6.254 Game Theory with Engineering Applications

Midterm

April 8, 2008

Problem 1 : (35 points) Consider a game with two players, where the pure strategy of each player is given by $x_i \in [0, 1]$. Assume that the payoff function u_i of player i is given by

$$u_i(x_1, x_2) = a_i x_i + \mathcal{I}\{x_i < x_j\},$$

where $0 < a_i \leq 1$ and $\mathcal{I}$ is the indicator function given by

$$\mathcal{I}\{x_i < x_j\} = \begin{cases} 1 & \text{if } x_i < x_j \\ 0 & \text{otherwise.} \end{cases}$$

(a) Does this game have a pure strategy Nash equilibrium? Verify your answer.

(b) Show that there cannot be an atom at any $x \in (0, 1]$ in either player's mixed strategy at an equilibrium.

(c) Find all mixed strategy Nash equilibria.

Problem 2 : (30 points) Consider the following oligopoly competition model: There are two firms. Each firm i simultaneously chooses price $p_i \in [0, 1]$. Assume that the demand function for firm i, given by $d_i(p_1, p_2)$, is twice differentiable. Each firm is interested in maximizing his profits (assume for simplicity that there is no cost).

(a) Formulate this problem as a strategic form game.

(b) Suppose first that the marginal revenue of firm i, i.e., the function $d_i(p_1, p_2) + p_i \frac{\partial d_i(p_1,p_2)}{\partial p_i}$, is nondecreasing in p_j for all $i = 1, 2$. Show that the resulting game is supermodular. Does the set of pure Nash equilibria have a smallest and a largest element? Briefly explain. Construct a learning algorithm that converges to the smallest Nash equilibrium.

(c) Now suppose that the marginal revenue of firm i, i.e., the function $d_i(p_1, p_2) + p_i \frac{\partial d_i(p_1,p_2)}{\partial p_i}$, is nonincreasing in p_j for all $i = 1, 2$. Does the set of Nash equilibria still have a smallest and a largest element? Verify your answer.

Problem 3 : (35 points) Consider a two-player game with the following payoff structure:

	A	B	C	D
A	$1, 1$	$2, 0$	$0, 2$	$-1, 3$
B	$3, -1$	$1, 1$	$1, 1$	$2, 0$
C	$2, 0$	$1, 1$	$-1, 3$	$0, 2$
D	$0, 2$	$1, 1$	$-2, 4$	$1, 1$

(a) Find all pure and mixed Nash equilibria of this game.

(b) Does fictitious play converge in the time-average sense for this game? Verify your answer.

(c) Find all correlated equilibrium payoffs, i.e., all possible payoffs at any correlated equilibrium.

6.254 Game Theory with Engineering Applications
Spring 2010