Basic Equations of Electrodynamics (2 pages)

Mathematical Identities

$v(t) = R_e\{\underline{V}e^{j\omega t}\}$ where $\underline{V} = |\underline{V}|e^{j\phi}$

$\nabla = \hat{x}\,\partial/\partial x + \hat{y}\,\partial/\partial y + \hat{z}\,\partial/\partial z$

$\overline{A}\cdot\overline{B} = A_xB_x + A_yB_y + A_zB_z$

$\nabla^2\phi = (\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)\phi$

$\sin^2\theta + \cos^2\theta = 1$

$\nabla\cdot(\nabla\times\overline{A}) = 0$

$\nabla\times(\nabla\times\overline{A}) = \nabla(\nabla\cdot\overline{A}) - \nabla^2\overline{A}$

$\int_V(\nabla\cdot\overline{G})dv = \oint_S\overline{G}\cdot\hat{n}da$

$\int_S(\nabla\times\overline{G})\cdot\hat{n}da = \oint_C\overline{G}\cdot d\hat{s}$

$e^{j\omega t} = \cos\omega t + j\sin\omega t$

$\cos\alpha + \cos\beta = 2\cos[(\alpha+\beta)/2]\cos[(\alpha-\beta)/2]$

$\underline{H}(\omega) = \int_{-\infty}^{+\infty} h(t)e^{-j\omega t}dt$

$e^x = 1 + x + x^2/2! + x^3/3! + \ldots$

$\sin\alpha = (e^{j\alpha} - e^{-j\alpha})/2j$

$\cos\alpha = (e^{j\alpha} + e^{-j\alpha})/2$

Electromagnetic Variables

$\overline{E}$ = electric field (V/m)

$\overline{H}$ = magnetic field (A/m)

$\overline{D}$ = electric displacement (C/m^2)

$\overline{B}$ = magnetic flux density (T)

Tesla (T) = Weber/m^2 = 10^4 gauss

ρ = charge density (C/m^3)

$\overline{J}$ = current density (A/m^2)

σ = conductivity (Siemens/m)

$\overline{J}_s$ = surface current density (A/m)

ρ_s = surface charge density (C/m^2)

Boundary Conditions

$\hat{n}\times(\overline{E}_1 - \overline{E}_2) = 0$

$\hat{n}\times(\overline{H}_1 - \overline{H}_2) = \overline{J}_s$

$\hat{n}\cdot(\overline{B}_1 - \overline{B}_2) = 0$

$\hat{n}\cdot(\overline{D}_1 - \overline{D}_2) = \rho_s$

$\underline{\overline{E}} = \underline{\overline{H}} = 0$ if $\sigma = \infty$

Maxwell's Equations, Force

$\nabla\times\overline{E} = -\partial\overline{B}/\partial t$

$\oint_C\overline{E}\cdot d\hat{s} = -\frac{d}{dt}\int_S\overline{B}\cdot\hat{n}da$

$\nabla\times\overline{H} = \overline{J} + \partial\overline{D}/\partial t$

$\oint_C\overline{H}\cdot d\bar{s} = \int_S\overline{J}\cdot\hat{n}da + \frac{d}{dt}\int_S\overline{D}\cdot\hat{n}da$

$\nabla\cdot\overline{D} = \rho \rightarrow \oint_s\overline{D}\cdot\hat{n}da = \int_V\rho dv$

$\nabla\cdot\overline{B} = 0 \rightarrow \oint_S\overline{B}\cdot\hat{n}da = 0$

$\nabla\cdot\overline{J} = -\partial\rho/\partial t$

$\overline{f} = q(\overline{E} + \overline{v}\times\mu_o\overline{H})$ [N]

Waves

$(\nabla^2 - \mu\varepsilon\,\partial^2/\partial t^2)\overline{E} = 0$

$(\nabla^2 + k^2)\,\underline{\overline{E}} = 0$, $\underline{\overline{E}} = \underline{\overline{E}}_o e^{-j\bar{k}\cdot\bar{r}}$

$k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda} = \sqrt{k_x^2 + k_y^2 + k_z^2}$

$v_p = \omega/k$, $v_g = \partial\omega/\partial k$

$E_x(z,t) = E_+(z-ct) + E_-(z+ct)$ [or $(\omega t-kz)$ or $(t-z/c)$]

$H_y(z,t) = (1/\eta_o)[E_+(z-ct) - E_-(z+ct)]$

$E_x(z,t) = R_e\{\underline{E}_x(z)e^{j\omega t}\}$

$<\overline{E}\times\overline{H}> = \frac{1}{2}\mathrm{Re}\{\underline{\overline{E}}\times\underline{\overline{H}}^*\}$

$\oint_s(\overline{E}\times\overline{H})\cdot\hat{n}da = -\frac{d}{dt}\int_v(\frac{1}{2}\varepsilon|\overline{E}|^2 + \frac{1}{2}\mu|\overline{H}|^2)dv - \int_v\overline{E}\cdot\overline{J}dv$

Constants

$\varepsilon_o = 8.85\times10^{-12}$ F/m

$\mu_o = 4\pi\times10^{-7}$ H/m

$c = 1/\sqrt{\varepsilon_o\mu_o} \cong 3\times10^8$ m/s

$h = 6.624\times10^{-34}$ Js

$e = 1.60\times10^{-19}$ [C]

$k_B = 1.38\times10^{-23}$ J/K

$\eta_o \cong 377$ ohms $= \sqrt{\mu_o/\varepsilon_o}$

$m_e = 9.1066\times10^{-31}$ kg

Media

$\overline{D} = \varepsilon\overline{E} = \varepsilon_o\overline{E} + \overline{P}$

$\nabla\cdot\overline{D} = \rho_f$

$\nabla\cdot\varepsilon_o\overline{E} = \rho_f + \rho_p$

$\underline{\overline{D}} = \underline{\overline{\overline{\varepsilon}}}\,\underline{\overline{E}}$, $\overline{J} = \sigma\overline{E}$

$\overline{B} = \mu\overline{H} = \mu_o(\overline{H} + \overline{M})$

Quasistatics

$\overline{E} = -\nabla\Phi$

$\nabla^2\Phi = -\rho/\varepsilon_o$

$\underline{\Phi}(\overline{r}) = \int_{V'} \{\underline{\rho}(\overline{r}')/4\pi\varepsilon|\overline{r}'-\overline{r}|\}dv'$

$\mu_o\overline{H} = \nabla\times\overline{A}$

$\nabla^2\overline{A} = -\mu_o\overline{J}$

$\underline{\overline{A}}(r) = \int_{V'} \{\mu_o\underline{\overline{J}}(\overline{r}')/4\pi|\overline{r}'-\overline{r}|\}dv'$

Circuit Elements

$C = \frac{Q}{V}$

$L = \frac{\Lambda}{I}$

$i(t) = C\frac{dv(t)}{dt}$

$v(t) = L\frac{di(t)}{dt}$

$\Lambda = \int_A \overline{B}\cdot d\hat{a}$ (per turn) $\cdot N$

$w_e(t) = \frac{1}{2}Cv^2(t)$

$w_m(t) = \frac{1}{2}Li^2(t)$

$\tau = RC, \quad \tau = \frac{L}{R}$

Electromagnetic Forces

$\overline{f} = q(\overline{E} + \overline{v}\times\mu_o\overline{H})$ [N]

$\overline{F} = \overline{I}\times\mu_o\overline{H}$ [N/m]

$\overline{E}_e = -v\times\mu_o\overline{H}$ (inside conductor)

$vi = \frac{dw}{dt} + f\frac{dz}{dt}$

$f_x = -\frac{dw_e}{dx}\Big|_{Q=\text{const.}}$

$f_x = -\frac{dw_m}{dx}\Big|_{\Lambda=\text{const.}}$

$\overline{T} = \overline{r}\times\overline{f}$

$T_\theta = -\frac{dw}{d\theta}\Big|_{Q \text{ or } \Lambda=\text{const}}$

$P_m\ [N/m^2] = \frac{1}{2}\mu_o H^2$

$P_e\ [N/m^2] = \frac{1}{2}\varepsilon_o E^2$