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6.055J / 2.038J The Art of Approximation in Science and Engineering
Spring 2008

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6.055J/2.038J (Spring 2008)

Homework 6

Do the following warmups and problems. Due in class on **Friday, 09 May 2008**.

Open universe: Collaboration, notes, and other sources of information are **encouraged**. However, avoid looking up answers until you solve the problem (or have tried hard). That policy helps you learn the most from the problems.

Bring a photocopy to class on the due date, trade it for a solution set, and figure out or ask me about any confusing points. Your work will be graded lightly: P (made a reasonable effort), D (did not make a reasonable effort), or F (did not turn in).

Warmups

1. Integrals

Use special cases of a to choose the correct formula for each integral.

a. $\int_{-\infty}^{\infty} e^{-ax^2} dx$

(1.) $\sqrt{\pi a}$ (2.) $\sqrt{\pi/a}$

b. $\int_{-\infty}^{\infty} \frac{1}{a^2 + x^2} dx$

(1.) πa (2.) π/a (3.) $\sqrt{\pi a}$ (4.) $\sqrt{\pi/a}$

2. Debugging

Use special (i.e. easy) cases of n to decide which of these two C functions correctly computes the sum of the first n odd numbers:

```
int sum_of_odds (int n) {
    int i, total = 0;
    for (i=1; i<=2*n+1; i+=2)
        total += i;
    return total;
}
```

or

```
int sum_of_odds (int n) {
    int i, total = 0;
    for (i=1; i<=2*n-1; i+=2)
        total += i;
    return total;
}
```

3. Reynolds numbers

Estimate the Reynolds number for:

- a falling raindrop;
- a flying mosquito;

4. Drag at low Reynolds number

At low Reynolds number, the drag on a sphere is

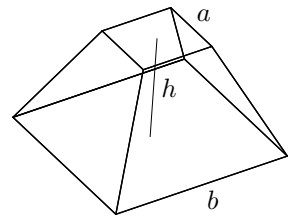
$$F = 6\pi\rho\nu r.$$

What is the drag coefficient c_d as a function of Reynolds number Re ?

Problems**5. Truncated pyramid**

In this problem you use special cases to find the volume of a truncated pyramid. It has a square base with side b , a square top with side a , and height h . So, use special cases of a and b to evaluate these candidates for the volume:

- $\frac{1}{3}hb^2$
- $\frac{1}{3}ha^2$
- $\frac{1}{3}h(a^2 + b^2)$
- $\frac{1}{2}h(a^2 + b^2)$



Which if any of these formulas pass all your special-cases tests? If no formula passes all tests, invent a formula that does. If you are stuck, find the volume by integration!

6. Fog

- Estimate the terminal speed of fog droplets (radius $\sim 10\ \mu\text{m}$). Use either the low- or high-Reynolds-number limit for the drag force, whichever you guess is the more likely to be valid.
- Use the speed to estimate the Reynolds number and check that you used the correct limit for the drag force. If not, try the other limit!
It is much less than 1, so the original assumption of low-Reynolds-number flow is okay.
- Fog is a low-lying cloud. How long would fog droplets take to fall 1 km (the height of a typical cloud)? What is the everyday effect of this settling time?

7. Tube flow

In this problem you study fluid flow through a narrow tube. The quantity to predict is Q , the volume flow rate (volume per time). This rate depends on:

l	the length of the tube
Δp	the pressure difference between the tube ends
r	the radius of the tube
ρ	the density of the fluid
ν	the kinematic viscosity of the fluid

- a. Find three independent dimensionless groups G_1 , G_2 , and G_3 from these six variables. *Hint 1:* One physically reasonable group is $G_2 = r/l$. *Hint 2:* Put Q in G_1 only! Then write the general form

$$G_1 = f(G_2, G_3).$$

[There are lots of choices for G_1 and G_3 .]

- b. Now imagine that the tube is very long and thin ($l \gg r$) and that the radius or flow speed are small enough to make the Reynolds number low. Then you can deduce the form of f using proportional reasoning.

You might think about these proportionalities:

1. How should Q depend on Δp ? For example, if you double the pressure difference, what should happen to the flow rate?
2. How should Q depend on l ? For example, if you keep the pressure difference the same but double the tube length, what happens to Q ? Or if you double Δp and double l , what happens to Q ?

Figure out the form of f to satisfy all your proportionality requirements.

If you get stuck going forward, instead work backward from the correct result. Look up Poiseuille flow, and use this result to deduce the preceding proportionalities; and then give reasons for why they are that way.

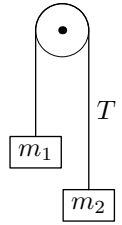
- c. *[optional]*

The dimensional analysis in the preceding parts does not tell you the dimensionless constant. Use a syringe and needle to estimate the constant. Compare your constant with the value of $\pi/8$ that comes from solving the equations of fluid mechanics honestly.

8. Atwood machine: Tension in the string

Here is the Atwood machine from lecture. The string and pulley are massless and frictionless. We used dimensional analysis and special cases to guess the acceleration of either mass. With the right choice of sign,

$$\frac{a}{g} = \frac{m_1 - m_2}{m_1 + m_2}.$$



In this problem you guess the tension in the string.

- The tension T , like the acceleration, depends on m_1 , m_2 , and g . Explain why these four variables result in two independent dimensionless groups.
- Choose two suitable independent dimensionless groups so that you can write an equation for the tension in this form:

$$\text{dimensionless group containing } T = f(\text{dimensionless group without } T).$$

The next part will be easier if you use a lot of symmetry in choosing the groups.

- Use special cases to guess f , and sketch f .
- Solve for T using the usual methods from introductory physics (8.01); then compare that answer with your answer from the preceding part.

Optional

9. Plant-watering system

The semester is over and you are going on holiday for a few weeks. But how will you water the house plants?! Design an unpowered slow-flow system to keep your plants happy.

10. Dimensional analysis for circuits

- Using Q as the dimension of charge, what are the dimensions of inductance L , capacitance C , and resistance R ?
- Show that the dimensions of L , C , and R contain two independent dimensions.
- In a circuit with one inductor, one capacitor, and one resistor, one dimensionless group should result from the three component values L , R , and C . What is physical interpretation of this group?