Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

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Problem Set 8 (due April 16, 2004) [1]

Problem 8.1

For the standard LTI feedback design setup defined by equations

$$\dot{x}(t) = ax(t) + u(t) + w_1(t),\ z(t) = \begin{bmatrix} x(t) \\ u(t) \end{bmatrix},\ y = \begin{bmatrix} \dot{x}(t) \\ x(t) + w_2(t) \end{bmatrix},$$

where $a \in \mathbf{R}$ is a parameter, find matrices T_0, T_1, T_2 defining a valid Q-parameterization of all closed loop transfer matrices $T:\ w \to z$ which can be achieved while using a finite order stabilizing dynamic feedback $u = Ky$.

Problem 8.2

For the standard discrete time LTI feedback design setup defined by equations

$$x[k+1] = -x[k] + u[k] + w_1[k],\ z[k] = \begin{bmatrix} ax[k] \\ u[k] \end{bmatrix},\ y[k] = x[k] + w_2[k],$$

where $a > 0$ is a parameter, find the H2 optimal feedbacl law by using a Tustin transformation to an equivalent continuous time problem. Also give explicit expressions for the equivalent CT setup, and for the corresponing CT H2 optimal feedback.

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[1]Version of April 9, 2004

Problem 8.3

Consider a system described by the hyperbolic partial differential equation

$$v_t = v_{xx} + rv, \quad v(0,t) = 0, \quad y(t) = v(1,t) + w(t), \; u(t) = v_x(1,t),$$

where $v = v(x,t)$, for fixed time, is a function of the spatial parameter $x \in [0,1]$, v_t denotes the time derivative of v, v_{xx} denotes the double spatial derivative of v, and $r > 0$ is a given parameter. The control action is the Dirichlet boundary condition $u(t) = v_x(1,t)$, while a noisy measurement of $y(t) = v(1,t) + w(t)$ is used as the sensor signal.

(a) Find an analytical expression for the transfer function $P = P_r(s)$ from u to y.

(b) For $r = 1$, find a good low order rational approximation $\hat{P}_1$ of P_1, such that $\Delta = P_1 - \hat{P}_1$ is stable, together with an upper bound $\|\Delta\|_\infty < \epsilon$.

(c) Using the results from (b), small gain theorem, and H-Infinity optimization, design a finite order stabilizing feedback $u = Ky$ for the original system, while trying to provide an upper bound for the closed loop H-Infinity norm $\|T_{wu}\|$ which is as small as possible. Note that this will only be possible when ϵ is small enough.