Problem 1 (35 points) For each one of the statements below, state whether it is true or false. If the answer is true, explain why. If the answer is false, give a counterexample. Explanations and counterexamples are required for full credit. (7 points each)

(1) In the following game, a strategy is strictly dominated if and only if it is a never-best response.

	A	B
C	$4,2$	$0,3$
D	$3,1$	$1,0$
E	$0,0$	$2,2$

(2) In the following game, the set of correlated equilibria is the same as the set of Nash equilibria.

	H	T
H	$1,-1$	$-1,1$
T	$-1,1$	$1,-1$

(3) Suppose that function f satisfies strictly increasing differences. That is, suppose that for all $x' > x$ and $\theta' > \theta$, we have $f(x',\theta') - f(x,\theta') > f(x',\theta) - f(x,\theta)$. Let $X^*(\theta) = \text{argmax}_{x\in\mathbb{R}}\{f(x,\theta) + g(x)\}$ be nonempty for each θ. For $\theta' > \theta$ if $z \in X^*(\theta)$ and $y \in X^*(\theta')$ then $y \geq z$.

(4) Fictitious play converges in the time-average sense for the following game.

	A	B	C
D	$-1,4$	$1,2$	$-2,5$
E	$3,6$	$-1,-3$	$1,3$
F	$5,-2$	$0,3$	$4,-1$

(5) For an infinite-horizon game, a strategy profile s^* is a subgame perfect equilibrium (SPE) if and only if the one stage deviation condition holds.

Problem 2 (30 points) *(Patent Race for a New Market)*

Consider a patent race game, where the players are 3 firms: Alps, Bees, and Caron, which we denote by A, B, and C respectively. Each of the firms chooses simultaneously spending budget on research $x_i \geq 0$ (i=A,B,C). The firms are risk neutral and there is no discounting. Innovation occurs at time $T(x_i)$, which is a function of the spending, and $T'(x) < 0$. The value of the patent is V, the cost to develop it is x_i, and if several players innovate simultaneously they share its value equally.

(1) (5 points) Formulate the situation as a strategic game by specifying the payoff functions π_i for firm $i = A, B, C$. You may use j and k to denote the other two firms.

(2) (10 points) Does this game have any pure strategy Nash Equilibrium? If it has, specify the equilibrium strategy profile. If it does not, demonstrate the reason.

(3) (15 points) Find a symmetric mixed strategy Nash Equilibrium of the game.

Problem 3 (35 points) *(Parameterized Prisoner's Dilemma)*

Consider the parameterized Prisoner's Dilemma game G, where the payoffs are given in the following matrix, where $x > 1$.

	Cooperate	*Defect*
Cooperate	$-1, -1$	$-x-2, 0$
Defect	$0, -x-2$	$-x, -x$

(1) (10 points) What is the sum of the payoffs in Nash Equilibrium for this game? What is the social optimal payoff for this game, i.e. the largest possible value for the sum of the payoffs?

(2) (20 points) Consider an infinitely-repeated version of this game with discount rate δ and perfect monitoring, we denote this game by $G^\infty(\delta)$. Assume that there is a common correlating device (e.g., a coin tossed at each stage before actions, and strategies can be conditioned on the history of the correlating device). Show that there exists some $\delta^* \leq 1$, such that for all $\delta \geq \delta^*$, there is a subgame perfect equilibrium of $G^\infty(\delta)$ with payoffs $(\frac{-x-2}{2}, \frac{-x-2}{2})$. Note that the payoffs are defined in the usual way as follows: for a sequence of action profiles $\mathbf{a} = \{a^t\}$, the payoff of player i is given by

$$u_i(\mathbf{a}) = (1-\delta)\sum_{t=0}^{\infty} \delta^t g_i(a_i^t, a_{-i}^t),$$

where $g_i(a^t)$ denotes the stage game payoff. Assume $x > 2$ for this part, what is a strategy profile for this equilibrium? What is the corresponding δ^*?

(3) (5 points) What is unattractive about this subgame perfect equilibrium?