MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 Electromagnetics and Applications – Quiz 2 Solutions

Problem 1. (28/100 points)

a) $\boxed{R_{Th} = 100 \text{ ohms}}$ since there is no "glitch" at t = 2τ and therefore no reflection at the source when the step returns at 2τ. Thus R_{Th} is matched to the line.

b) Line length $\boxed{D = c\tau/2}$

c) An $\boxed{\text{R and C in parallel}}$ at the end of the line produces this response; the C in parallel looks like a short circuit, yielding the zero at t = τ, and the R in parallel prevents the voltage V from returning to the source voltage A = 2 volts.

Problem 2. (22/100 points)

A certain parallel-plate TEM transmission line is filled with μ_o and $\varepsilon = 9\varepsilon_o$.

a) A 1-GHz signal on this TEM line has $\boxed{\lambda = 10 \text{ cm}}$ since $\lambda = v_p/f = 1/10^9\sqrt{\mu_o 9\varepsilon_o} = 0.1[\text{m}]$.

b) $|\underline{V}(z)| = V_o e^{-\alpha z}$. $V_o e^{-j\underline{k}z} = V_o e^{-j\omega\sqrt{\mu_o 9\varepsilon_o(1+0.01j)}z} = V_o e^{-j\frac{3\omega}{c}\sqrt{1+0.01j}\,z}$, $\sqrt{1+0.01j} = \pm(1+0.005j)$, but the "-" solution propagates in the wrong direction. Therefore we have $e^{-j\frac{3\omega}{c}\sqrt{1+0.01j}\,z} = e^{-j\frac{6\pi 10^9}{3\times 10^8}\sqrt{1+0.01j}\,z} = e^{-20j\pi(1+0.005j)\,z} = e^{-20j\pi(1+0.005j)z} \Rightarrow e^{0.1\pi z} = e^{-\alpha z}$

$\Rightarrow \boxed{\alpha = -\pi/10 = -0.314[\text{m}^{-1}]}$ Therefore this transmission line amplifies.

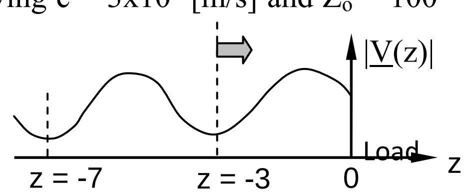
Problem 3. (22/100 points)

a) $\overline{\underline{H}} = \hat{x}\, H_o e^{-jz - 0.6y}$ is a $\boxed{\text{TM wave}}$ because H is ⊥ z axis, the axis of propagation.

b) $(\omega\,[\text{r/s}]/c)^2 = k_o^2 = k_z^2 + k_y^2 = 1^2 + (0.6j)^2 = 0.64$. So $\boxed{\omega = 0.8c = 2.4 \times 10^8 \,[\text{r/s}].}$

Problem 4. (28/100 points)

(a) Numerical value of the inductance L [Hy/m] on a line having c = 3×10^8 [m/s] and $Z_o = 100$ is $L = \sqrt{\frac{L}{C}}\sqrt{LC} = 100/(3\times10^8) = \boxed{3.33 \times 10^{-7} \,[\text{Hy/m}]}$



(b) $|\underline{\Gamma}| = (\text{VSWR} - 1)/(\text{VSWR} + 1) = 2/4$, so $\boxed{F = |\underline{\Gamma}|^2 = 1/4.}$

(c) $\underline{Z}$(z = -3 meters) = real and minimum since $|\underline{V}|$ = minimum. $\underline{Z}_n$ (z = -3) = $(1 - |\underline{\Gamma}|)/(1 + |\underline{\Gamma}|)$ = 1/3, so Z (z = -3) = 100/3 = $\boxed{33.3\ \Omega.}$