



Mathematics for Computer Science
MIT 6.042J/18.062J

Expected Number of Heads




Albert R Meyer, May 8, 2013 lec 12F.1




Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?


$$E[\# \text{ Heads}] = E[B_{n,p}]$$


Albert R Meyer, May 8, 2013 lec 12F.3




Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?


$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k (1-p)^{n-k}$$


Albert R Meyer, May 8, 2013 lec 12F.4




Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?


$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$


Albert R Meyer, May 8, 2013 lec 12F.5




Expected #Heads

Binomial theorem and differentiating gives a closed formula




Albert R Meyer, May 8, 2013 lec 12F.6




Binomial Expectation

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

take $\partial / \partial x$:


$$n(x + y)^{n-1} = \frac{1}{x} \sum_{k=0}^n k \binom{n}{k} x^k y^{n-k}$$


Albert R Meyer, May 8, 2013 lec 12F.7




Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$


$$n(x + y)^{n-1} = \frac{1}{x} \sum_{k=0}^n k \binom{n}{k} x^k y^{n-k}$$


Albert R Meyer, May 8, 2013 lec 12F.8



Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$

$$n(p + q)^{n-1} = \frac{1}{p} \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$



Albert R Meyer, May 8, 2013 lec 12F.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$

$$n = \frac{1}{p} \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$


Albert R Meyer, May 8, 2013
lec 12F.10


6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^n k \binom{n}{k} p^k q^{n-k}$$

$$n = \frac{1}{p} E[B_{n,p}]$$

$np = E[B_{n,p}]$


Albert R Meyer, May 8, 2013
lec 12F.11

MIT OpenCourseWare
<http://ocw.mit.edu>

6.042J / 18.062J Mathematics for Computer Science
Spring 2015

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.