

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Mathematics for Computer Science
 MIT 6.042J/18.062J

Simple Graphs: k-Connectivity

Albert R Meyer, April 5, 2013

k-connect.1

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Edge Connectedness

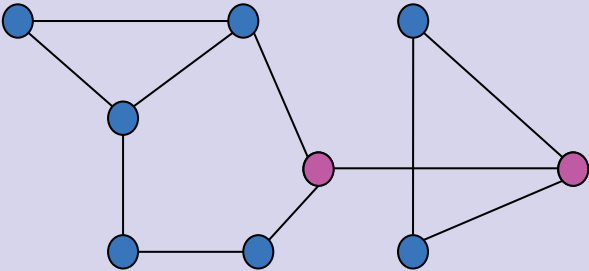
Def: vertices v, w are
k-edge connected
 if they remain connected
 whenever **fewer than k**
 edges are deleted.

Albert R Meyer, April 5, 2013

k-connect.2

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

k-edge Connectedness



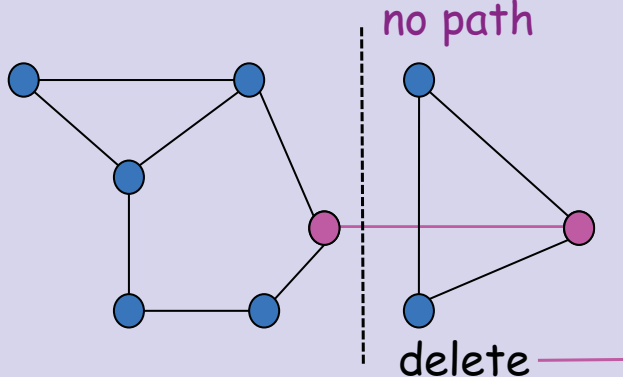
1-edge connected

Albert R Meyer, April 5, 2013

k-connect.3

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

k-edge Connectedness



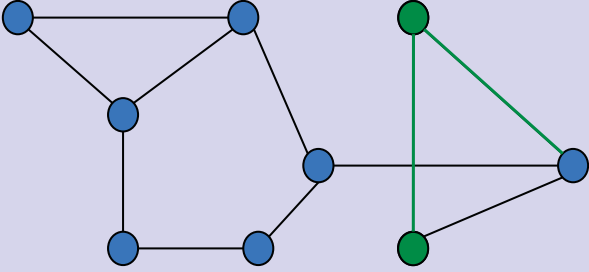
1-edge connected

Albert R Meyer, April 5, 2013

k-connect.4

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Edge Connectedness

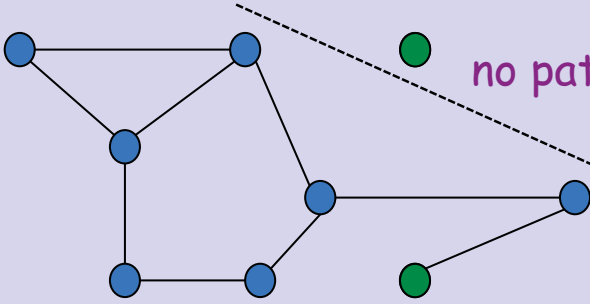


2-edge connected

Albert R Meyer, April 5, 2013 k-connect.5

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Edge Connectedness



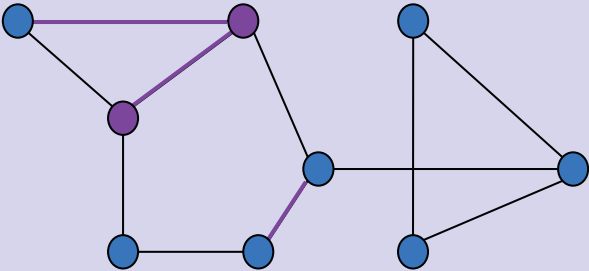
no path

2-edge connected

Albert R Meyer, April 5, 2013 k-connect.6

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Edge Connectedness

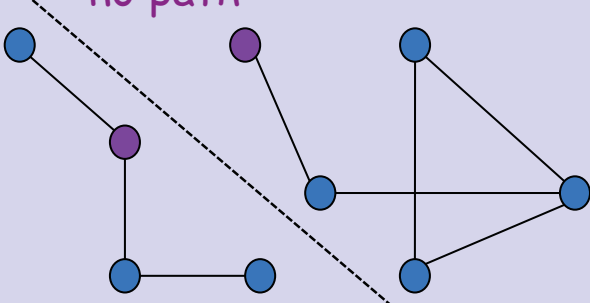


3-edge connected

Albert R Meyer, April 5, 2013 k-connect.7

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Edge Connectedness



no path


3-edge connected

Albert R Meyer, April 5, 2013 k-connect.8

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

k-edge Connectedness

Def: A graph is **k-edge connected** iff every two vertices are **k-edge connected**.




Albert R Meyer, April 5, 2013

k-connect.9

| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Edge Connectedness

Connectivity measures **fault tolerance** of a network:
how many connections can fail without cutting off communication?



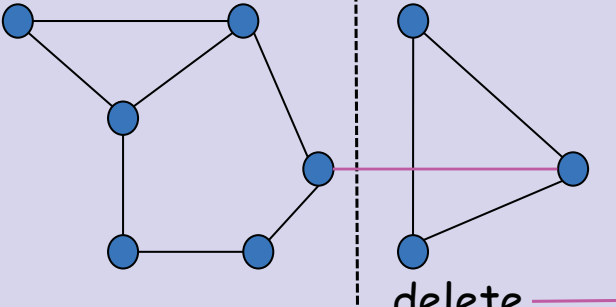
Albert R Meyer, April 5, 2013

k-connect.10


| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

k-edge Connectedness

this whole graph is



1-edge connected



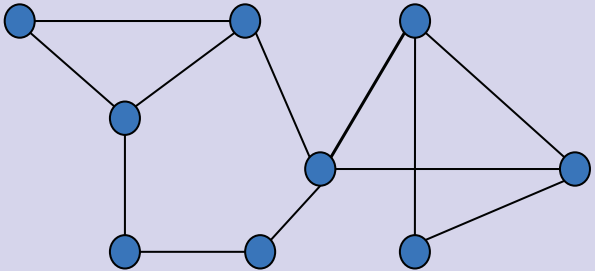
Albert R Meyer, April 5, 2013

k-connect.11


| | | | |
|----|----|----|----|
| 6 | 9 | 13 | 7 |
| 12 | 10 | 5 | |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

k-edge Connectedness

this whole graph is





2-edge connected




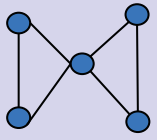
Albert R Meyer, April 5, 2013


k-connect.12



k-vertex Connectedness
k-vertex
connectedness
 defined similarly



 Albert R Meyer, April 5, 2013 k-connect.13



k-vertex Connectedness
k-vertex connected
 IMPLIES
k-edge connected
 not conversely:



2-edge connected
1-vertex connected


 Albert R Meyer, April 5, 2013 k-connect.14


k-vertex Connectedness
 K_n is the **complete**
 graph on n vertices.
 K_n is $(n-1)$ -vertex
 connected.


 Albert R Meyer, April 5, 2013 k-connect.15


k-vertex Connectedness
 The **n -dimensional**
hypercube H_n
 $V(H_n) ::= \{0,1\}^n$
 $\langle u-v \rangle$ an edge IFF u,v
 differ in **1** place


 Albert R Meyer, April 5, 2013 k-connect.16

| | | |
|----|----|----|
| 6 | 13 | 7 |
| 12 | 10 | 5 |
| 3 | 1 | 14 |
| 15 | 8 | 11 |

k -vertex Connectedness

H_n is n -vertex
connected.
(class problem)



Albert R Meyer, April 5, 2013

k-connect.17

| | | |
|----|----|----|
| 6 | 13 | 7 |
| 12 | 10 | 5 |
| 3 | 1 | 14 |
| 15 | 8 | 11 |

Menger's Theorem

k -connected vertices
will be connected by k
 $non-overlapping$ paths



Albert R Meyer, April 5, 2013

k-connect.18

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