Recitation 15
April 20, 2006

1. Let X and Y be random variables, and let a and b be scalars; X takes nonnegative values.

   (a) Use the Markov inequality on the random variable e^{sY} to show that

   $$P(Y \geq b) \leq e^{-sb} M_Y(s),$$

   for every $s > 0$, where $M_Y(s)$ is the transform of Y.

2. Joe wishes to estimate the true fraction f of smokers in a large population without asking each and every person. He plans to select n people at random and then employ the estimator $F = S/n$, where S denotes the number of people in a size-n sample who are smokers. Joe would like to sample the minimum number of people, but also guarantee an upper bound p on the probability that the estimator F differs from the true value f by a value greater than or equal to d i.e., for a given accuracy d and given confidence p, Joe wishes to select the minimum n such that

   $$\mathbf{P}(|F - f| \geq d) \leq p \quad .$$

   For $p = 0.05$ and a particular value of d, Joe uses the Chebyshev inequality to conclude that n must be at least 50,000. Determine the new minimum value for n if:

   (a) the value of d is reduced to half of its original value.

   (b) the probability p is reduced to half of its original value, or $p = 0.025$.

3. Let $X_1, X_2, \ldots$ be a sequence of independent random variables that are uniformly distributed between 0 and 1. For every n, we let Y_n be the median of the values of $X_1, X_2, \ldots, X_{2n+1}$. [That is, we order $X_1, \ldots, X_{2n+1}$ in increasing order and let Y_n be the $(n+1)$st element in this ordered sequence.] Show that the sequence Y_n converges to 1/2, in probability.