6.01 Midterm 2: Fall 2010

Name:	**Section:**

Solutions: Not correct for the make-up exam.

Enter all answers in the boxes provided.

During the exam you may:

- read any paper that you want to
- use a calculator

You may not

- use a computer, phone or music player

For staff use:

1.	/14
2.	/16
3.	/13
4.	/16
5.	/14
6.	/15
7.	/12
total:	/100

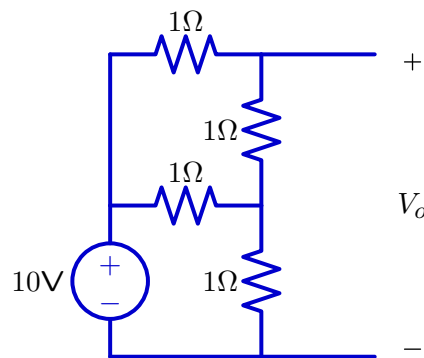# 1 Analyzing Circuits (14 points)

Determine the indicated parameter for each of the following circuits. Write your answers (either a number or numerical expression) in the box provided. For partial credit, you may also provide a brief explanation of your reasoning.
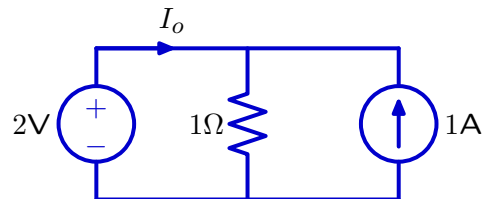
$$I_0 = 1A$$

Reasoning:

The voltage across the resistor is 2V, making the current through the resistor 2A. Applying KCL at the top node leads to $I_o + 1A = 2A$. So $I_o = 1A$.


$$V_0 = 8V$$

Reasoning:

Label the node at the center-right as V_1, then:
● KCL at top-right node: $(V_o - 10) + (V_o - V_1) = 0$.
● KCL at center-right node: $(V_1 - V_o) + (V_1 - 10) + V_1 = 0$.
Solve to find that $V_0 = 8V$ and $V_1 = 6V$.
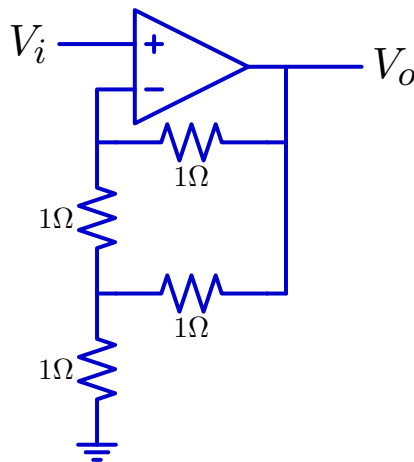
2 Op Amps (16 points)

Determine the indicated parameter for each of the following circuits. You may assume the ideal op-amp model. Write your answers (either a number or numerical expression) in the box provided. For partial credit, you may also provide a brief explanation of your reasoning.
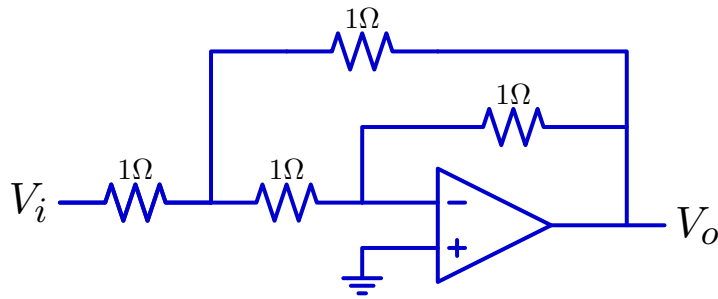
$$\frac{V_0}{V_i} = -\frac{1}{4}$$

Reasoning:

The ideal op-amp model determines $V_- = 0$, because $V_+ = 0$ and there is no voltage difference between these nodes. Then the node between V_i and V_- is $V_x = -V_o$, by applying KCL at the negative input to the op amp. Applying KCL at that node yields $(V_x - V_i) + (V_x - V_o) + V_x = 0$, which shows that $V_i = -4V_o$.

$$\frac{V_0}{V_i} = \frac{5}{4}$$

Reasoning:

Label the node between V_- and ground as V_x.
- KCL at V_-: $(V_- - V_o) + (V_- - V_x) = 0$
- KCL at V_x: $(V_x - V_-) + (V_x - V_o) + V_x = 0$
Solving yields $V_- = \frac{4}{5}V_o$. But $V_- = V_+ = V_i$. Thus $V_o = \frac{5}{4}V_i$.

3 Designing Circuits (13 points)

Determine the indicated parameters for each of the following circuits. Write your answers (either a number or numerical expression) in the box provided. For partial credit, you may also provide a brief explanation of your reasoning.

Part a. Determine values of R_1, R_2, R_3, R_4, and R_5 so that $V_1 = 1V$, $V_2 = 2V$, $V_3 = 4V$, and $V_4 = 8V$. The answer to this part is not unique. You will get full credit for any valid solution.

$+10V$

R_5

$V_4 = 8V$

R_4

$V_3 = 4V$

R_3

$V_2 = 2V$

R_2

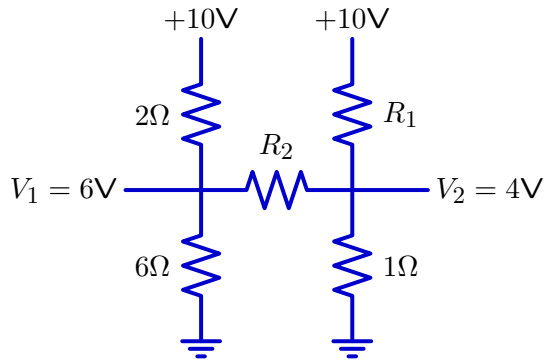$V_1 = 1V$

R_1
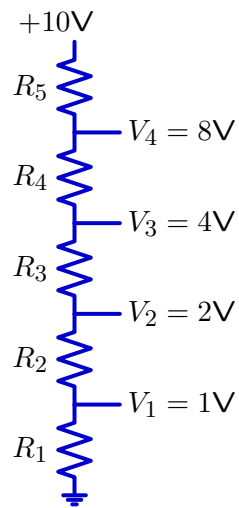
$R_5 = 2R_0$

$R_4 = 4R_0$

$R_3 = 2R_0$

$R_2 = R_0$

$R_1 = R_0$

Reasoning:

Because the resistors are in series, the resistance between successive nodes will be proportional to the voltage between the nodes. Thus $R_1 \propto V_1 = 1V$; $R_2 \propto V_2 - V_1 = 1V$; $R_3 \propto V_3 - V_2 = 2V$; $R_4 \propto V_4 - V_3 = 4V$; $R_5 \propto V_5 - V_4 = 2V$.

Part b. Determine values of R_1 and R_2 so that $V_1 = 6V$ and $V_2 = 4V$.



$R_1 = 2\Omega$

$R_2 = 2\Omega$

Reasoning:

KCL at the left-center node determines that a 1A current flows rightward through R_2. To make $V_1 - V_2 = 2V$, it follows that $R_2 = 2\Omega$.

KCL at the right-center node then determines that a 3A current flows downward through R_1. To make $10V-V_2 = 6V$, it follows that $R_1 = 2\Omega$.
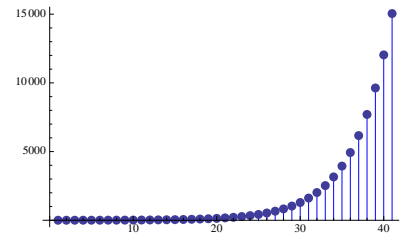
4 Pole Position (16 points)

Consider eight poles located at the following locations in the z plane. The plots below show the unit-sample responses of eight linear, time-invariant systems. Match them with the dominant pole for each system (remember thta the system may have more than just one pole).
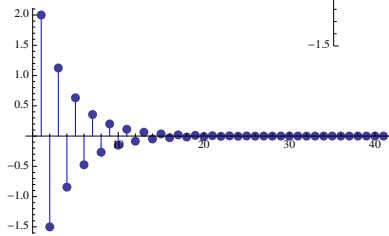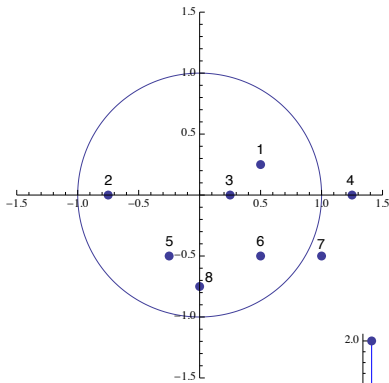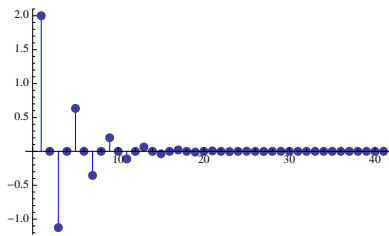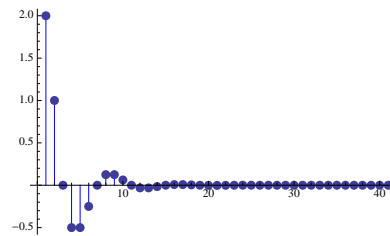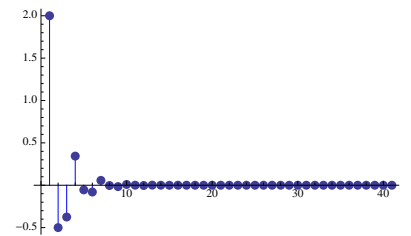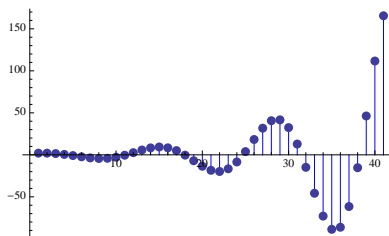

A = 2
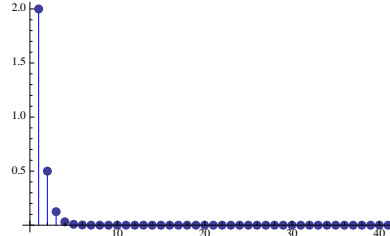
B = 3

C = 4

D = 8

E = 6

F = 5

G = 7
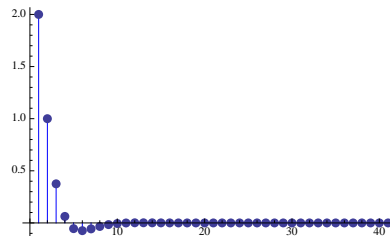
H = 1

5 System functions (14 points)

Let H_1 represent a subsystem that is part of the larger system shown below.
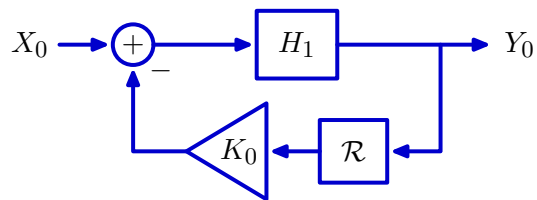
The system function for the larger system can be written as

$$H_0 = \frac{Y_0}{X_0} = \frac{H_1}{1 + K_0 R H_1} \ .$$

Assume that $H_1 = H_{1B} = \frac{Y_1}{X_1}$ as shown below.
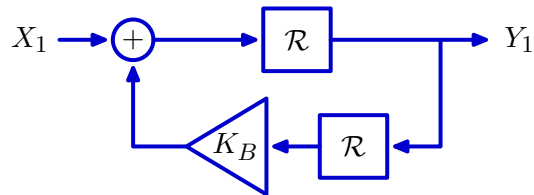
What is the system function for H_{1B}?

$$H_{1B} = \frac{\mathcal{R}}{1 - K_B \mathcal{R}^2}$$

Determine the system function H_0 for the larger system when $H_1 = H_{1B}$.

$$H_0 = \frac{\mathcal{R}}{1 + (K_0 - K_B) R^2}$$

Under what conditions on K_0 and K_B is this system stable? Explain.

There are poles at $z = \pm\sqrt{K_B - K_0}$. To be stable, the poles should all have magnitued less than 1. Thus the system is stable if $|K_B - K_0| < 1$.

Under what conditions on K_0 and K_B does the unit-sample response decay monotonically? Explain.

None. For monotonic convergence, **both** poles must have magnitudes between 0 and 1 (since there are two poles of equal magnitude). If $K_B < K_0$ then the poles have non-zero imaginary parts, and the response oscillates. If $K_B > K_0$ then one pole is on the positive real axis and one is on the negative real axis. The pole on the negative real axis causes the unit sample response to alternate. Thus there are no values of K_O and K_B for which there is monotonic decay.

6 State machines (15 points)

You are going to design a state machine that takes a string of characters as input, and outputs
at each time step the largest number of times a character is repeated in a row in the sequence as
observed so far. The state variable will have the following form:

```
[lastCharacter, timesSeenConsecutively, best]
```

that is, the state will be a list of the previous input character (except for the first time step, see
below), the number of times that character has occurred consecutively up to this point, and the
length of the longest sequence of repeated characters observed so far.

Complete the following definition:

```
Class SubSeq(sm.SM):
    startState = None
    def getNextValues(self, state, inp):
        if state == None:
            #part 1
        elif state[0] == inp:
            #part 2
        else:
            #part 3
```

so that it has the following example behavior:

```
>>> test = SubSeq()
>>> test.transduce('abaabcdccccc')
[1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 4, 5]
```

Supply the code for part 1:

```
        return ((inp, 1, 1), 1)
```

Supply the code for part 2:

```
        if state[1] == state[2]:
            return ((inp, state[1]+1, state[1]+1), state[1]+1)
        else:
            return ((inp, state[1]+1, state[2]), state[2])
```

Supply the code for part 3:

```
        return ((inp, 1, state[2]), state[2])
```

7 Equivalent Circuits (12 points)

Find the Thevenin and Norton equivalent circuits for the circuit shown in the lower left panel, looking into the port labeled $n+$ and $n-$. Determine values for V_T, R_T, I_N and R_N in terms of V_x and R_x. For partial credit, you may also provide a brief explanation of your reasoning.

Original   Thevenin   Norton
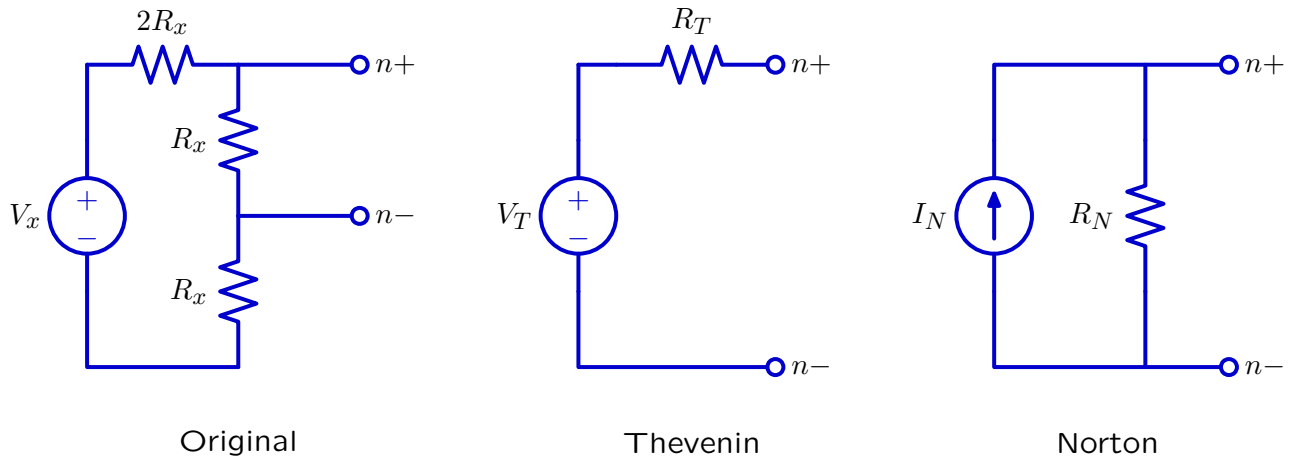
$$V_T = \frac{V_x}{4} \qquad\qquad I_N = \frac{V}{3R}$$

$$R_T = \frac{3R_x}{4} \qquad\qquad R_N = \frac{3R}{4}$$

Reasoning:

Open circuit equations

$\frac{V_x - n_+}{2R} = \frac{n_+ - n_-}{R}$ and $\frac{n_+ - n_-}{R} = \frac{n_-}{R}$

Solving these equations leads to difference between n_+ and n_- of $\frac{V_x}{4}$.

Closed circuit gives current of $\frac{-V_x}{3R}$, since only one path in circuit.

Solve linear equation between two points, find slope is $\frac{4}{3R}$, so reciprocal is resistance of $\frac{3R}{4}$

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