Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Recitation 07 (Answers)
March 07, 2006

1. (a) $\mathbf{E}[X] = \frac{1}{\lambda}$, $\text{var}(X) = \frac{1}{\lambda^2}$ and $\mathbf{P}(X \geq \mathbf{E}[X]) = \frac{1}{e}$

   (b)

$$\mathbf{P}(X > t + k | X > t) \quad = \quad e^{-\lambda(k)}$$

   Note: the exponential random variable is memoryless

2. We first compute the CDF $F_X(x)$ and then obtain the PMF as follows

$$p_X(k) = \begin{cases} F_X(k) - F_X(k-1) & \text{if } k = 3, ...10, \\ 0, & \text{otherwise.} \end{cases}$$

   We have,

$$F_X(k) = \begin{cases} 0, & k < 3, \\ \frac{k}{10}\frac{k-1}{9}\frac{k-2}{8} & 3 \leq k \leq 10, \\ 1 & 10 \leq k. \end{cases}$$

3. (a)

$$\begin{aligned} \mathbf{P}(\text{error}) &= \mathbf{P}(R_1|S_0)\mathbf{P}(S_0) + \mathbf{P}(R_0|S_1)\mathbf{P}(S_1) \\ &= \mathbf{P}(Z - 1 > a)(p) + \mathbf{P}(Z + 1 < a)(1 - p) \\ &= p \cdot \left(1 - \Phi\left(\frac{a - (-1)}{\sigma}\right)\right) + (1 - p) \cdot \Phi\left(\frac{a - 1}{\sigma}\right) \\ &= p - p \cdot \Phi\left(\frac{a + 1}{\sigma}\right) + (1 - p) \cdot \left(1 - \Phi\left(\frac{1 - a}{\sigma}\right)\right) \\ &= 1 - p \cdot \Phi\left(\frac{a + 1}{\sigma}\right) - (1 - p) \cdot \Phi\left(\frac{1 - a}{\sigma}\right) \end{aligned}$$

   (b) $\mathbf{P}(\text{error}) = 1 - 0.4 \cdot \Phi(\frac{3/2}{1/2}) - 0.6 \cdot \Phi(\frac{1/2}{1/2})$