Problem Set 5 Solutions

6.101 Analog Electronics Lab

April 9, 2007

Problem 1 (a)

$$i_1 = \frac{v_1}{R_1} = i_2 = -\frac{v_o}{R_2} Rightarrow \frac{v_o}{v_1} = -\frac{R_2}{R_1}$$

(b)

$$i_2 = i_1 = \frac{v_1}{R_1} = i_3 + \frac{v_o}{R_L} = i_3 + \frac{1}{R_L}\left(-\frac{R_2}{R_1}v_1\right)$$

Then,

$$i_3 = \frac{v_1}{R_1}\left(1 + \frac{R_2}{R_L}\right)$$

Problem 2 (1)

$$\frac{v_{I1} - v_A}{R_1 + R_2} = \frac{v_A - v_B}{R_v} + \frac{v_A - v_o}{R_2}$$

(2)

$$\frac{v_{I2} - v_B}{R_1 + R_2} = \frac{v_B - v_A}{R_v} + \frac{v_B}{R_2}$$

(3)

$$v_- = \left(\frac{R_1}{R_1 + R_2}\right) v_A + \left(\frac{R_2}{R_1 + R_2}\right) v_{I1}$$

(4)

$$v_+ = \left(\frac{R_1}{R_1 + R_2}\right) v_B + \left(\frac{R_2}{R_1 + R_2}\right) v_{I2}$$

Now, $v_- = v_+ \Rightarrow R_1 v_A + R_2 v_{I1} = R_1 v_B + R_2 v_{I2}$
So that $v_a = v_B + \frac{R_2}{R_1}(v_{I2} - v_{I1})$
(1)

$$\frac{v_{I1}}{R_1 + R_2} = v_A\left(\frac{1}{R_1 + R_2} + \frac{1}{R_v} + \frac{1}{R_2}\right) - \frac{v_B}{R_v} - \frac{v_o}{R_2}$$

(2)

$$\frac{v_{I2}}{R_1 + R_2} = v_B\left(\frac{1}{R_1 + R_2} + \frac{1}{R_v} + \frac{1}{R_2}\right) - \frac{v_A}{R_v}$$

Then,
(1)

$$\frac{v_{I1}}{R_1+R_2} = v_B\left(\frac{1}{R_1+R_2}+\frac{1}{R_v}+\frac{1}{R_2}\right)-\frac{v_B}{R_v}-\frac{v_o}{R_2}+\left(\frac{R_2}{R_1}\right)\left(\frac{1}{R_1+R_2}+\frac{1}{R_v}+\frac{1}{R_2}\right)(v_{I2}-v_{I1})$$

(2)

$$\frac{v_{I2}}{R_1+R_2} = v_B\left(\frac{1}{R_1+R_2}+\frac{1}{R_v}+\frac{1}{R_2}\right)-\frac{1}{R_v}\left[v_B+\frac{R_2}{R_1}(v_{I2}-v_{I1})\right]$$

Subtract (2) from (1)

$$\frac{1}{R_1+R_2}(v_{I1}-v_{I2}) = \frac{R_2}{R_1}\left(\frac{1}{R_1+R_2}+\frac{1}{R_v}+\frac{1}{R_2}\right)(v_{I2}-v_{I1})-\frac{v_o}{R_2}+\frac{1}{R_v}\frac{R_2}{R_1}(v_{I2}-v_{I1})$$

$$\frac{v_o}{R_2} = (v_{I2}-v_{I1})\left[\left(\frac{R_2}{R_1}\right)\left(\frac{1}{R_1+R_2}+\frac{1}{R_v}+\frac{1}{R_2}\right)+\frac{1}{R_1+R_2}+\frac{1}{R_v}\frac{R_2}{R_1}\right]$$

$$v_o = (v_{I2}-v_{I1})\left(\frac{R_2}{R_1}\right)\left[\frac{R_2}{R_1+R_2}+\frac{R_2}{R_v}+1+\frac{R_1}{R_1+R_2}+\frac{R_2}{R_v}\right]$$

$$v_o = \frac{2R_2}{R_1}\left(1+\frac{R_2}{R_v}\right)(v_{I2}-v_{I1})$$

Problem 3 (a)

$$v_{OB} = \left(1+\frac{40}{12}\right)v_I = 2.1667\sin\omega t$$

(b)

$$v_{OC} = -\frac{30}{12}v_I = -1.25\sin\omega t$$

(c)

$$V_o = v_{OB}-v_{OC} = 3.417\sin\omega t$$

(d)

$$\frac{v_o}{v_i} = \frac{3.417}{0.5} = 6.83$$

Problem 4

$$v_o = \left(\frac{333}{20}\right)(v_{o1}-v_{o2}) = 16.65(v_{o1}-v_{o2})$$

$$v_{o1} = v_{BE1} = -V_T\ln\left(\frac{i_{C1}}{I_S}\right)$$

$$v_{o2} = v_{BE2} = -V_T \ln\left(\frac{i_{C2}}{I_S}\right)$$

$$v_{o1} - v_{o2} = -V_T \ln\left(\frac{i_{C1}}{i_{C2}}\right) = V_T \ln\left(\frac{i_{C2}}{i_{C1}}\right)$$

$$i_{C2} = \frac{v_2}{R_2} \ , \ i_{C2} = \frac{v_1}{R_1}$$

So $v_{o1} - v_{o2} = V_T \ln\left(\frac{v_2}{R_2}\frac{R_1}{v_1}\right)$. Then,

$$v_o = (16.65)(0.026) \ln\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$$

$$v_o = 0.4329 \ln\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$$

$$v_o = \log_{10}\left(\frac{v_2}{v_1}\frac{R_1}{R_2}\right)$$