Mapping and Navigation

January 6th, 2005

Edwin Olson

Why build a map?

- Time!
  - Playing field is big, robot is slow
  - Driving around perimeter takes a minute!
  - Scoring takes time… often ~20 seconds to “line up” to a mouse hole.
- Exploration round gives advantage to robots that can map

Attack Plan

- **Motivation: why build a map?**
- Terminology, basic concepts
- Mapping approaches
  - Metrical
    - State Estimation
    - Occupancy Grids
  - Topological
- Data Association
- Hints and Tips

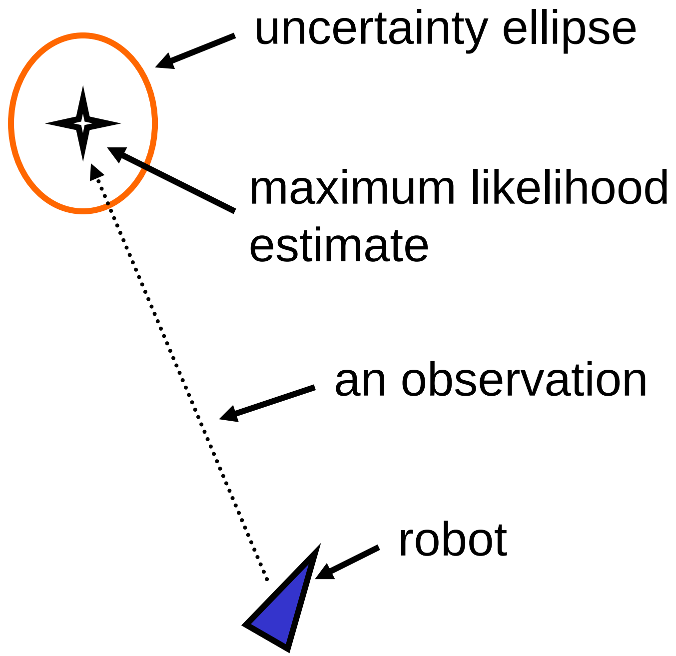
What is a feature?

- An object/structure in the environment that we will represent in our map
- Something that we can observe multiple times, from different locations

Bunker Hill Monument
(Image courtesy of H. Oestreich and stock.xchng)

What is an Observation?

- Where do we get observations from?
  - Camera
    - Range/bearing to ticks and landmarks
  - Corners detected from camera, range finders
- For now, let's assume we get these observations plus some noise estimate.



Data Association

- The problem of recognizing that an object you see now is the same one you saw before
  - Hard for simple features (points, lines)
  - Easy for “high-fidelity” features (barcodes, bunker hill monuments)
- With perfect data association, most mapping problems become “easy”

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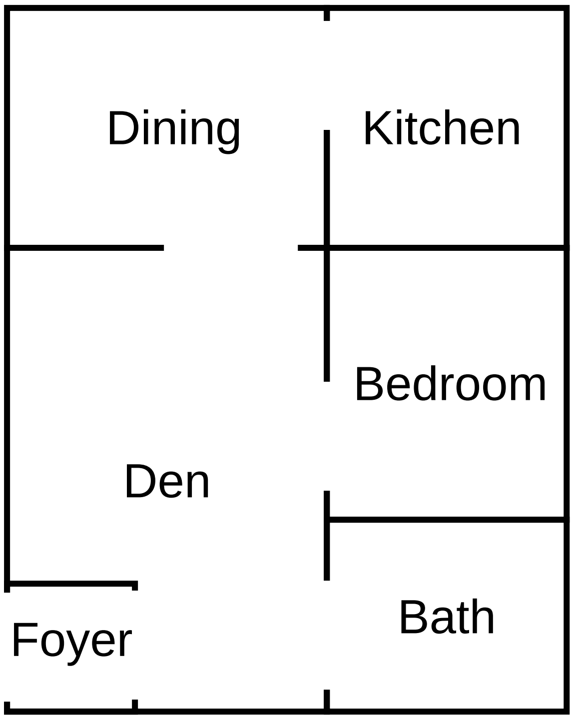
Metrical Maps

- Try to estimate actual locations of features and robot
- “The robot is at (5,3) and feature 1 is at (2,2)”
  - Both “occupancy grid” and discrete feature approaches.
- Relatively hard to build
- Much more complete representation of the world



Metrical Maps

- State Estimation
  - Estimate discrete quantities: “If we fit a line to the wall, what are its parameters y=mx+b?”
  - Often use probabilistic machinery, Kalman filters

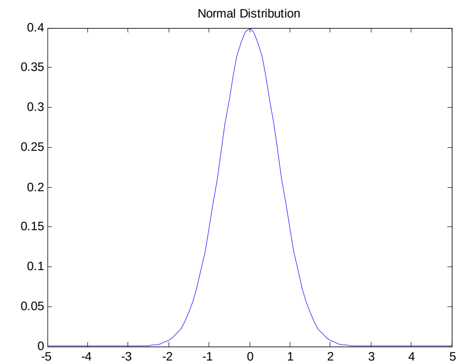
- Occupancy Grid
  - Discretize the world. “I don’t know what a wall is, but grids 43, 44, and 45 are impassable.”

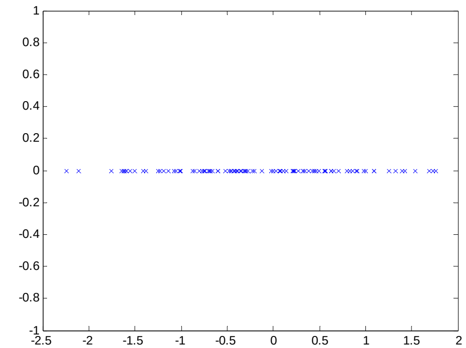
Bayesian Estimation

- Represent unknowns with probability densities
  - Often, we assume the densities are Gaussian

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/\sigma^2}$$

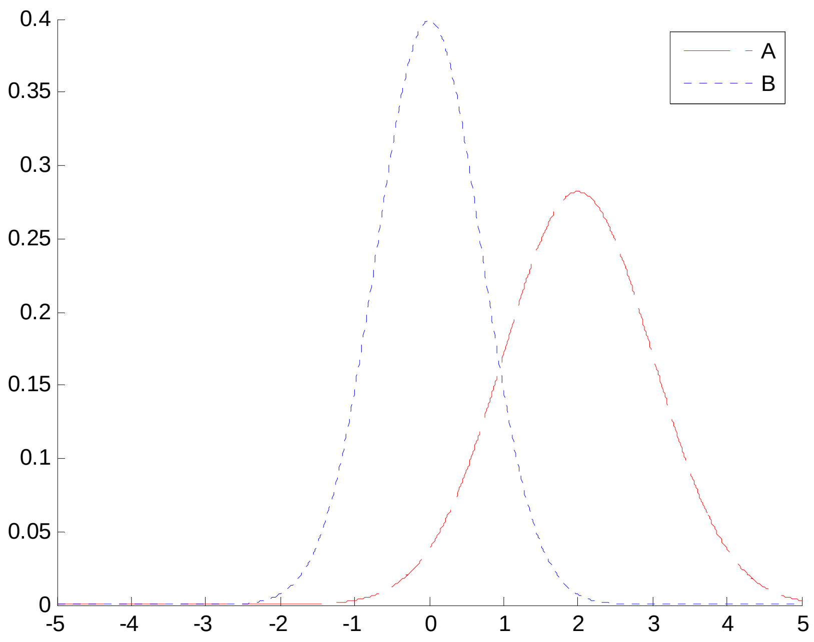
  - Or we represent arbitrary densities with particles
    - We won't cover this today





Bayesian Data Fusion

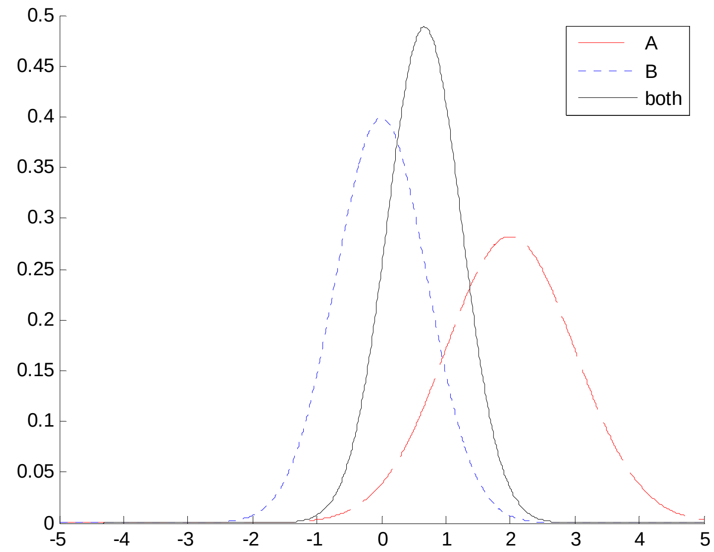
- Example: Estimating where Jill is standing:
  - Alice says: x=2
    - We think σ^2 =2; she wears thick glasses
  - Bob says: x=0
    - We think σ^2 =1; he's pretty reliable
- How do we combine these measurements?



Simple Kalman Filter

- Answer: algebra (and a little calculus)!
  - Compute mean by finding maxima of the log probability of the product $P_A P_B$.
  - Variance is messy; consider case when $P_A=P_B=N(0,1)$

- *Try deriving these equations at home!*

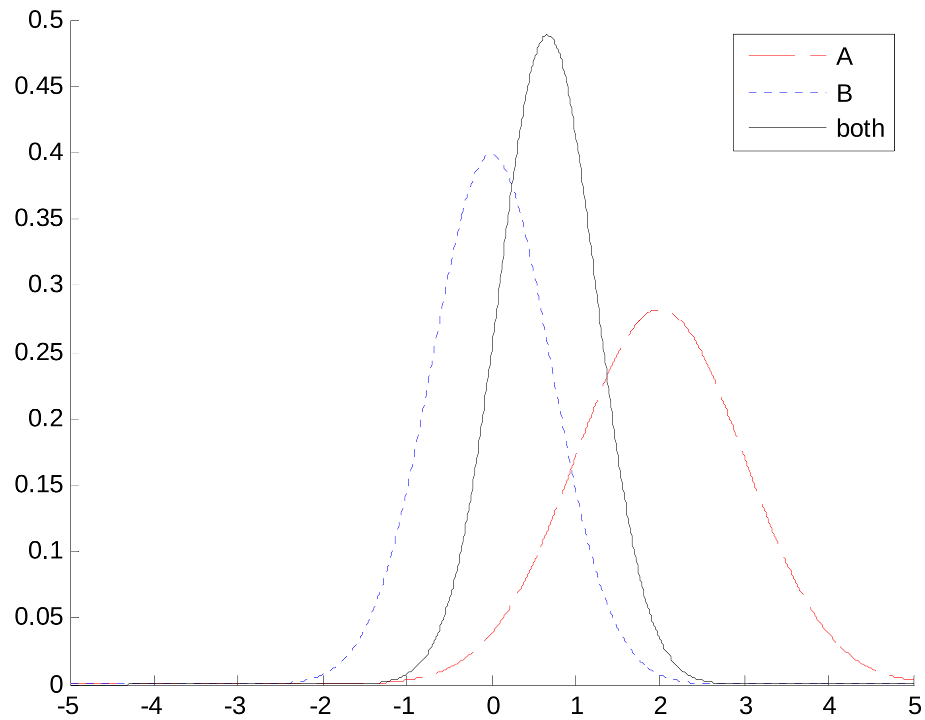


$$\frac{1}{\sigma^2} = \frac{1}{\sigma_A{}^2} + \frac{1}{\sigma_B{}^2}$$

$$\mu = \frac{\mu_A \sigma_B{}^2 + \mu_B \sigma_A{}^2}{\sigma_A{}^2 + \sigma_B{}^2}$$

Kalman Filter Example

- We now think Jill is at:
  - $x=0.66$
  - $\sigma^2 = 0.66$



Kalman Filter: Properties

- You incorporate sensor observations one at a time.
- Each successive observation is the same amount of work (in terms of CPU).
- *Yet, the final estimate is the* ***global*** *optimal solution.*

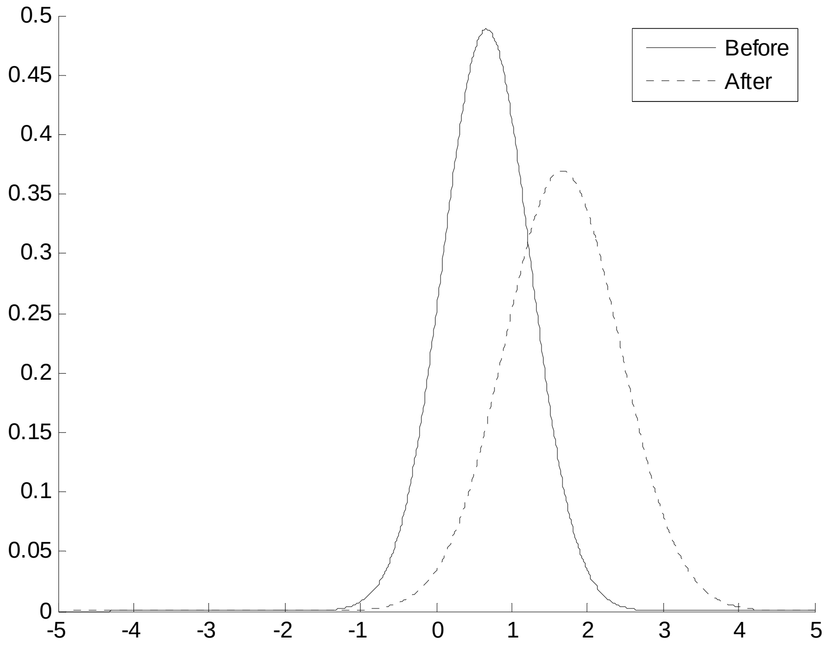
The Kalman Filter is an *optimal,* recursive estimator.

Kalman Filter: Properties

- Observations *always* reduce the uncertainty.

Kalman Filter

- Now Jill steps forward one step
- We think one of Jill's steps is about 1 meter, $\sigma^2 = 0.5$
- We estimate her position:
  - $x = x_{before} + x_{change}$
  - $\sigma^2 = \sigma_{before}^2 + \sigma_{change}^2$
- Uncertainty *increases*



State Vector

- We're going to estimate robot location and orientation (x_r, x_y, x_t), and feature locations (f_{nx}, f_{ny}).

$$x = [\ x_r\ x_y\ x_t\ f_{1x}\ f_{1y}\ f_{2x}\ f_{2y}\ \ldots\ f_{nx}\ f_{ny}\ ]^T$$

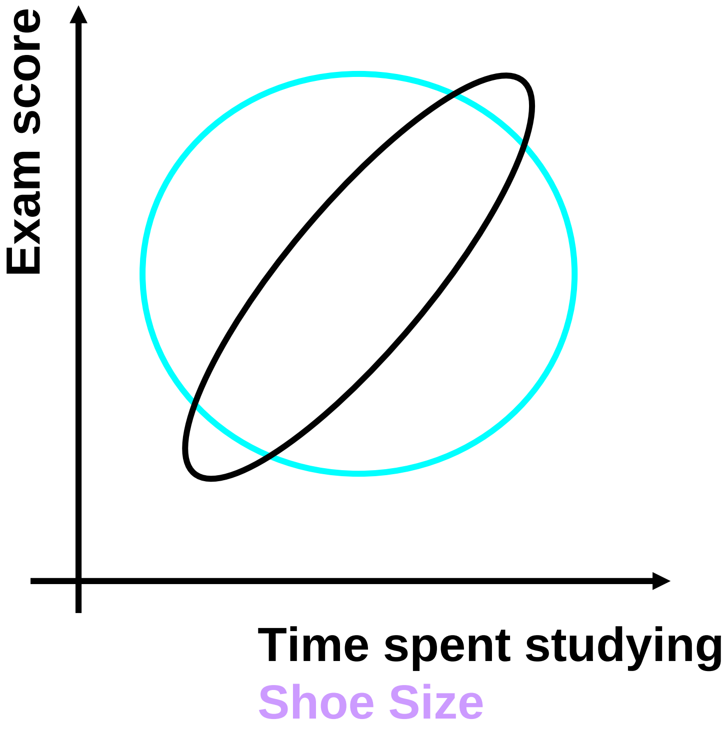
- We *could* try to estimate each of these variables independently
  - But they're correlated!

State Correlation/Covariance

- We observe features relative to the robot's current position
  - Therefore, feature location estimates *covary* (or correlate) with robot pose.
- Why do we care?
  - We need to track covariance so we can correctly propagate new information:
  - Re-observing one feature gives us information about robot position, and therefore also all other features.

Correlation/Covariance

- In multidimensional Gaussian problems, equal-probability contours are ellipsoids.
- Shoe size doesn't affect grades:
  P(grade,shoesize)=P(grade)P(shoesize)
- Studying helps grades:
  P(grade,studytime)!=P(grade)P(studytime)
  - We must consider P(x,y) jointly, respecting the correlation!
  - If I tell you the grade, you learn something about study time.



Kalman Filters and Multi-Gaussians

- We use a Kalman filter to estimate the whole state vector *jointly*.

  $$x = [\ x_r\ x_y\ x_t\ f_{1x}\ f_{1y}\ f_{2x}\ f_{2y}\ \ldots\ f_{nx}\ f_{ny}\ ]^T$$

- State vector has N elements.
- We don't have a scalar variance σ^2, we have NxN *covariance matrix* Σ.
  - Element (i,j) tells us how the uncertainties in feature i and j are related.

Kalman Filters and Multi-Gaussians

- Kalman equations tell us how to incorporate observations
  - Propagating effects due to correlation
- Kalman equations tell us how to add new uncertainty due to robot moving
  - We choose a Gaussian noise model for this too.

System Equations (EKF)

- Consider range/bearing measurements, differentially driven robot
- Let $x_k=f(x_{k-1},u_{k-1}, w_{k-1})$ u=control inputs, w=noise
- Let $z_k=h(x_k,v_k)$ v=noise

$$f = \begin{pmatrix} x' = x + (u_d + w_d)\cos(\theta + w_\theta) \\ y' = y + (u_d + w_d)\sin(\theta + w_\theta) \\ \theta' = \theta + u_\theta + w_\theta \end{pmatrix}$$

$$h = \begin{pmatrix} z_d = [(x_f - x_r)^2 + (y_f - y_r)^2]^{1/2} + v_d \\ z_\theta = \arctan 2(y_f - y_r, x_f - x_r) - x_\theta + v_\theta \end{pmatrix}$$

EKF Update Equations

- Time update:
  - x'=f(x,u,0)
  - $P=APA^T+WQW^T$
- Observation
  - $K=PH^T(HPH^T + VRV^T)^{-1}$
  - x'=x+K(z-h(x,0))
  - P=(I-KH)P

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- P is your covariance matrix
- They look scary, but once you compute your Jacobians, it just works!
  - A=df/dx W=df/dw H=dh/dx V=dh/dv
  - Staff can help… (It's easy except for the atan!)

EKF Jacobians

$$f = \begin{pmatrix} x' = x + (u_d + w_d)\cos(\theta + w_\theta) \\ y' = y + (u_d + w_d)\sin(\theta + w_\theta) \\ \theta' = \theta + u_\theta + w_\theta \\ x_1' = x_1 \\ y_1' = y_1 \end{pmatrix}$$

$$d = [(x_f - x_r)^2 + (y_f - y_r)^2]^{1/2}$$
$$d_x = x_f - x_r$$
$$d_y = y_f - y_r$$

$$A = \begin{vmatrix} 1 & 0 & -u_d \sin(\theta) & 0 & 0 \\ 0 & 1 & u_d \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$W = \begin{vmatrix} \cos(\theta) & -u_d \sin(\theta) \\ \sin(\theta) & u_d \cos(\theta) \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{vmatrix}$$

$$Q = \begin{vmatrix} {\sigma_{w_d}}^2 & 0 \\ 0 & {\sigma_{w_\theta}}^2 \end{vmatrix}$$

EKF Jacobians

$$h = \begin{pmatrix} z_d = [(x_f - x_r)^2 + (y_f - y_r)^2]^{1/2} + v_d \\ z_\theta = \arctan 2(y_f - y_r, x_f - x_r) - x_\theta + v_\theta \end{pmatrix}$$

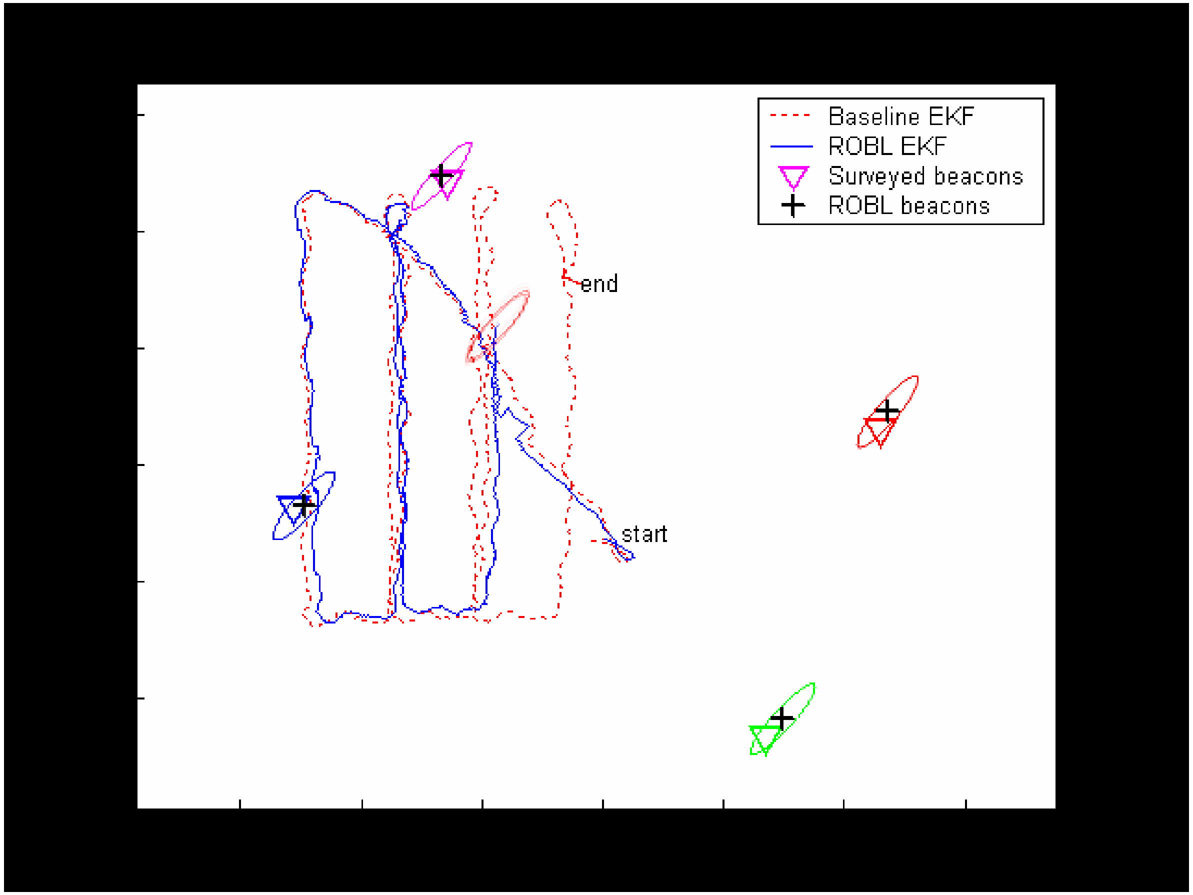
$$\lambda = 1/(1 + (d_y / d_x)^2$$

$$H = \begin{vmatrix} -d_x / d & -d_y / d & 0 & d_x / d & d_y / d \\ \lambda d_y / d_x{}^2 & -\lambda / d_x & -1 & -\lambda d_y / d_x{}^2 & \lambda / d_x \end{vmatrix} \qquad VRV^T = \begin{vmatrix} \sigma_{v_d}{}^2 & 0 \\ 0 & \sigma_{v_\theta}{}^2 \end{vmatrix}$$

Kalman Filter: Properties

- In the limit, features become highly correlated
  - Because observing one feature gives information about other features
- Kalman filter computes the *posterior pose*, but **not** the posterior *trajectory*.
  - If you want to know the path that the robot traveled, you have to make an extra “backwards” pass.

Kalman Filter: a movie

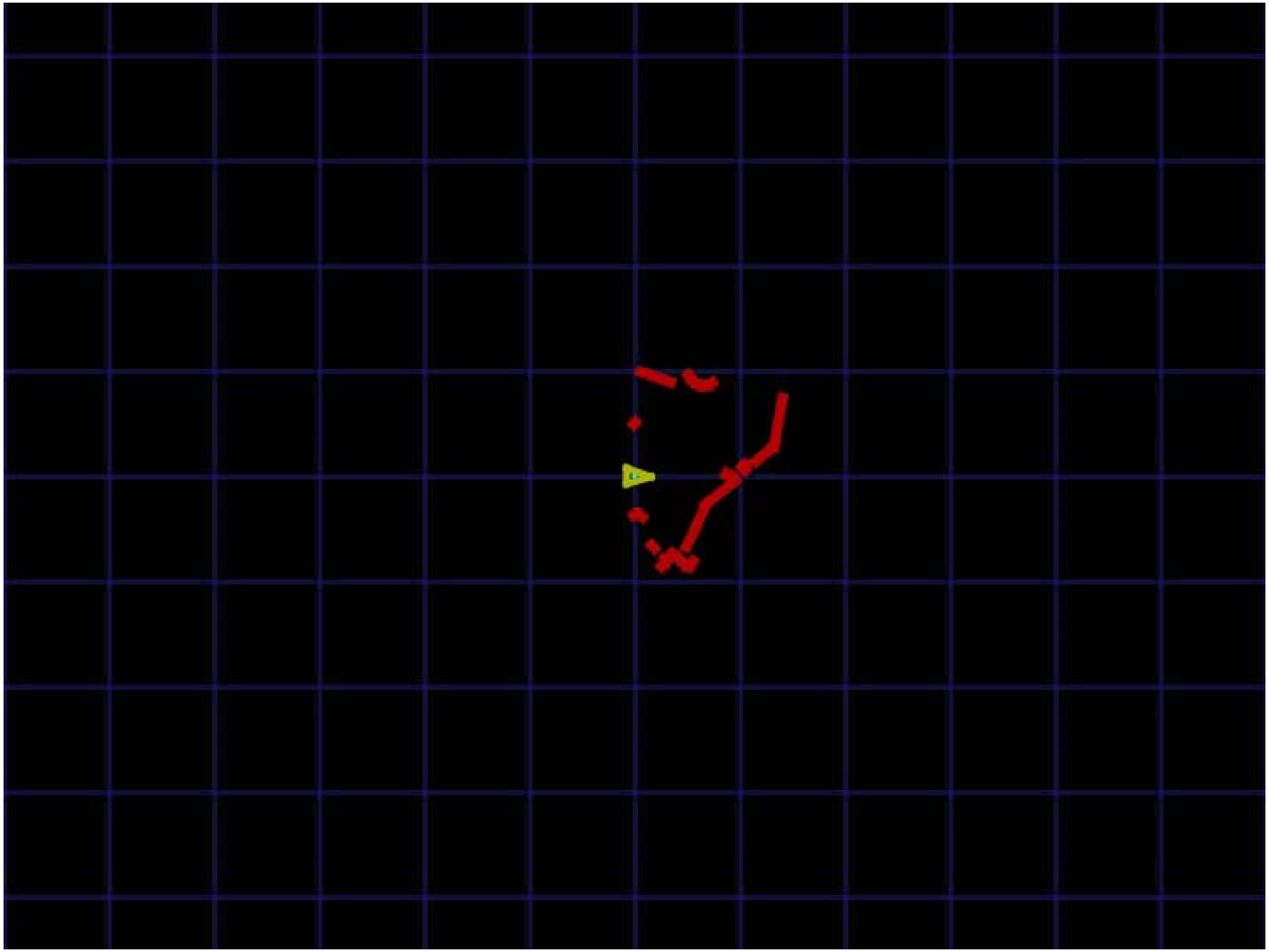


Kalman Filters’ Nemesis

- With N features, update time is $O(N^2)$!
- For Maslab, N is small. Who cares?
- In the “real world”, N can be 10^6.

- Current research: lower-cost mapping methods

Non-Bayesian Map Building



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    - **Occupancy Grids**
  - Topological
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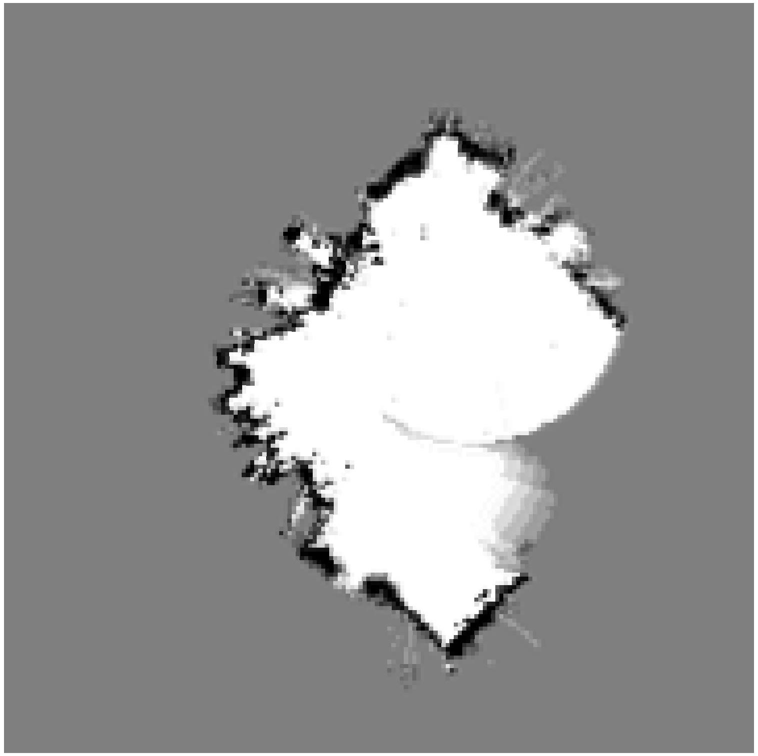
Occupancy Grids

- Another way of mapping:
- Divide the world into a grid
  - Each grid records whether there's something there or not
  - Use current robot position estimate to fill in squares according to sensor observations



Occupancy Grids

- Easy to generate, hard to maintain accuracy
  - Basically impossible to “undo” mistakes
- Occupancy grid resolution limited by the robot’s position uncertainty
  - Keep dead-reckoning error as small as possible
  - When too much error has accumulated, save the map and start over. Use older maps for reference?

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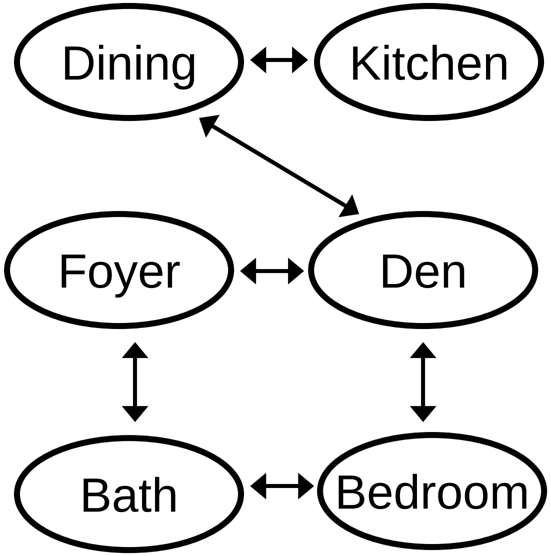
Topological Maps

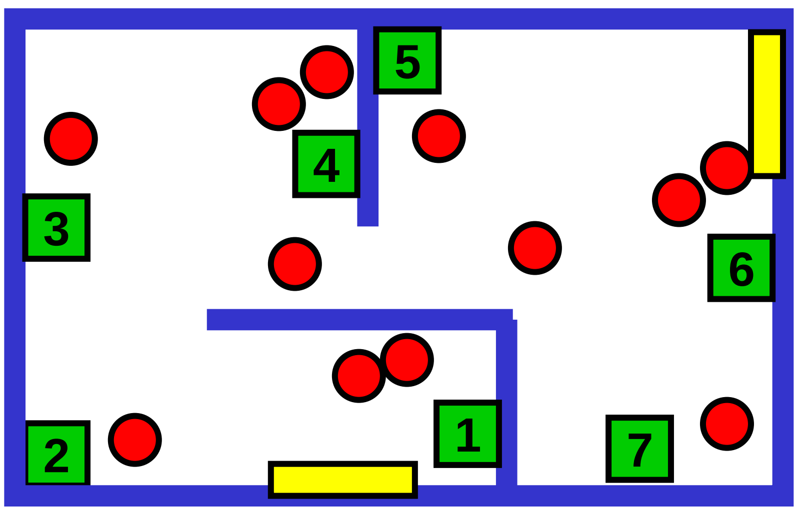
- Try to estimate how locations are related
- "There's an easy (straight) path between feature 1 and 2"
- Easy to build, easy to plan paths
- Only a partial representation of the world
  - Resulting paths are suboptimal

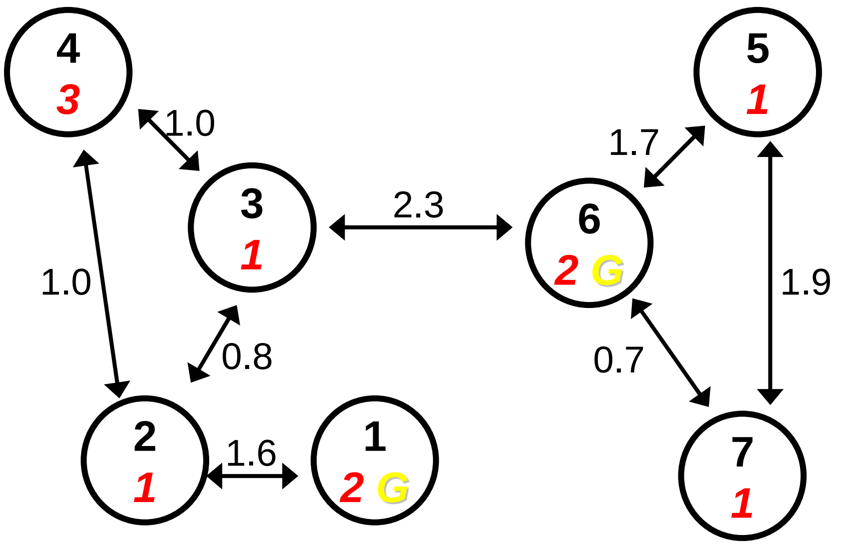


Topological Maps

- *Much* easier than this metrical map stuff.
- Don’t even try to keep track of *where* features are. Only worry about *connectivity.*

Topological Map Example





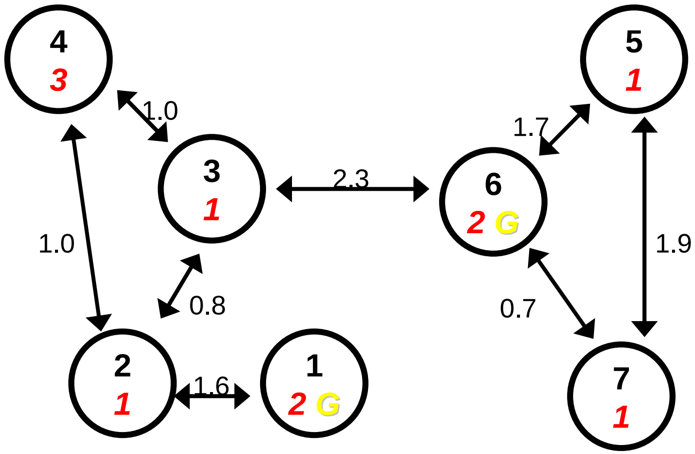
- Note that the way we draw (where we draw the nodes) does not contain information.

Topological Map-Building Algorithm

- Until exploration round ends:
  - Explore until we find a previously unseen barcode
  - Travel to the barcode
  - Perform a 360 degree scan, noting the barcodes, balls, and goals which are visible.
  - Build a tree
    - Nodes = barcode features
    - Edges connect features which are “adjacent”
    - Edge weight is distance

Topological Maps: Planning

- Graph is easy to do process!
- If we're lost, go to nearest landmark.
  - Nodes form a "highway"
- Can find "nearest" goal, find areas of high ball density
  - A* Search



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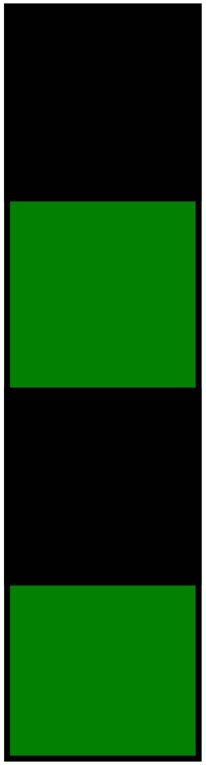
Data Association

- If we can’t tell when we’re reobserving a feature, we don’t learn anything!
  - We need to observe the same feature *twice* to generate a constraint

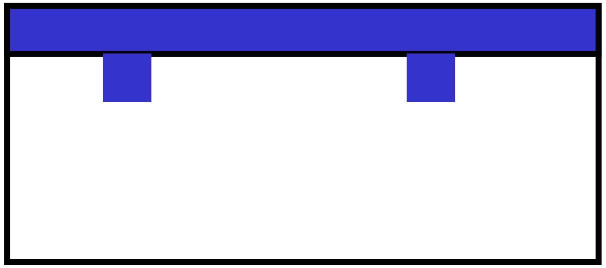
Data Association: Bar Codes

- Trivial!
- The Bar Codes have unique IDs; read the ID.



Data Association: Tick Marks

- The blue tick marks can be used as features too.
  - You only need to reobserve the same feature *twice* to benefit!
  - If you can track them over short intervals, you can use them to improve your dead-reckoning.

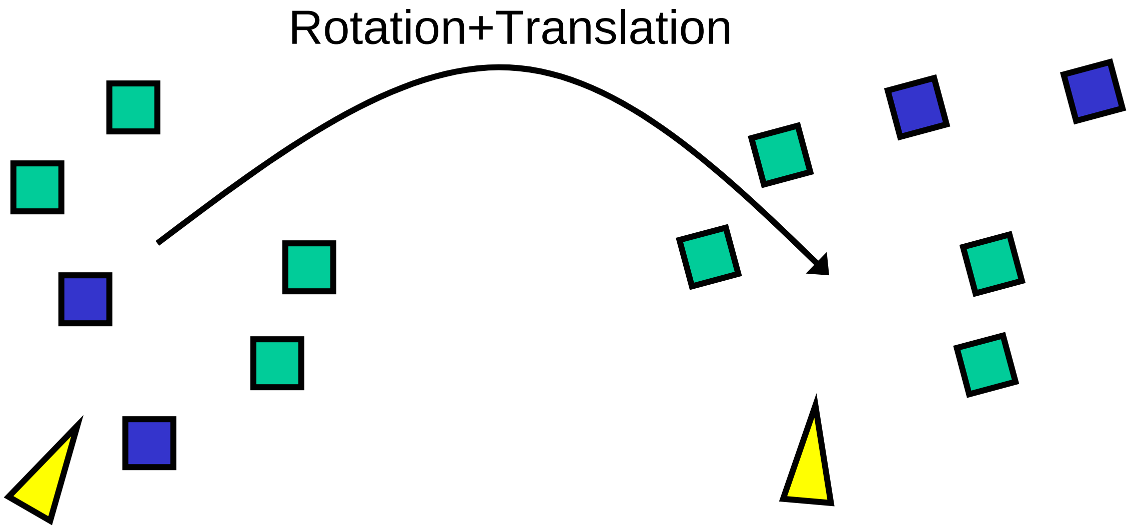


Data Association: Tick Marks

- Ideal situation:
  - Lots of tick marks, randomly arranged
  - Good position estimates on all tick marks
- Then we search for a *rigid-body-transformation* that best aligns the points.

Data Association: Tick Marks

- Find a rotation that aligns the most tick marks…
  - Gives you data association for matched ticks
  - Gives you rigid body transform for the robot!



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Using the exploration round

- Contest day:
  1. During exploration round, build a map.
  2. Write map to a file.

  3. During scoring round, reload the map.
  4. Score lots of points.

- Use two separate applications for explore/score rounds.
- Saving state to a file will ease testing:
  - You can test your scoring code without having to re-explore
  - You can hand-tweak the state file to create new test conditions or troubleshoot.

Debugging map-building algorithms

- You can’t debug what you can’t see.
- Produce a visualization of the map!
  - Metrical map: easy to draw
  - Topological map: draw the graph (using graphviz/dot?)
  - Display the graph via BotClient
- Write movement/sensor observations to a file to test mapping independently (and off-line)

Course Announcements

- Gyros:
  - Forgot to mention that your first gyro costs ZERO sensor points.
  - Gyro mounting issues: axis of rotation
- Lab checkoffs
  - Only a couple checkoffs yesterday

Today's Lab Activities

- No structured activities today
- Work towards tomorrow's check-off:
  1. Robot placed in playfield
  2. Find and approach a red ball.
  3. Stop.
- Keep it simple!
  - Random walks are *fine*!
  - Status messages must be displayed on OrcPad or BotClient