SINUSOIDAL RADIATION BY ANTENNAS

Poisson equations: $\nabla^2\Phi = -\dfrac{\rho}{\varepsilon}$ $\nabla^2\bar{A} = -\mu_o\bar{J}$ $(\bar{B} \triangleq \nabla\times\bar{A})$

Retarded potentials:

$$\Phi_p = \frac{1}{4\pi\varepsilon_o}\int_{V_q}\frac{\rho_q\left(t - r_{pq}/c\right)}{r_{pq}}dV_q \qquad \bar{A}_p = \frac{\mu_o}{4\pi}\int_{V_q}\frac{\bar{J}_q\left(t - r_{pq}/c\right)}{r_{pq}}dV_q$$

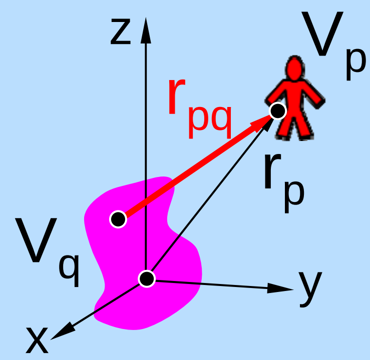
$$\underline{\Phi}(\omega) = \frac{1}{4\pi\varepsilon_o}\int_{V_q}\frac{\underline{\rho}(\omega)}{r_{pq}}e^{-jkr_{pq}}\ dV_q \qquad \underline{\bar{A}}(\omega) = \frac{\mu_o}{4\pi}\int_{V_q}\frac{\underline{\bar{J}}(\omega)}{r_{pq}}e^{-jkr_{pq}}\ dV_q$$

Radiation $\cong$ static solution if $kr_{pq} = \dfrac{2\pi r_{pq}}{\lambda} << 1$, or $r_{pq} << \dfrac{\lambda}{2\pi}$

Finding fields from potentials:

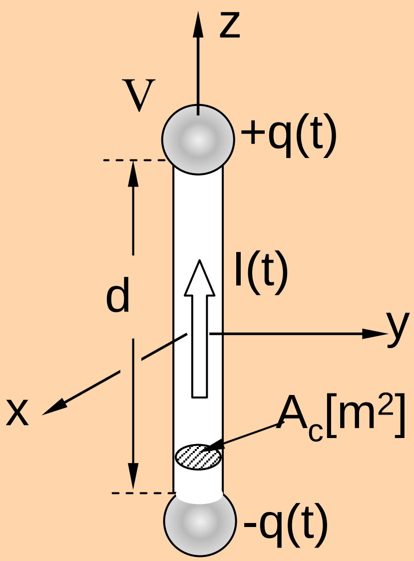
$$\underline{\bar{B}} = \nabla\times\underline{\bar{A}}\ , \qquad \underline{\bar{E}} = -\nabla\underline{\Phi} - j\omega\underline{\bar{A}} = -\nabla\times\frac{\bar{H}}{j\omega\varepsilon}$$

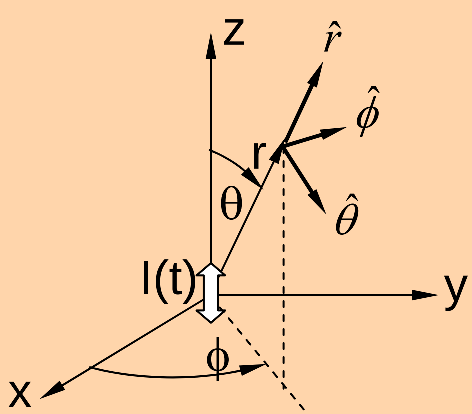
$$k = \omega\sqrt{\mu_o\varepsilon_o} = \frac{2\pi}{\lambda_o}$$



Algorithm for general problem: $\{\underline{\rho},\underline{\bar{J}}\} \Rightarrow \{\underline{\Phi},\underline{\bar{A}}\} \Rightarrow \{\underline{\bar{E}},\underline{\bar{H}}\}$

HERTZIAN DIPOLE RADIATOR





$$\overline{A}_p(\omega) = \frac{\mu_o}{4\pi}\int_{V_q} \frac{\overline{J}_q(\omega)}{r_{pq}} e^{-jkr_{pq}}\, dV_q$$

$$\cong \underbrace{(\hat{r}\cos\theta - \hat{\theta}\sin\theta)}_{\hat{z}} \frac{\mu_o \underline{I}(\omega) d}{4\pi r_{pq}} e^{-jkr_{pq}}$$

Where: $\overline{J}_q = \hat{z}\underline{J}_q$ for $-\frac{d}{2} < z < \frac{d}{2}$

And provided that d << r
(so r can come outside the integral)

$$\overline{H} = \frac{\nabla\times\overline{A}}{\mu_o} = \frac{1}{\mu_o r^2 \sin\theta} \det\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

SOLVING FOR HERZIAN DIPOLE FIELDS

$$\underline{\overline{H}} = \frac{\nabla \times \underline{\overline{A}}}{\mu_o} = \hat{\phi}\frac{jk\underline{I}d}{4\pi r}e^{-jkr}\left[1+\frac{1}{jkr}\right]\sin\theta$$

$$\underline{\overline{E}} = \frac{\nabla \times \underline{\overline{H}}}{j\omega\varepsilon_o}$$

$$= j\frac{k\underline{I}d\eta_o}{4\pi r}e^{-jkr}\left\{\hat{r}\left[\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]2\cos\theta+\hat{\theta}\left[1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]\sin\theta\right\}$$

Far field: $r >> \frac{\lambda}{2\pi}$

$$\underline{\overline{H}}_{ff} \cong \hat{\phi}\frac{jk\underline{I}d}{4\pi r}e^{-jkr}\sin\theta$$

$$\underline{\overline{E}}_{ff} \cong \hat{\theta}\frac{jk\underline{I}d\eta_o}{4\pi r}e^{-jkr}\sin\theta$$

Near field: $r << \frac{\lambda}{2\pi}$

$$\underline{\overline{H}}_{nf} \cong \hat{\phi}\frac{\underline{I}d}{4\pi r^2}e^{-jkr}\sin\theta$$

(yields Biot-Savart law)

$$\underline{\overline{E}}_{nf} \cong -j\frac{\underline{I}d\eta_o}{4\pi kr^3}e^{-jkr}(\hat{r}2\cos\theta+\hat{\theta}\sin\theta)$$

$\underline{\overline{H}}(\overline{r},t) = \oiiint_{V'}\frac{\hat{\phi}\underline{J}\sin\theta}{4\pi|\overline{r}-\overline{r}'|^2}dv.$ Since $\hat{\phi}\underline{J}\sin\theta = \underline{\overline{J}}\times\frac{(\overline{r}-\overline{r}')}{|\overline{r}-\overline{r}'|}$, $\underline{\overline{H}}(\overline{r},t) = \oiiint_{V'}\frac{\underline{\overline{J}}\times(\overline{r}-\overline{r}')}{4\pi|\overline{r}-\overline{r}'|^3}dv$

NEAR AND FAR FIELDS OF HERTZIAN DIPOLE

Far field: $r >> \frac{\lambda}{2\pi}$ (radiation)

$$\underline{\overline{E}}_{ff} \cong \hat{\theta}\frac{jk\underline{I}d\eta_o}{4\pi r}e^{-jkr}\sin\theta$$

$$\underline{\overline{H}}_{ff} \cong \hat{\phi}\frac{jk\underline{I}d}{4\pi r}e^{-jkr}\sin\theta$$

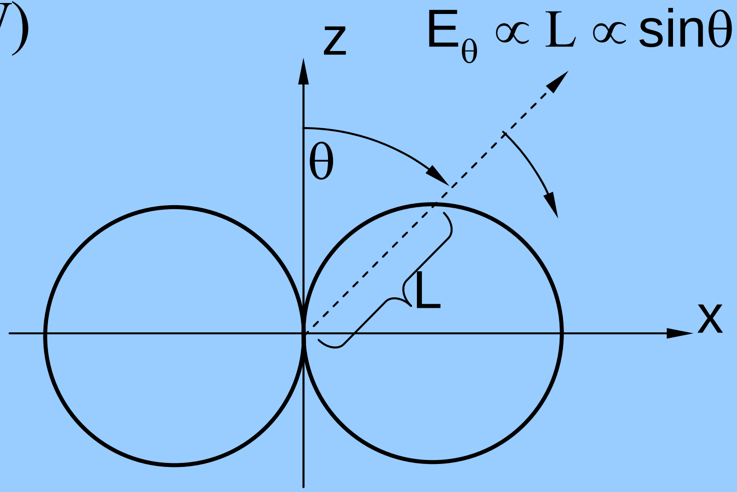
Near field: $r << \frac{\lambda}{2\pi}$ (quasistatic)

$$\underline{\overline{E}}_{nf} \cong -\frac{j\underline{I}d\eta_o}{4\pi k r^3}e^{-jkr}(\hat{r}2\cos\theta + \hat{\theta}\sin\theta)$$

$$\underline{\overline{H}}_{nf} \cong \hat{\phi}\frac{\underline{I}d}{4\pi r^2}e^{-jkr}\sin\theta$$

$$\underline{\overline{E}}_{ff} = \eta_o \underline{\overline{H}}_{ff} \times \hat{r} \quad \text{(UPW)}$$

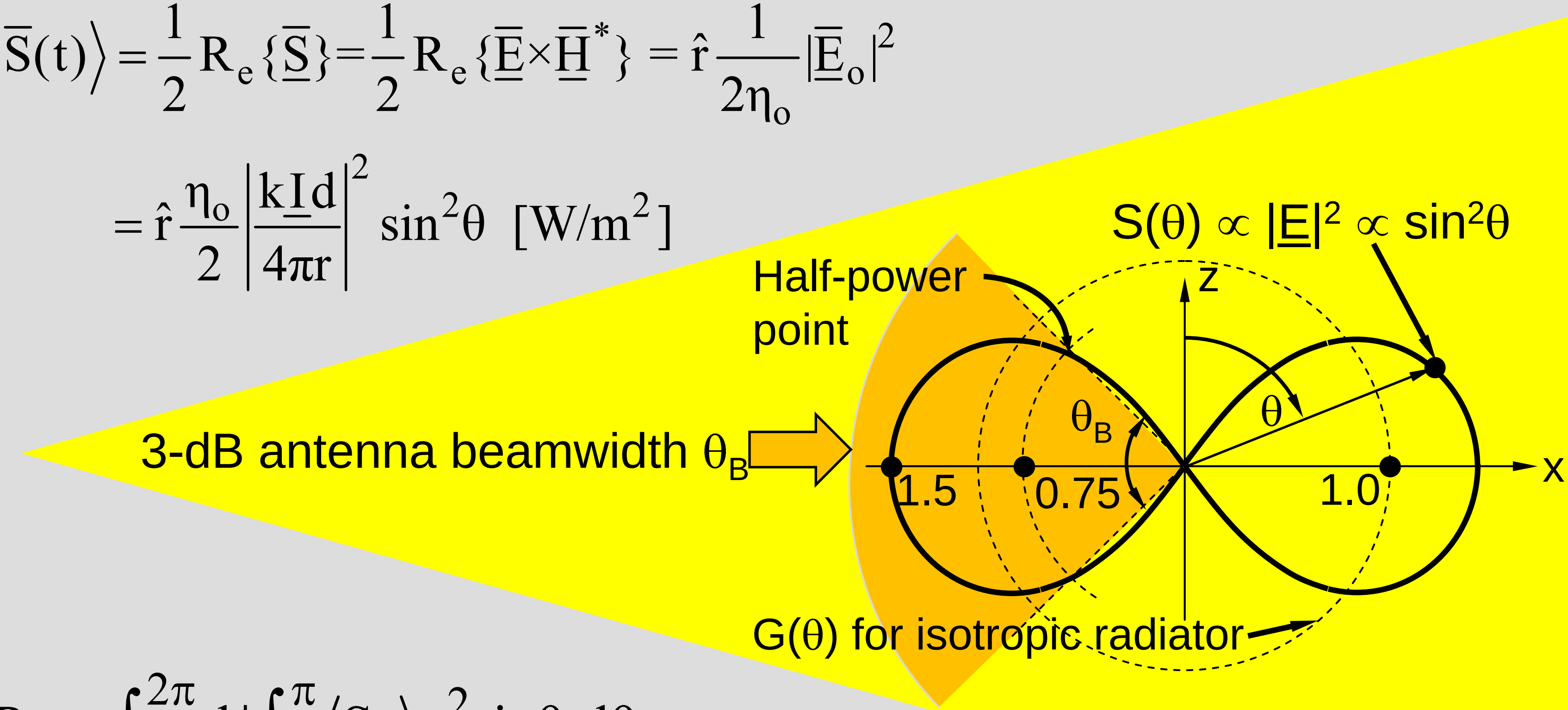
$$\left(\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 377\Omega\right)$$



POWER RADIATED BY HERTZIAN DIPOLE

$$\langle \overline{S}(t) \rangle = \frac{1}{2} R_e \{\underline{\overline{S}}\} = \frac{1}{2} R_e \{\underline{\overline{E}} \times \underline{\overline{H}}^*\} = \hat{r} \frac{1}{2\eta_o} |\underline{\overline{E}}_o|^2$$

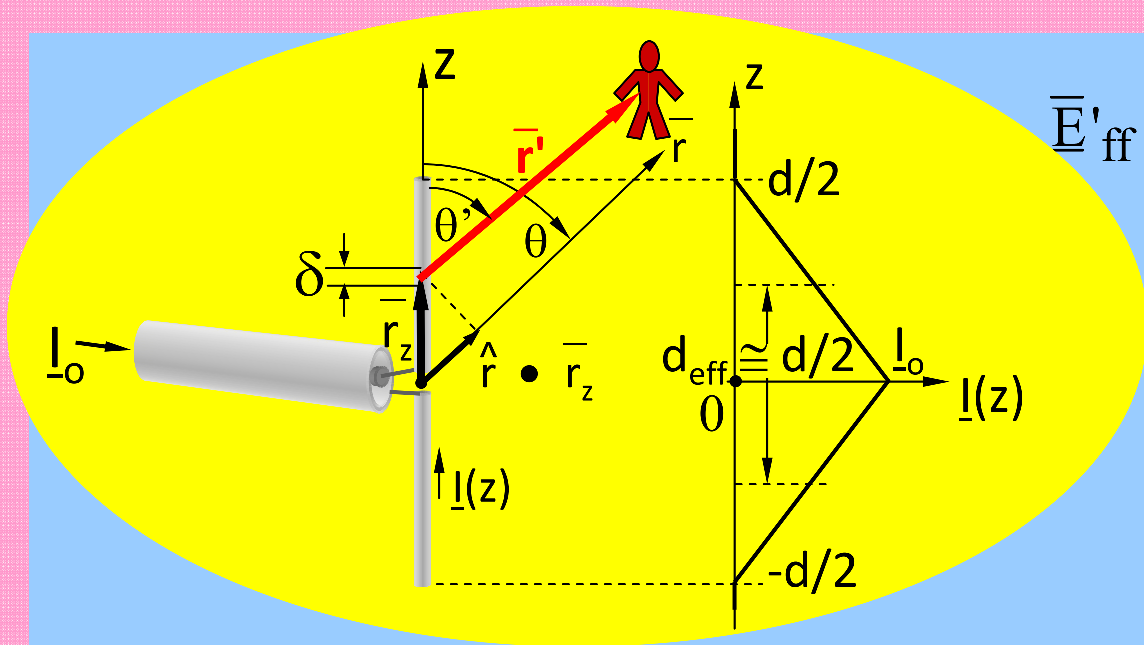
$$= \hat{r} \frac{\eta_o}{2} \left| \frac{k\underline{I}d}{4\pi r} \right|^2 \sin^2\theta \ \ [\mathrm{W/m^2}]$$



$$P_R = \int_0^{2\pi} d\phi \int_0^{\pi} \langle S_r \rangle r^2 \sin\theta \ d\theta$$

$$= \pi\eta_o \left| \frac{k\underline{I}d}{4\pi} \right|^2 \int_0^{\pi} \sin^3\theta \ d\theta = \frac{\eta_o}{12\pi} |k\underline{I}d|^2 \cong 395 \left| \frac{\underline{I}d}{\lambda} \right|^2 \ [\mathrm{W}]$$

SHORT DIPOLE ANTENNAS



$$\underline{\overline{E}}'_{ff} \cong \hat{\theta}' \frac{jk\underline{I}(z)\delta\eta_o}{4\pi r'} e^{-jkr'} \sin\theta'$$

Wire ≅ TEM line open-circuited, so current is a truncated sinusoid ⇒ triangle.

$$\underline{\overline{E}}_{ff} = \int_{-d/2}^{d/2} \underline{\overline{E}}_{ff}{}'(r',\theta')dz = \frac{jk\eta_o}{4\pi} \int_{-d/2}^{d/2} \hat{\theta}' \frac{\sin\theta'}{r'} \underline{I}(z) e^{-jkr'} dz$$

$$\cong \hat{\theta}\sin\theta \frac{jk\eta_o}{4\pi r} e^{-jkr} \int_{-d/2}^{d/2} \underline{I}(z)dz \cong \hat{\theta}\sin\theta \frac{jk\eta_o}{4\pi r} \underline{I}_o d_{eff}\, e^{-jkr}$$

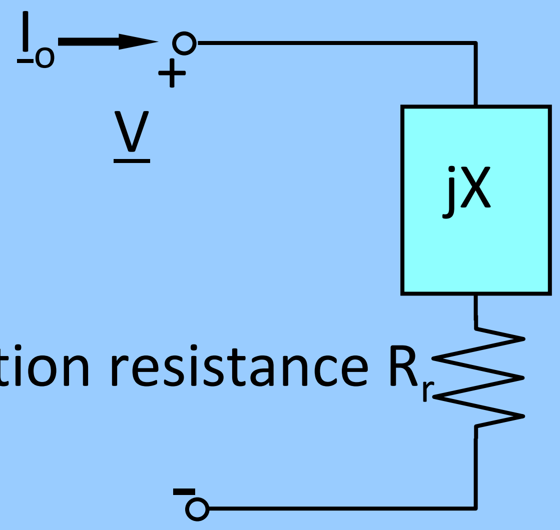
Has same far fields as does a Hertzian dipole, except $\underline{I}d \rightarrow \underline{I}_o d_{eff}$

ANTENNA RADIATION RESISTANCE

$$P_T = \frac{\eta_o}{12\pi}\left|k\underline{I}_o d_{eff}\right|^2 \cong 395\left|\frac{\underline{I}_o d_{eff}}{\lambda}\right|^2 \quad [W]$$

$$= \frac{1}{2}\left|\underline{I}_o\right|^2 R_r \quad \Rightarrow \quad \text{Radiation resistance } R_r = \frac{2P_T}{\left|\underline{I}_o\right|^2} = \frac{2\eta_o\pi}{3}\left(\frac{d_{eff}}{\lambda}\right)^2 \text{ ohms}$$

For a triangular current distribution, center fed, $d_{eff} \cong d/2$.



Short dipole antenna example:
AM radio at 1 MHz has λ = 300 m.
Car antenna 2m long has:

$$R_r = \frac{2 \bullet 377 \bullet 3.14}{3}\left(\frac{1}{300}\right)^2 \cong 0.02 \text{ ohms}$$

jX can similarly be found by measuring or computing $\underline{S}$ into an antenna; it results from net electric or magnetic energy in the near field.

ANTENNA GAIN OVER ISOTROPIC

$$G(\theta,\phi) \equiv \frac{P(r,\theta,\phi)}{\left(P_A/4\pi r^2\right)} = \frac{|\underline{E}(r,\theta,\phi)|^2/2\eta_o}{\left(P_A/4\pi r^2\right)} = \frac{\left|\sin\theta\ \frac{jk\eta_o}{4\pi r}\underline{I}_o d_{eff}\right|^2/2\eta_o}{\frac{\eta_o}{12\pi}\left|k\underline{I}_o d_{eff}\right|^2/4\pi r^2} = \frac{3}{2}\sin^2\theta$$

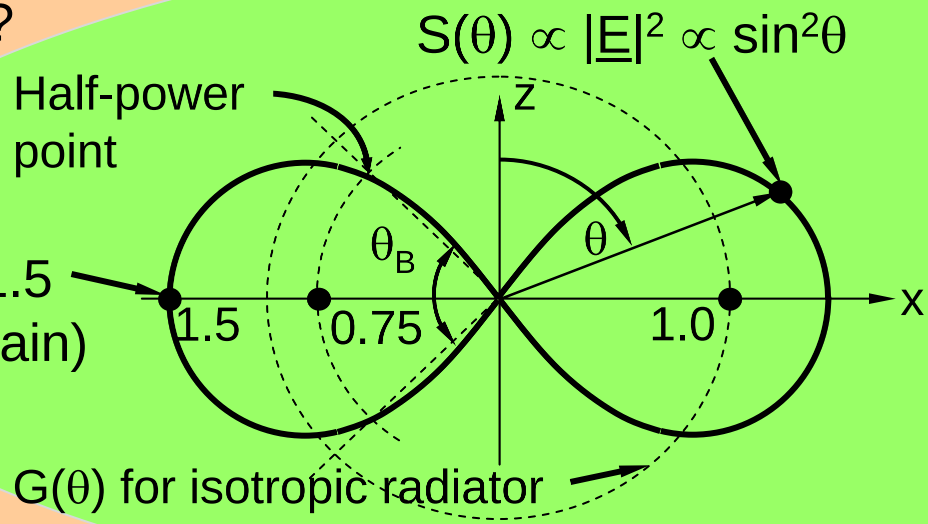
Example: What maximum $\overline{E}$ [V/m] is produced at 10 km by a short dipole radiating P_A = 1 kW?

Solution:

$$\frac{|\overline{\underline{E}}(r,\theta,\phi)|^2}{2\eta_o} = P(r,\theta,\phi)$$

$$= \frac{(1.5\sin^2\theta)P_A}{4\pi r^2}$$

$$\Rightarrow \quad |\overline{\underline{E}}(r,\theta,\phi)| = \sqrt{(1.5\sin^2\theta)10^3\eta_o/2\pi 10^8} = 30 \text{ mv/m}$$



ANTENNA DIRECTIVITY AND RADIATION EFFICIENCY

Gain over isotropic

$$G(\theta,\phi) \equiv \frac{P(r,\theta,\phi)}{\left(P_A / 4\pi r^2\right)}$$

Antenna directivity

$$D(\theta,\phi) \equiv \frac{P(r,\theta,\phi)}{\left(P_T / 4\pi r^2\right)}$$

$$\frac{G(\theta,\phi)}{D(\theta,\phi)} = \frac{P_T}{P_A} = \frac{\text{Power transmitted}}{\text{Power available}} \triangleq \eta_r \quad \text{"radiation efficiency"} \leq 1$$

$$\oint_{4\pi} D(\theta,\phi) \sin\theta \, d\theta \, d\phi = 4\pi \quad \text{since} \quad \oint_{4\pi} P(r,\theta,\phi) r^2 \sin\theta \, d\theta \, d\phi = P_T$$

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