Slide 1

Mathematics for Computer Science
MIT 6.042J/18.062J

Simple Graphs:
k-Connectivity

Slide 2

Edge Connectedness
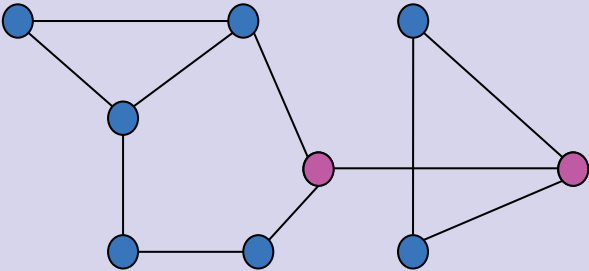
Def: vertices v, w are
k-edge connected
if they remain connected
whenever fewer than k
edges are deleted.

Slide 3
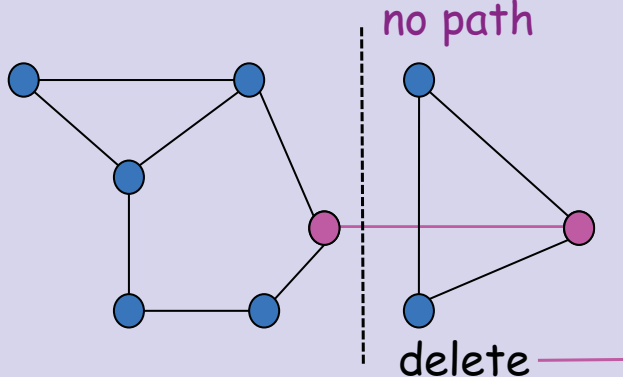
k-edge Connectedness

1-edge connected

Slide 4

k-edge Connectedness

no path

delete ——

1-edge connected

1
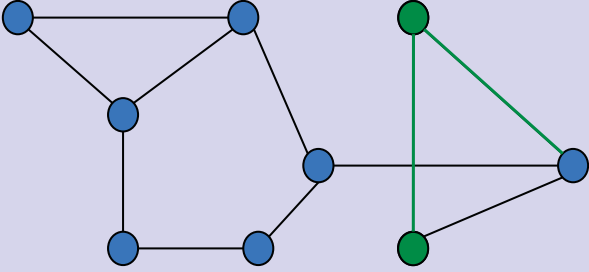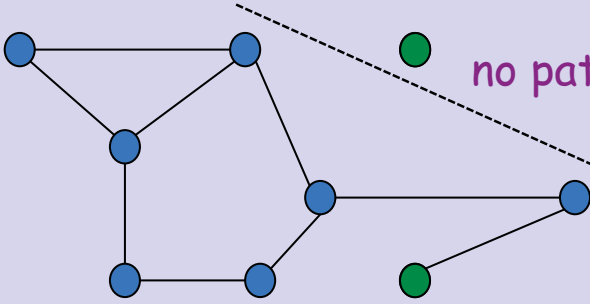
Edge Connectedness

2-edge connected

Albert R Meyer, April 5, 2013

k-connect.5

Edge Connectedness
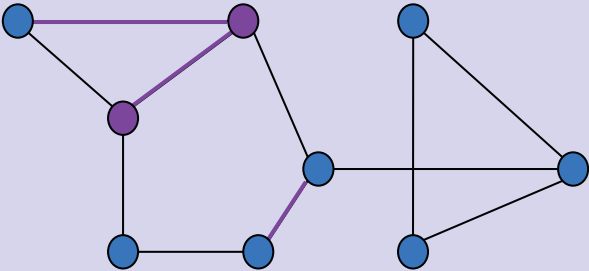
no path

2-edge connected

Albert R Meyer, April 5, 2013

k-connect.6
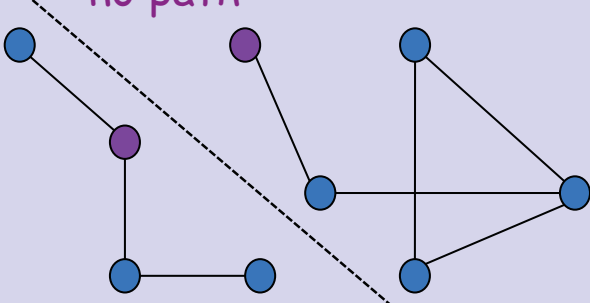
Edge Connectedness

3-edge connected

Albert R Meyer, April 5, 2013

k-connect.7

Edge Connectedness

no path

3-edge connected

Albert R Meyer, April 5, 2013

k-connect.8

k-edge Connectedness

Def: A graph is
    k-edge connected
iff every two vertices
are k-edge connected.

Edge Connectedness

Connectivity measures fault tolerance of a network: how many connections can fail without cutting off communication?

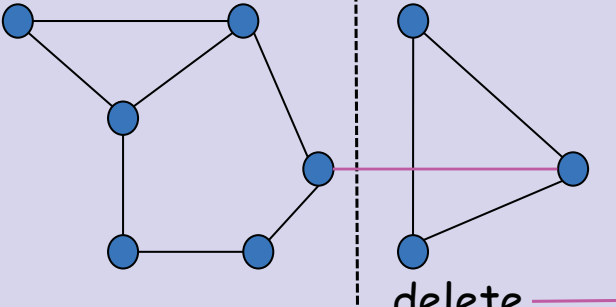**k-edge Connectedness**
this whole graph is

delete ——
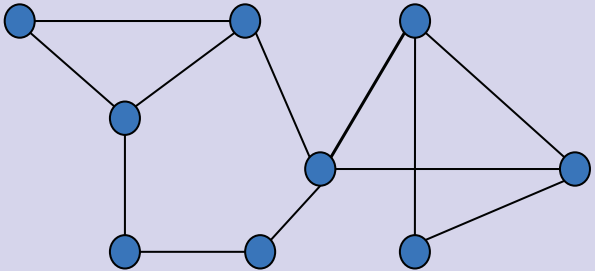
1-edge connected

k-edge Connectedness
this whole graph is

2-edge connected

k-vertex Connectedness

k-vertex connectedness defined similarly

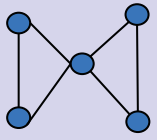**k-vertex Connectedness**

k-vertex connected

IMPLIES

k-edge connected
not conversely:

2-edge connected
1-vertex connected

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k-vertex Connectedness

K_n is the complete graph on n vertices.
K_n is $(n-1)$-vertex connected.

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k-vertex Connectedness

The n-dimensional hypercube H_n

$V(H_n) ::= \{0,1\}^n$

$\langle u - v \rangle$ an edge IFF u, v differ in 1 place

k-vertex Connectedness

H_n is n-vertex connected.
(class problem)

Menger's Theorem

k-connected vertices will be connected by k non-overlapping paths

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