RECEIVING ANTENNAS

Reciprocity:

$$A_e(\theta,\phi) = \frac{\lambda^2}{4\pi} G(\theta,\phi)$$

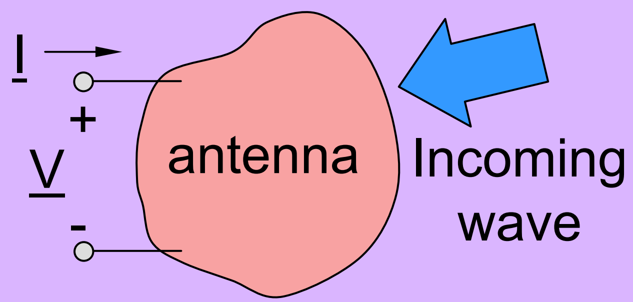
Not yet proved here; sometimes untrue (when?)

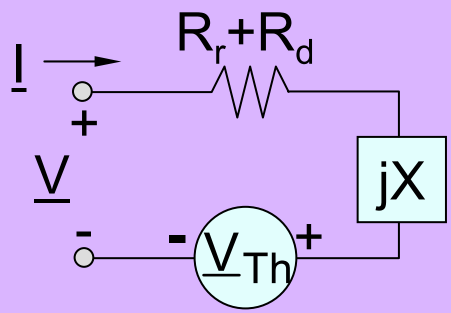
Approach:

Study small simple antennas; d << λ $\Rightarrow$ quasistatics

Seek V_{Th} in equivalent circuit = open-circuit voltage

Examples: small loop antenna and short dipole



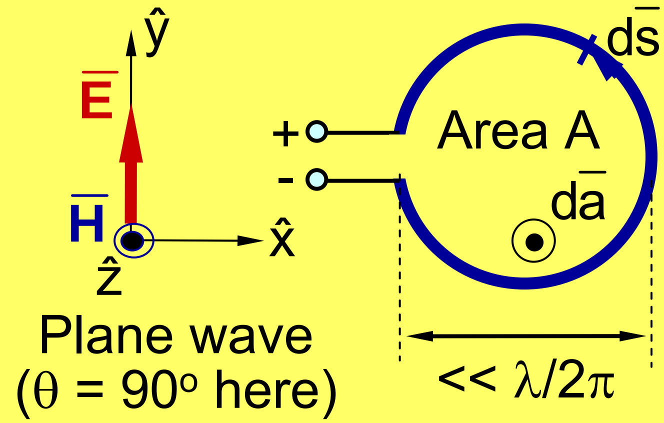


SMALL LOOP ANTENNA

Quasistatic Limit:

$\nabla \times \underline{\overline{E}} = -j\omega \underline{\overline{B}}$ (Faraday's Law)

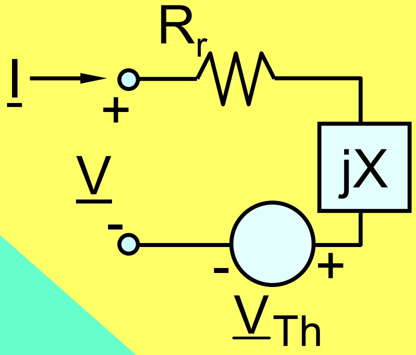
$\Rightarrow \oint_c \underline{\overline{E}} \bullet d\overline{s} = -j\omega \iint_A \underline{\overline{B}} \bullet d\overline{a} = \underline{V}_{Th}$



Open Circuit Voltage:

$\underline{V}_{Th} = -j\omega AN\mu_o \underline{\overline{H}} \bullet \hat{z} \propto \sin\theta$

θ = angle of incidence relative to z axis



Radiation Resistance:

$P_{rec}(\theta) = \frac{1}{8R_r}|\underline{V}_{Th}|^2$ [W] $\Rightarrow$ $R_r = \frac{|\underline{V}_{Th}|^2}{8P_{rec}}$

$P_{rec}(\theta) = I(\theta)A_e(\theta)$ $[\propto \sin^2\theta]$ $= \frac{1}{2}\eta_o|\underline{\overline{H}}(\theta)|^2 \; G(\theta)\frac{\lambda^2}{4\pi}$

$G(\theta) = \frac{3}{2}\sin^2\theta$ $[\propto A_e(\theta) \propto \sin^2\theta \; ; \int_{4\pi} G(\theta)d\Omega = 4\pi\ ]$

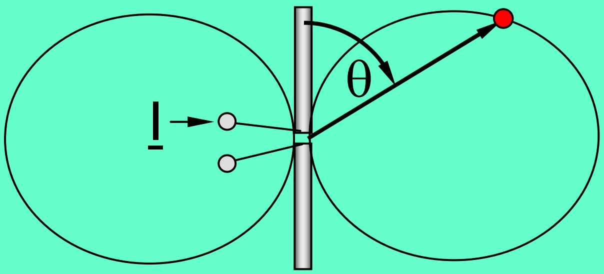
$$\therefore R_r = \frac{|-j\omega AN\mu_o \underline{\overline{H}} \sin\theta|^2}{8[\frac{1}{2}\eta_o |\underline{\overline{H}}|^2][\frac{3}{2}\sin^2\theta \frac{\lambda^2}{4\pi}]} = \frac{2\pi}{3\eta_o}\left(\frac{\omega\mu_o AN}{\lambda}\right)^2 \text{ ohms for N-turn coil}$$

SHORT DIPOLE RADIATION

Far field radiation:

$$\underline{\overline{E}}_{ff} = \hat{\theta}\,\frac{j\eta_o}{2\lambda r}\,\underline{I}\,d_{eff}\,\sin\theta\; e^{-jkr} \quad [\mathrm{V\ m^{-1}}]$$

G = $1.5\sin^2\theta$



Far field intensity:

$$\overline{I}(r,\theta) = \hat{r}\,\frac{|\underline{\overline{E}}_{ff}|^2}{2\eta_o} \quad [\mathrm{W\ m^{-2}}]$$

Total power radiated:

$$P_T = \int_0^{2\pi}\int_0^{\pi}[\overline{I}(r,\theta)\bullet\hat{r}]\,r^2\sin\theta\,d\theta\,d\phi = \pi\eta_o\left|\frac{\underline{I}\,d_{eff}}{2\lambda}\right|^2\int_0^{\pi}\sin^3\theta\,d\theta$$

$$= \frac{\eta_o\pi}{3}\left|\frac{\underline{I}\,d_{eff}}{\lambda}\right|^2 \quad [\mathrm{W}]$$

Antenna gain:

$$G(\theta,\phi) \triangleq \frac{I(r,\theta)}{[P_T/4\pi r^2]} = \frac{3}{2}\sin^2\theta$$

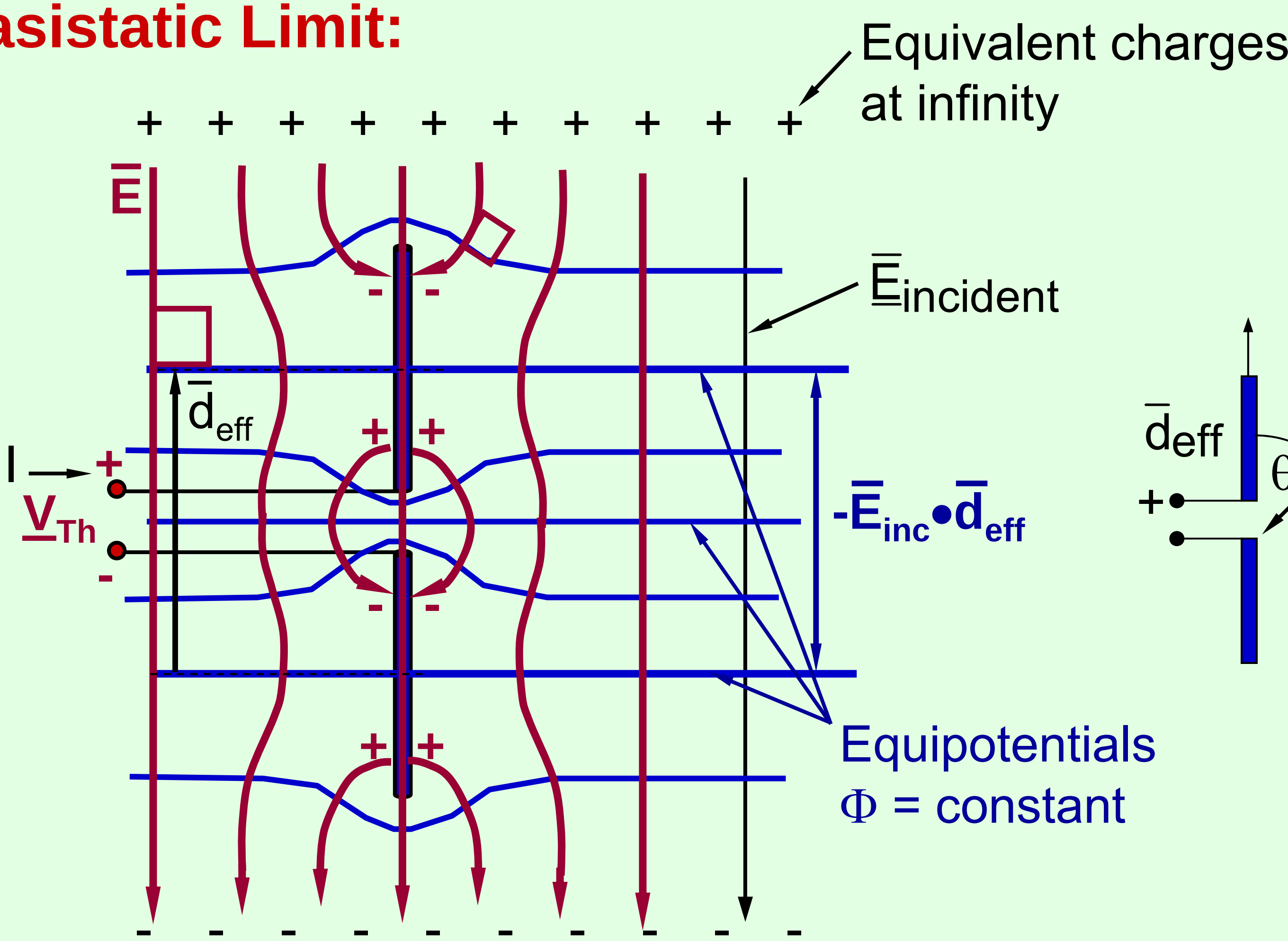
Radiation resistance:

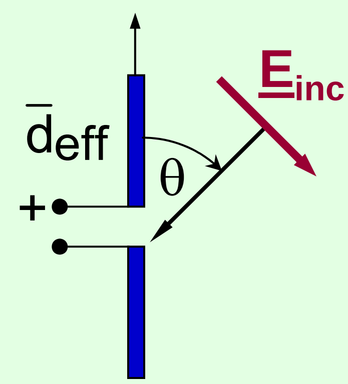
$$P_T = \frac{1}{2}|\underline{I}|^2 R_r$$

$$R_r = \frac{2\pi\eta_o}{3}\left(\frac{d_{eff}}{\lambda}\right)^2 \quad [\Omega] \quad \text{(short dipole antenna)}$$

SHORT DIPOLE ANTENNA

Quasistatic Limit:

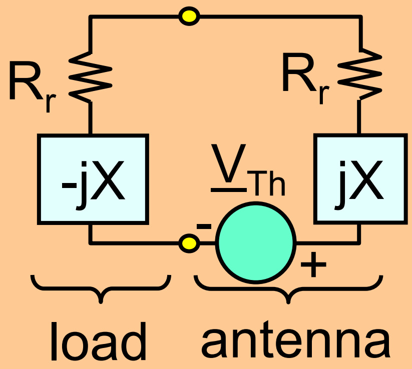




$$\underline{V}_{Th} = -\bar{E}_{inc} \bullet \bar{d}_{eff} = -\underline{E}_{inc} d_{eff} \sin\theta$$

ANTENNA EFFECTIVE AREA A_e

Antenna Equivalent Circuit:



$$\underline{V}_{Th} = -\underline{E}_{inc} d_{eff} \sin\theta$$

$$P_{rec} = \frac{|\underline{V}_{Th}/2|^2}{2R_r}$$

$$= |\underline{E}_{inc}|^2 \frac{d_{eff}^2}{8R_r} \sin^2\theta$$

$$= I(\theta)\eta_o \sin^2\theta \, \frac{d_{eff}^2}{4R_r} \quad \left[I(\theta) = \frac{|\underline{E}_{inc}|^2}{2\eta_o} \right]$$

$$R_r = \frac{2\pi\eta_o}{3}\left(\frac{d_{eff}}{\lambda}\right)^2$$

$$P_{rec} = I(\theta)\sin^2\theta \, \frac{3\lambda^2}{8\pi}$$

Antenna Effective Area A_{eff}:

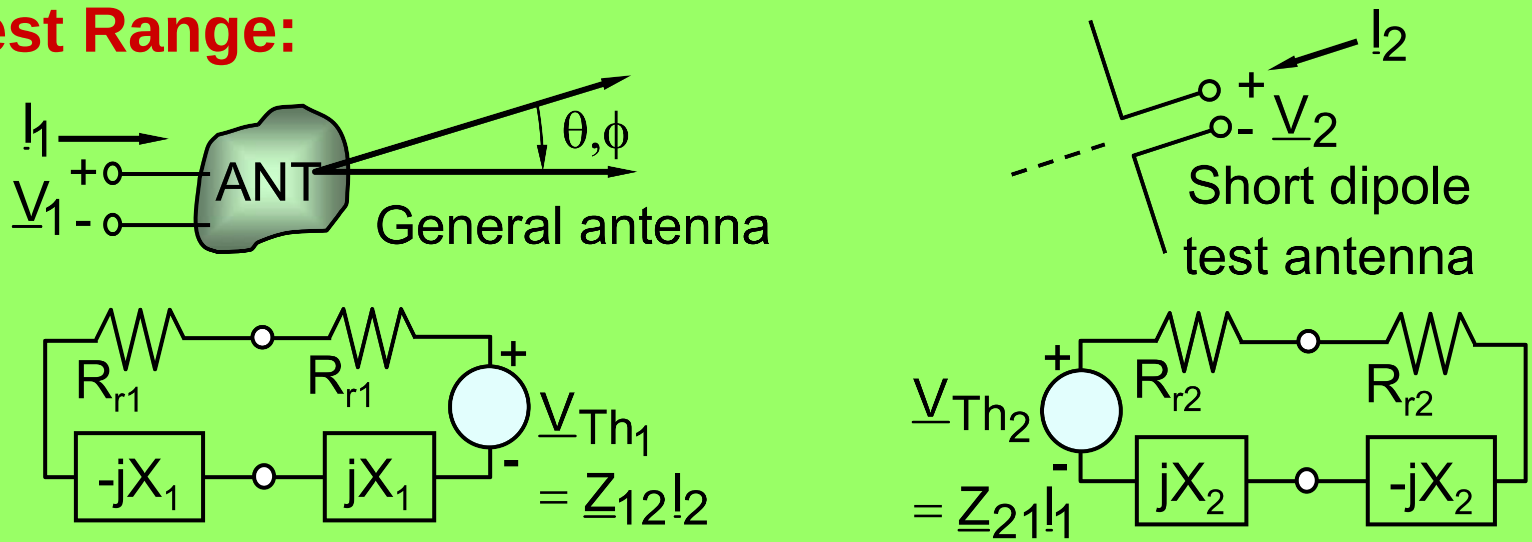
$$A_e(\theta) \triangleq \frac{P_{rec}}{I(\theta)}$$

True for most antennas

$$A_e(\theta) = \frac{\lambda^2}{4\pi} 1.5 \sin^2\theta = \overbrace{\frac{\lambda^2}{4\pi} G(\theta) = A_e(\theta)}$$

PROOF: $A = G\lambda^2/4\pi$ (If reciprocity)

Test Range:



Proof:

$$P_{rec_1} = \frac{|\underline{Z}_{12}\underline{I}_2|^2}{8R_{r_1}} = P_{t_2}\frac{G_2}{4\pi r^2}A_1 \qquad P_{rec_2} = \frac{|\underline{Z}_{21}\underline{I}_1|^2}{8R_{r_2}} = P_{t_1}\underbrace{\frac{G_1}{4\pi r^2}}_{Wm^{-2}}A_2$$

$$\frac{P_{rec_2}}{P_{rec_1}} = \frac{G_1 A_2 P_{t_1}}{G_2 A_1 P_{t_2}} \Rightarrow \frac{A_1}{G_1} = \frac{A_2}{G_2}\left[\frac{P_{t_1}}{P_{t_2}} \bullet \frac{P_{rec_1}}{P_{rec_2}}\right]$$

But $$\frac{P_{rec_1}}{P_{rec_2}} = \frac{|\underline{Z}_{12}\underline{I}_2|^2}{|\underline{Z}_{21}\underline{I}_1|^2} \bullet \frac{R_{r_2}}{R_{r_1}} = \frac{|\underline{Z}_{12}|^2}{|\underline{Z}_{21}|^2} \bullet \frac{P_{t_2}}{P_{t_1}}$$

If $|\underline{Z}_{12}|^2 = |\underline{Z}_{21}|^2$, then

$$\frac{A_1}{G_1} = \frac{A_2}{G_2} = \frac{\lambda^2}{4\pi} \quad \text{Q.E.D.}$$

NON-RECIPROCAL DEVICES

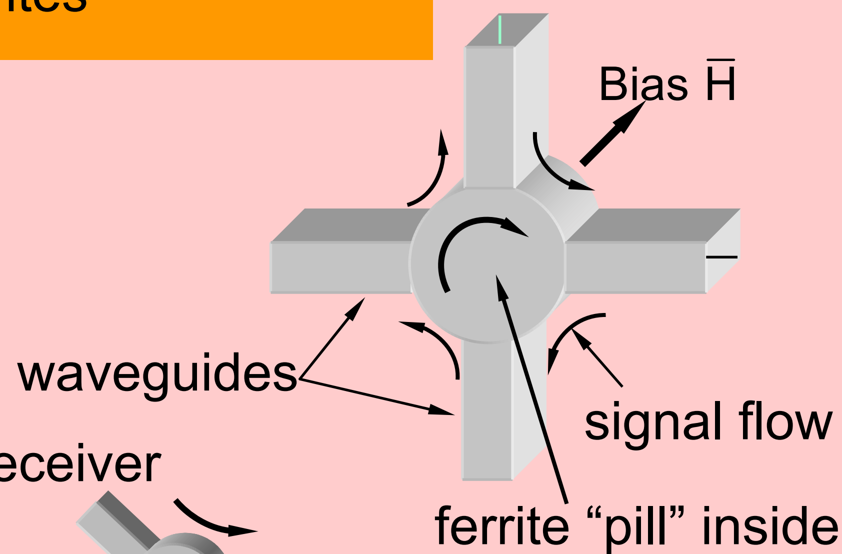
Reciprocity:

$|\underline{Z}_{12}|^2 = |\underline{Z}_{21}|^2$ if $\bar{\bar{\varepsilon}} = \bar{\bar{\varepsilon}}^t$, $\bar{\bar{\mu}} = \bar{\bar{\mu}}^t$ everywhere.

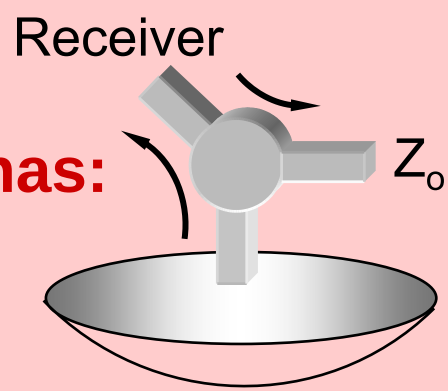
Exceptions: magnetized plasmas,
magnetized ferrites

Non-reciprocal Devices:

4-Port Circulators
2 and 3-port circulators
Isolators



Non-reciprocal Antennas:

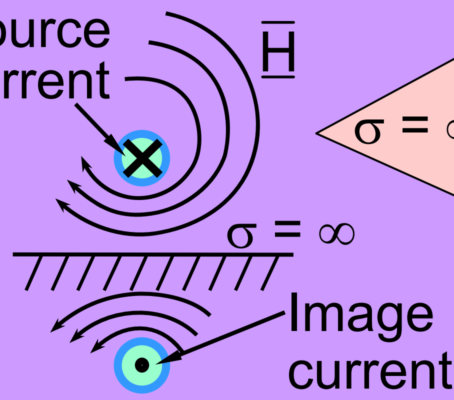


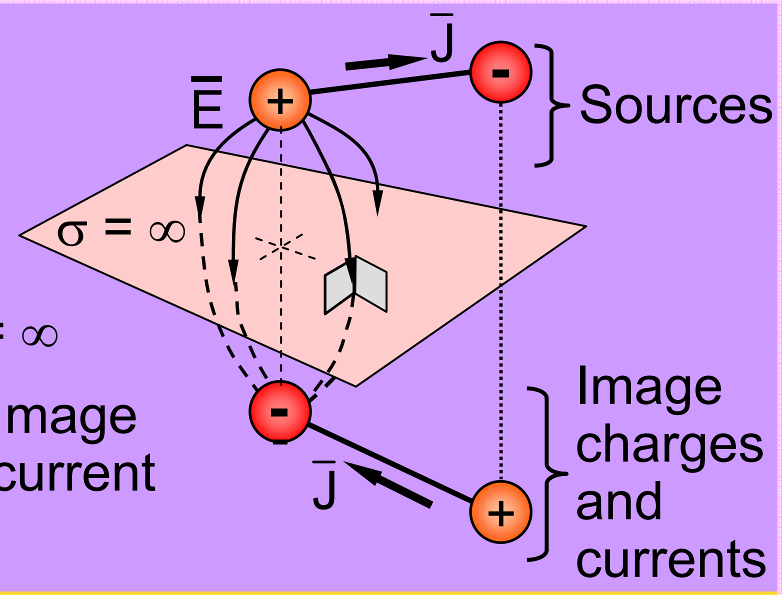
MIRROR IMAGES

Mirror Images:

$\bar{E}$ must be ⊥ to mirror
$\bar{H}$ must be parallel

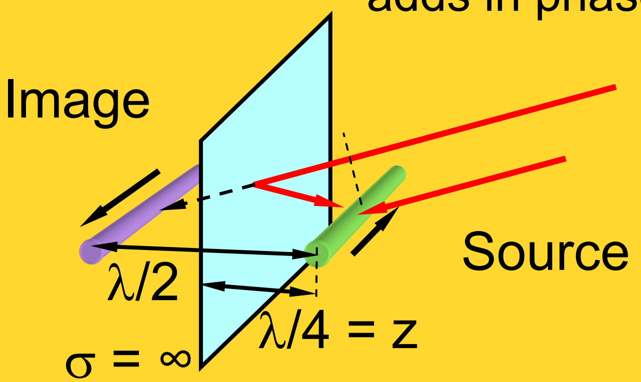
Fields above mirror equal
Fields with image sources



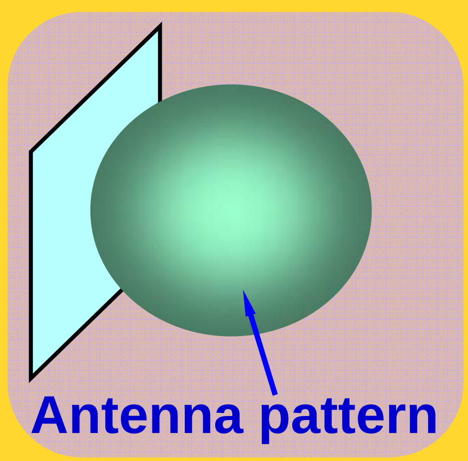


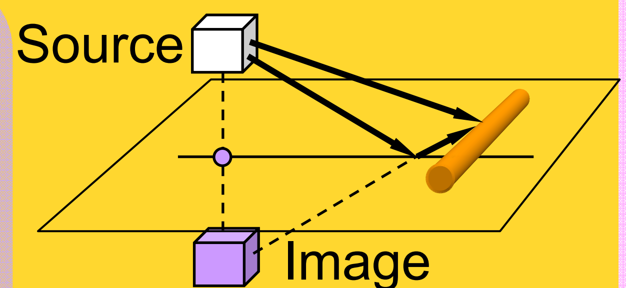
Examples:

Path at normal incidence is λ/2 longer, but adds in phase due to mirror's phase reversal



Normal gain has nulls at z = 0, λ/2, λ, etc.





Antenna height above ground matters!