0
-100
-200
-300
-400
Phase (deg); Magnitude (dB)
10^2
10^3
10^4
10^5
10^6
Frequency (rad/sec)

* C:\MSimEv_8\Projects\Boost_c4.sch

Date/Time run: 04/01/99 23:21:54

Temperature: 27.0



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Time: 23:56:43

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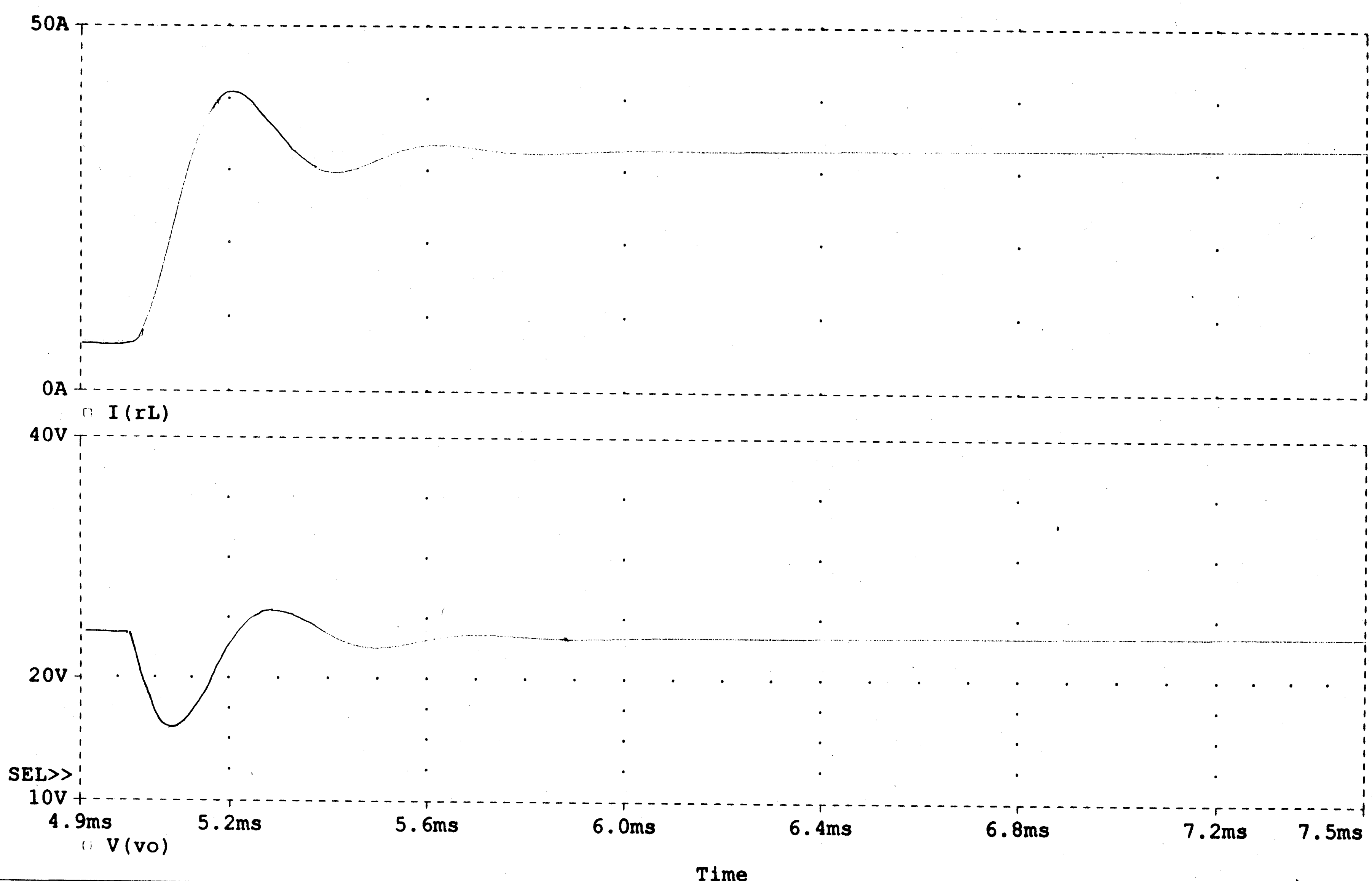
* C:\MSimEv_8\Projects\BOOST_AVG_Switch_c2.sch

Date/Time run: 04/01/99 23:12:43

Temperature: 27.0

(A) BOOST_AVG_Switch_c2.dat



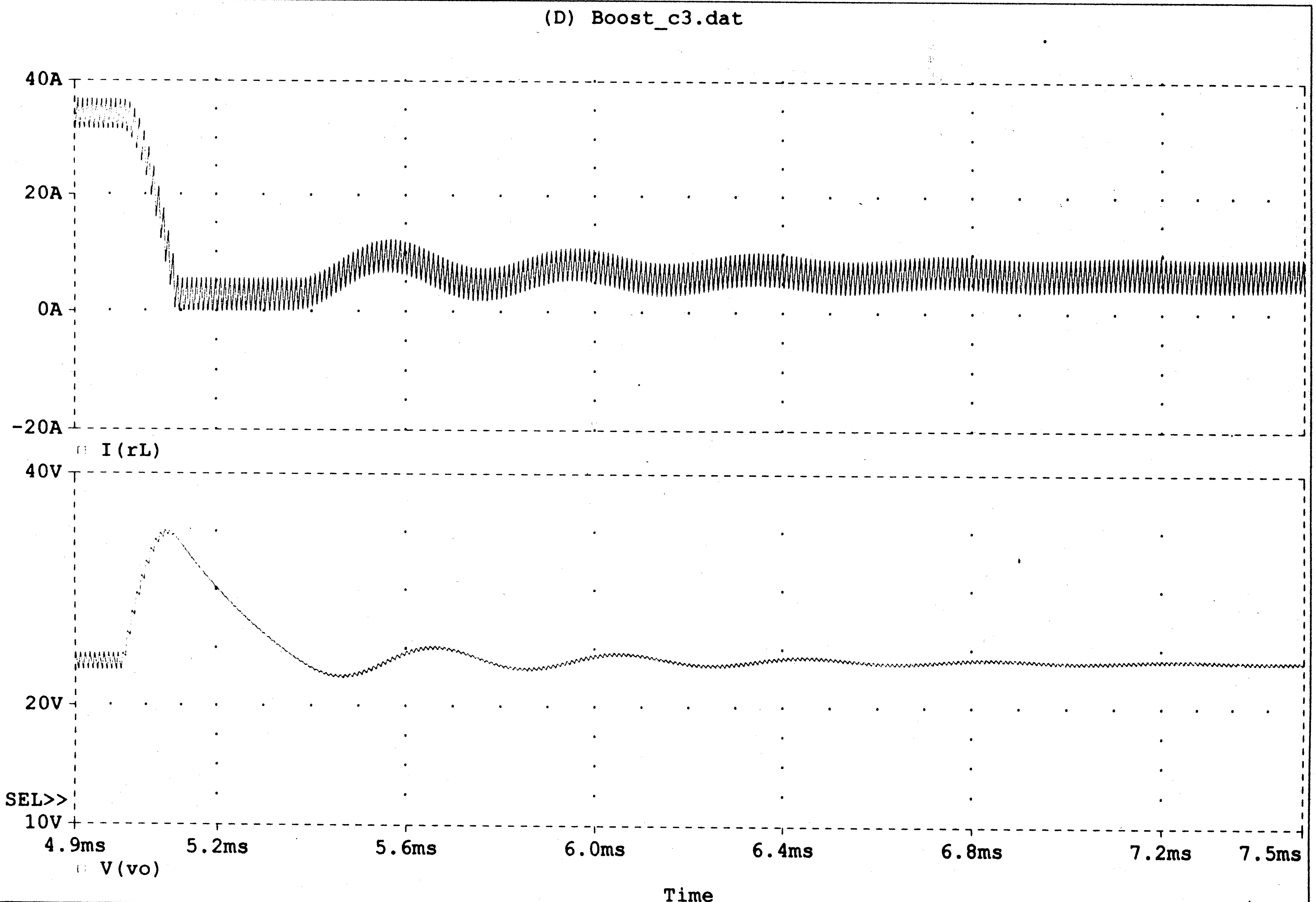
Date: April 01, 1999

Time: 23:56:43

* C:\MSimEv_8\Projects\Boost_c3.sch

Date/Time run: 04/01/99 23:18:45

Temperature: 27.0



Date: April 01, 1999

Time: 23:56:46

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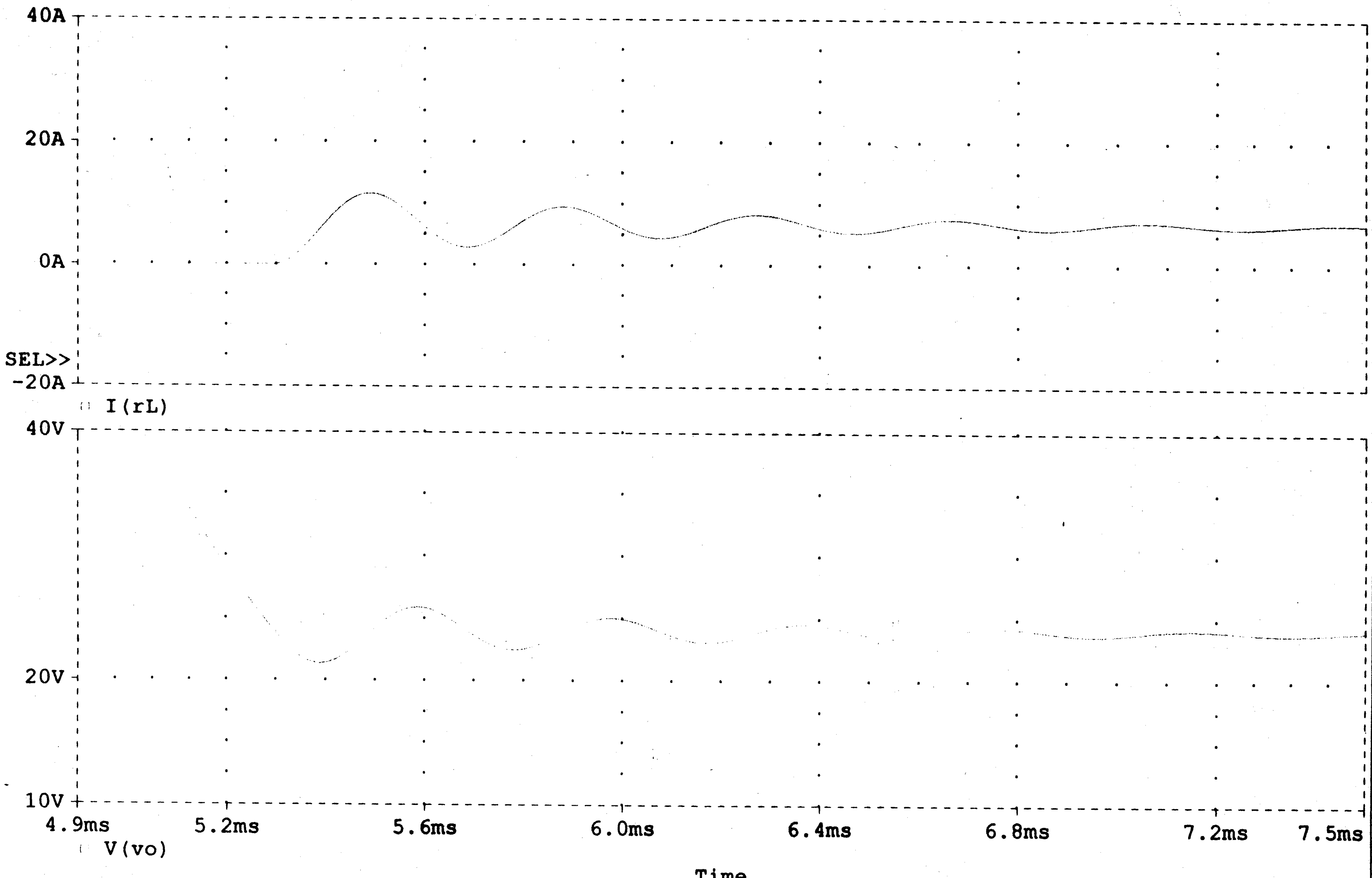
* C:\MSimEv_8\Projects\BOOST_AVG_Switch_cl.sch

Date/Time run: 04/01/99 22:36:37 Temperature: 27.0

(C) BOOST_AVG_Switch_cl.dat



Date: April 01, 1999 Page 3 Time: 23:56:46

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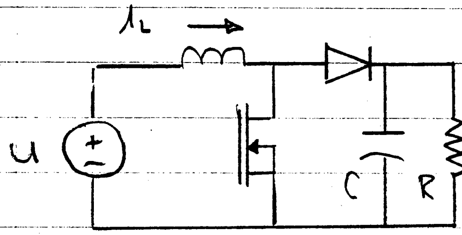
Power Electronics Notes - D. Perreault

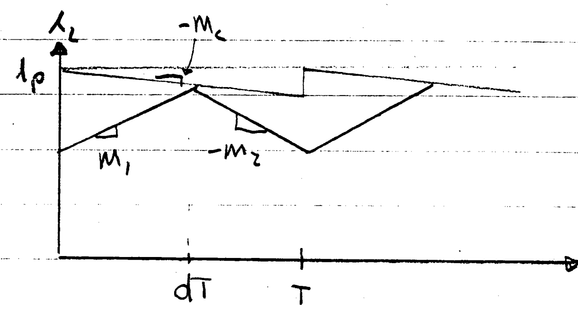
★ Current-mode control

Review: Duty-ratio control of boost converter (or buck-boost) is problematic because of RHP zero + lightly-damped poles in averaged, linearized model.

Solution: Current-mode control! (add inner feedback loop which controls current.)

- concept: Control inductor current in inner feedback loop, + set duty ratio implicitly





In Boost:

$M_1 = \frac{u}{L}$

$M_2 = \frac{V-u}{L}$

1. → switch turns on at beginning of each cycle
2. → switch turns off when i_L reaches $i_p - M_c(t-nT)$ (will see why M_c is used shortly)
3. → control output voltage by controlling i_p.

Advantages:

1. → direct current control in system (input current for boost)
2. → better dynamics (will see shortly)

Problems:

1. Need to measure current (but typically do anyways)
2. Ripple instability!

Under some conditions, the system will not settle to a single duty ratio, but may oscillate subharmonically or chaotically!

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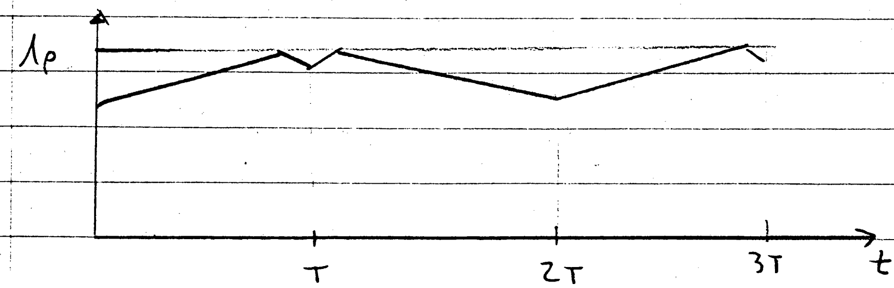
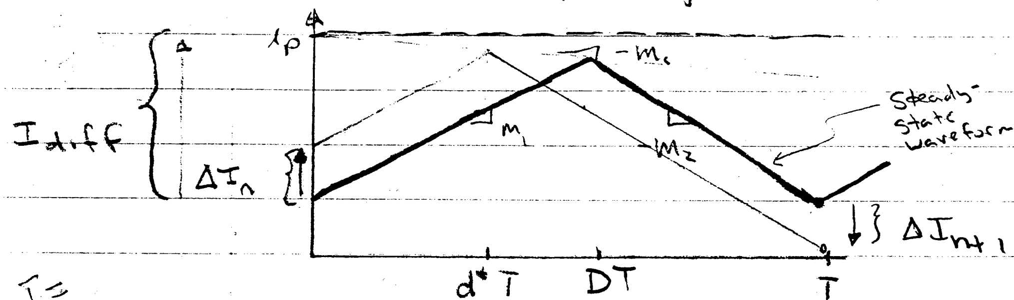
Ex/ Ripple Instability (no compensating ramp)



this is bad because: 1. subharmonics (low-frequency) ripple (below f_{sw})
2. larger ripple than necessary
3. Control is "jittery"

Solution: Compensating ramp. (Properly chosen compensating ramp stabilizes ripple instability)

⇒ Lets analyze ripple dynamics (can't use averaged model)



Start w/ ΔI offset from S.S.

$M_1 D = M_2 (1-D)$

$\Delta I = \Delta_{dynamic} - \Delta_{ss}$

look at cycle-by-cycle deviation of current from S.S.:

$$\begin{cases} I_{diff} = (M_1 + m_c) D T \\ \text{also} \quad I_{diff} = \Delta I_n + (M_1 + m_c) d^* T \end{cases} \Rightarrow \boxed{\Delta I_n = (M_1 + m_c)(D - d^*) T}$$

now $$\Delta I_{n+1} = m_c (D - d^*) T - M_2 (D - d^*) T$$
$$= (M_c - m_2)(D - d^*) T$$

combining $$\Delta I_{n+1} = (M_c - m_2) T \cdot \frac{\Delta I_n}{(M_1 + m_c) T}$$

$$\Rightarrow \quad \Delta I_{n+1} = \left(- \frac{m_2 - m_c}{m_1 + m_c} \right) \Delta I_n$$

$$\longrightarrow \quad \Delta I_n = \left(- \frac{m_2 - m_c}{m_1 + m_c} \right)^n \Delta I_0$$

unstable if $\left| \frac{m_2 - m_c}{m_1 + m_c} \right| \geq 1$!!

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So: unstable if $\left|\frac{m_2 - m_c}{m_1 + m_c}\right| > 1$

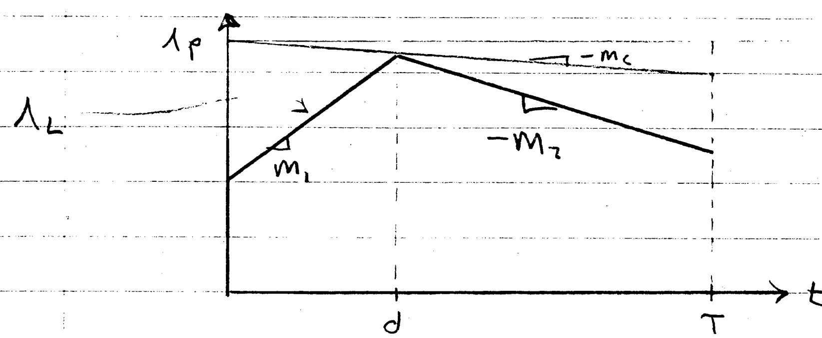
@ $m_c = 0 \rightarrow \left|\frac{m_2}{m_1}\right| > 1 \xrightarrow[m_2 D' = m_1 D]{} \left|\frac{D}{1-D}\right| > 1 \therefore$ unstable for $D \geq 0.5$

So: choose m_c so that $\left|\frac{m_2 - m_c}{m_1 + m_c}\right| < 1$ ($m_2 = m_c \rightarrow$ deadbeat control.)

note: <u>m_c choice affects converter dynamics!</u>

Now that ripple instability is fixed, let's consider system dynamics.

Simplest method: Start with duty ratio-based equations, (from last time) find relation between $d(t)$, $i_p(t)$, $\overline{i_L}$



look at 1-cycle window, and make <u>geometric</u> <u>approximation</u>:

$$\overline{i_L} \cong (i_p - m_c dT) - \frac{1}{T}\left[\frac{1}{2} m_1 d^2 T^2 + \frac{1}{2} m_2 (1-d)^2 T^2\right]$$

$$\therefore \boxed{\overline{i_L} \cong i_p - m_c dT - \frac{1}{2} m_1 d^2 T - \frac{1}{2} m_2 (1-d)^2 T} \leftarrow \text{general eqn for various converters.}$$

for Boost $m_1 = \frac{u}{L}$, $m_2 = \frac{v-u}{L}$

$$\overline{i_L} = i_p - m_c dT - \frac{1}{2L}\bar{u} d^2 T - \frac{1}{2L}(\bar{v} - \bar{u})(1-d)^2 T$$

linearize + solve for $\hat{d}$

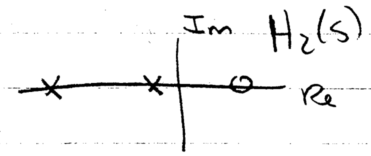
$$(*) \quad \boxed{\hat{d} = \frac{1}{m_c T}(\hat{i}_p - \hat{i}_L) - \frac{D^2 - D'^2}{2L m_c}\hat{u} - \frac{D'^2}{2L m_c}\hat{v}} \leftarrow \text{for Boost converter}$$

Substitute (*) into linearized, state space averaged model of Boost converter from before, eliminate $\tilde{d}$, and have new control variable $\hat{i}_p$

From new model, we can derive $H_2(s)$

$$\hat{i}_p \longrightarrow \boxed{H_2(s)} \longrightarrow \tilde{v}_o$$

$H_2(s)$ has: 1.) RHP zero
2.) 2 real-axis poles
(1 low freq, 1 high-freq)



because the poles are overdamped (instead of lightly damped), we can achieve much better control dynamics!!