Mathematics for Computer Science

MIT 6.042J/18.062J

Bijections for Counting

Example: Counting Passwords

Password conditions:

- characters are digits & letters
- between 6 & 8 characters long
- starts with a letter
- case sensitive

Counting Passwords

$L ::= \{a,b,\ldots,z,A,B,\ldots,Z\}$

$D ::= \{0,1,\ldots\ldots,9\}$

$P_n ::=$ length n words starting w/letter

$$= L \times (L \cup D)^{n-1}$$

Counting Passwords

$$|L \times (L \cup D)^{n-1}|$$
$$= |L| \cdot |(L \cup D)|^{n-1}$$
$$= |L| \cdot (|L| + |D|)^{n-1}$$
$$= 52 \cdot (52+10)^{n-1}$$

Counting Passwords

set of passwords:

$P ::= P_6 \cup P_7 \cup P_8$

$|P| = |P_6| + |P_7| + |P_8|$

$= 52 \cdot (62^5 + 62^6 + 62^7)$

$\approx 19 \cdot 10^{14}$

4-digit nums w/ $\geq$ one 7

cases by 1st occurrence of 7:

x: any digit o: any digit $\neq 7$

7xxx or o7xx or oo7x or ooo7

$10^3 + 9 \cdot 10^2 + 9^2 \cdot 10 + 9^3$

$= 3439$

at least one 7: another way

|4-digit nums w/ $\geq$ one 7|

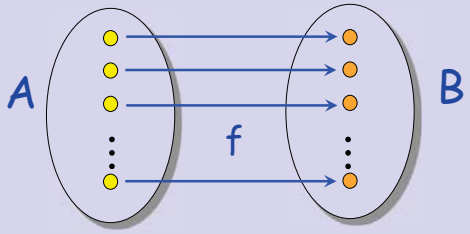
= |4-digit nums|

— |those w/ no 7|

= $10^4 - 9^4 = 3439$

Mapping Rule: Bijections

If f is a bijection from A to B, then |A| = |B|



Bijection: $\mathcal{P}(A)$ and Binary Strings

A: $\{a_1, a_2, a_3, a_4, a_5, \ldots, a_n\}$

subset: $\{a_1, \quad a_3, a_4, \quad \ldots, a_n\}$

string: 1 0 1 1 0 ... 1

This is a bijection, so

$$\underbrace{|n\text{-bit binary strings}|}_{2^n} = |\mathcal{P}(A)|$$

Size of P(A)

$$|\mathcal{P}(A)| = 2^{|A|}$$

Counting Doughnut Selections

From 5 kinds of doughnuts select a dozen.

let A ::= all selections of 12 doughnuts

$\underbrace{00}_{\text{chocolate}}$ $\underbrace{(\text{none})}_{\text{lemon}}$ $\underbrace{000000}_{\text{sugar}}$ $\underbrace{00}_{\text{glazed}}$ $\underbrace{00}_{\text{plain}}$

Counting Doughnut Selections

B::= 16-bit words with four 1's

0011000000100100

$\underbrace{00}_{\text{chocolate}}\,1\;\underbrace{\;}_{\text{lemon}}\;1\underbrace{000000}_{\text{sugar}}\,1\,\underbrace{00}_{\text{glazed}}\,\underbrace{100}_{\text{plain}}$

Bijection from A to B

c chocolate, l lemon, s sugar, g glazed, p plain

maps to

$$0^c10^l10^s10^g10^p$$

so $|A| = |B|$

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