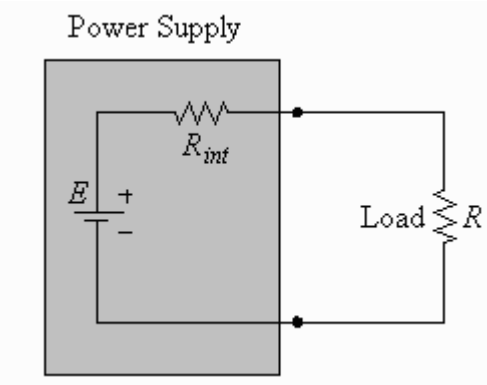

Maximum Power Transfer in a Circuit

Written by Eric Hiob, September 27, 1997

Problem

A power supply can be represented by an ideal voltage source of E volts in series with an internal resistance R_{int}. If a load is connected to the power supply, show that the maximum power that can be supplied to the load is achieved when the resistance R of the load is chosen to equal the internal resistance R_{int} of the power supply.

Solution

We will express the power P dissipated in the resistor R as a function of R and then find where this function has its maximum. The power P dissipated in a resistor R is given by the formula $P = I^2 R$, where I is the current flowing through the resistor. Expressing the current I as a function of R (using Ohm's law) gives:

$$I = \frac{E}{R + R_{\text{int}}}$$

Now we can express the power P as a function of R:

$$P = E^2 \frac{R}{\left(R + R_{\text{int}}\right)^2}$$

Notice that this function is zero when R is zero, is positive for all positive values of R, and decreases like $1/R$ when R is very large. Clearly P must have a maximum. Now calculate the derivative dP/dR:

$$\frac{dP}{dR} = E^2 \frac{\left(R + R_{\text{int}}\right)^2 \times 1 - R \times 2\left(R + R_{\text{int}}\right)^1 \times 1}{\left(R + R_{\text{int}}\right)^4}$$

$$= E^2 \frac{R_{\text{int}} - R}{\left(R + R_{\text{int}}\right)^3}$$

This derivative is equal to zero when $R = R_{int}$. From the above discussion it is clear that the derivative is zero here because this is the value of the resistance R at which the power P is a **maximum** (as opposed to say a minimum).
The above from: http://commons.bcit.ca/math/examples/elex/differential_calc/index.html

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Some practical applications of the Maximum Power Transfer principle include:

1. Transmitting telecommunications signals over long distances of cable requires impedance matching [source=load] to prevent standing waves on the cable as well as signal losses.

2. RF front-end design [radio receivers] depends on impedance matching to the antenna, and to the first amplifier stage for maximum signal power transfer to keep the signal as strong as possible with respect to noise generated in the resistors and the amplifier.

3. Extracting the maximum power available from a photovoltaic array.

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Please see Fig. 2-13 in Turner, Rufus P. *Impedance*. New York, NY: Tab Books, 1976.