Mathematics for Computer Science
MIT 6.042J/18.062J

Expected Number of Heads

Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?

$$E[\#\text{ Heads}]$$
$$= E[B_{n,p}]$$

Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?

$$E[B_{n,p}] ::= \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$$

Expected #Heads

n independent flips of a coin with bias p for Heads. How many Heads expected?

$$E[B_{n,p}] ::= \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}$$

Expected #Heads

Binomial theorem and differentiating gives a closed formula

Binomial Expectation

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

take $\partial / \partial x$:

$$n(x+y)^{n-1} = \frac{1}{x} \sum_{k=0}^{n} k \binom{n}{k} x^k y^{n-k}$$

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}$$

$$n(x+y)^{n-1} = \frac{1}{x} \sum_{k=0}^{n} k \binom{n}{k} x^k y^{n-k}$$

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}$$

$$n(p+q)^{n-1} = \frac{1}{p} \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}$$

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}$$

$$n \qquad = \frac{1}{p} \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}$$

Binomial Expectation

$$E[B_{n,p}] ::= \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}$$

$$n \qquad = \frac{1}{p} E[B_{n,p}]$$

$$np \quad E[B_{n,p}]$$